The Simple Macroeconometrics of the Quantity Theory and the Welfare Cost of Inflation

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Abstract

The quantity theory of money is studied using bounds tests for a levels relationship that are robust to the univariate integration properties of the variables, and is found to be generally supported by a century of U.S. data. The classic quantity theory proposition of one-for-one associations between prices and money or income is best satisfied when M2 is the monetary aggregate. With structural change treated by indicator saturation, the welfare cost of inflation is estimated to have ranged between 0.362 and 1.326 percent of national income at interest rates experienced by the United States during the past century.

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Disclosure Statement

At some level, we all believe in the quantity theory of money: the average level of prices is determined by the amount of money in circulation relative to people's desire to hold it for facilitating transactions. Going hand-in-hand with this are related propositions, such as the common-sense notion that prices should be homogeneous of degree one in money: a doubling of the quantity of money in circulation should double prices. Yet, as is perhaps not unusual with things many of us believe about economics, it can sometimes be surprisingly difficult to find the quantity theory in the data.

In this paper I argue that the quantity theory of money (QTM) emerges quite clearly from the simplest of dynamic econometric frameworks, a single-equation autoregressive distributed lag (ADL) model and its error correction mechanism (ECM) reparameterization. This emergence occurs when the model is estimated over a suitably long sample period—annual U.S data over the past century—and for a suitable monetary aggregate, which I find to be M2. The equilibrium error of the ECM is the cointegrating relationship between money, prices, income, and (possibly) interest rates that is predicted by the QTM. Because the integration properties of interest rates may be in dispute, I argue that the bounds tests of Pesaran, Shin, and Smith (2001) are the natural tool for studying the existence of an equilibrium levels relationship.

I implement this modeling approach using three elementary money demand specifications that have long been used in the empirical literature, and for which convenient formulas for the welfare cost of inflation have been derived. For the specification that the data most strongly support—an ADL/ECM based on money demand that is loglinear in the interest rate, and permitting structural change determined by indicator saturation—I obtain welfare costs in the range of 0.362 to 1.326 percent of income, for inflation rates of the order that have been experienced over the past century. These are slightly lower than the estimates obtained by Lucas (2000), but not as low as those of Ireland (2009).

The only thing surprising about this simple analysis is that it runs counter to other recent approaches to studying related issues. Benati, Lucas, Nicolini, and Webber (2017, 2021; henceforth BLNW) use the aggregate NewM1 advocated by Lucas and Nicolini (2015) to estimate long run money demand. They interpret the relevant cointegrating relationship to be between velocity and the interest rate, which in turn presumes that these variables are I(1) in their univariate behavior. A similar view is taken by Benati (2020). In contrast, I argue that nothing in the QTM requires or implies that velocity or interest rates be integrated individually, or that they be cointegrated. Because this may elicit some controversy, it is useful to set the stage and establish notation by reminding ourselves of the essentials of the QTM.

1   Background

The quantity theory is, of course, not so much a "theory of money" as an explanation of the determination of the price level P_t and its growth rate, the rate of inflation, in terms of the quantity of money in circulation M_t relative to real transactions Y_t. The theory is often interpreted in terms of the notion of the velocity of circulation of money V_t, the ratio of nominal transactions $P_t Y_t$ to the money stock: $V_t = P_t Y_t / M_t$. But, needless to say, definitions tell us nothing about cause and effect.

1.1   Money Demand and the Quantity Theory

As a *theory* of price level determination, the quantity theory is formulated in terms of behavior: people's willingness to hold money—money demand—relative to the stock of money in circulation available to be held—money supply. Were our interest in the short-term high-frequency dynamics of the variables involved, quite complicated specifications for each of these could be entertained. Money supply M_t^{S} would be specified as interest-elastic, because commercial banks reduce excess reserves in response to higher interest rates. And, similarly, much could be said about the demand for money M_t^{D}, on which there is of course an enormous and longstanding literature. Indeed, in contemporary models of the short term dynamics of interest rates and prices, as surveyed by McCallum and Nelson (2011, sec. 8.1), the dominant factors are such that money may be viewed as not playing an important role at all.

At its most essential, however, the QTM is a theory of price level determination in the

long run, because it attributes inflation to money supply growth, and inflation is by its nature a long run phenomenon. By definition inflation is an ongoing and sustained growth in prices over an extended period of time, not merely a one-time or temporary jump in prices. So too is the welfare cost of inflation an inherently long run phenomenon, because— as the literature conventionally defines it—that welfare cost arises from people economizing on their money holdings in response to expectations of inflation. And for expectations of inflation to develop, an inflation must be sustained.

This emphasis on the long run is consistent with the evidence. Stock and Watson (1993, p. 811) found that "The empirical analysis suggests that the precise estimation of long-run M1 demand requires a long span of data: estimates over the full 90 years are considerably more precise than over the first half of the century alone, and when used in isolation the data since 1946 contain quite limited information about long-run M1 demand."

In this conception of the QTM, the longer time frame to which it applies simplifies rather than complicates its empirical implementation. With respect to money supply, the quantity of money in circulation can be taken to be determined by the policies of the monetary authority, and so is treated as exogenous: $M_t^{\mathrm{S}} = M_t$. With respect to money demand, people desire to hold money, first and foremost, to facilitate their nominal transactions $P_t Y_t$. As a secondary consideration, they are influenced in this willingness by the opportunity cost of holding wealth in the form of money, given by the nominal interest rate r_t. The relevant interest rate is a short term one, because liquid interest-bearing assets like treasury bills or commercial paper are the most direct substitute for money.

This essential money demand function may be denoted

$$M_t^{\mathrm{D}} = f(P_t Y_t, r_t).$$

The equilibration of supply and demand determines the price level P_t, which adjusts to bring people's willingness to hold money into conformity with the quantity available to be held, $M_t^{\mathrm{S}} = M_t^{\mathrm{D}}$. In this time frame, P_t is the sole endogenous variable of the equilibrium condition

$$M_t = f(P_t Y_t, r_t). \tag{1}$$

In addition to the money supply being exogenous, the inflationary expectations incorporated into the nominal interest rate r_t are historically given at any point in time, and the real component of r_t is determined by flows of real saving and investment in financial markets.

The notion of a dichotomy between the real and nominal sectors is taken to be a reasonable approximation to reality: transactions Y_t are determined by real economic activity, and so are exogenous with respect to the monetary sector.

That, from this long run perspective, the price level P_t may be viewed as the sole endogenous variable of the single equation (1) has important implications for the empirical implementation of the QTM. Vector error correction systems in which some of the I(1) variables are exogenous have been studied rigorously by Pesaran, Shin, and Smith (2000, 2001; henceforth PSS). PSS (2001) focuses on the special case in which a single endogenous variable is related to others that are exogenous, so that the equation describing their relationship can be decomposed from the rest of the system. They present bounds tests for the existence of a levels relationship among the variables, tests that are robust to the univariate integration properties of the variables individually. Let us now consider how this relates to the traditional literature on the demand for money.

1.2 Empirical Implementation

Given the central role of money demand in the quantity theory, empirical implementations of the QTM typically proceed using assumed specifications for the money demand function. A common assumption is to specify money demand as homogeneous in prices and income, in general not necessarily of the same degree:

$$M_t = k(r_t)P_t^{\theta_p}Y_t^{\theta_y}. \tag{2}$$

Here $k(r_t)$ is a behavioral specification for the reciprocal of velocity. Isolating the endogenous variable P_t, this describes prices as being determined according to

$$P_t = k(r_t)^{-1/\theta_p}M_t^{1/\theta_p}Y_t^{-\theta_y/\theta_p}.$$

Under this parameterization, the long run equilibrium relationship determining prices is loglinear in money and income,

$$p_t = \frac{1}{\theta_p}m_t - \frac{\theta_y}{\theta_p}y_t - \frac{1}{\theta_p}\log k(r_t),$$

where lower case p_t, m_t, and y_t denote the logarithms of the respective variables. In a stochastic setting this long run equilibrium relationship translates into a stationary equilibrium error

$$p_t + \lambda_m m_t + \lambda_y y_t + \lambda_k \log k(r_t), \tag{3}$$

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where $\lambda_m = -1/\theta_p$, $\lambda_y = \theta_y/\theta_p$, and $\lambda_k = 1/\theta_p$. If all four of these variables, including $\log k(r_t)$, are I(1), then the vector $\lambda = [1, \lambda_m, \lambda_y, \lambda_k]$ is the cointegrating vector. However this framework is not limited to that circumstance. It could be that just two or three of the variables are I(1), money and prices, say, or money, prices, and income, so that it is this subset that is cointegrated. In this respect the only real requirement for the equilibrium error (3) to be stationary is that if any of the variables is I(1), at least one other must be as well.

Which of these possibilities actually holds in practice will depend on the historical time period. For example, during the heyday of the gold standard between the end of the Civil War and the beginning of World War I, the quantity of money in circulation was fairly stable and so may have been best described as I(0). Yet the American economy grew dramatically, and so income may have been I(1) with positive drift. The result was that the price level declined, and so may have been I(1) with negative drift. Famously, the prices of agricultural commodities fell while farmers were locked into fixed-rate mortgages. If, being a function of interest rates, velocity was stationary, this would be a case where the cointegrating relationship within the equilibrium error (3) was between income and prices, with money and interest rates stationary.

Turning to more recent economic history, with the demise of the gold standard in the last century prices p_t have tended to increase with the money supply m_t. So too have aggregate transactions y_t increased with economic growth. All three variables are therefore plausibly I(1) with drift. The appropriate specification for nominal interest rates is less clear. They increased during the 1970s, incorporating the rising inflation of that era, and so during that period would most likely be best approximated as I(1) with drift. But in the long run to which the quantity theory applies "...interest rates are almost certainly stationary in levels. Interest rates were about 6% in ancient Babylon; they are about 6% now. The chances of a process with a random walk component displaying this behavior are infinitesimal." (Cochrane 1991, p. 208). This suggests that, in the equilibrium error (3), any cointegrating relationship is between p_t, m_t, and y_t.

To prevent any misunderstanding that all four variables in the equilibrium error (3) are necessarily assumed to be integrated, I call λ the *equilibrium vector* rather than the cointegrating vector. Because, as these examples illustrate, the integration properties of the variables individually during any particular historical time period may be in dispute,

section 2.2 advocates the PSS (2001) bounds tests as the appropriate empirical tool.

Above and beyond the long run equilibrium relationship between the levels of the variables captured by the equilibrium error (3), the quantity theory further hypothesizes that short run price movements are driven in the direction needed to establish equilibrium. That is, the equilibrium error is embedded within some broader dynamic model, which in our case will be an ADL/ECM.

In addition to following a long empirical tradition, loglinearity of money demand—and, in turn, the equilibrium error—has some theoretical appeal. Although income Y_t is conventionally measured by real GDP, and P_t is therefore the GDP deflator, this is just a proxy for a much larger volume of actual transactions that includes intermediate goods and services, not just final ones. But if GDP is proportional to actual transactions then, in a loglinear specification, the difference is irrelevant because it will be absorbed into the intercept (which, in the equilibrium error (3), enters via the specification of $k(r_t)$). Interpreted in terms of the money demand function (2), the income elasticity θ_y is invariant to multiplicative data scalings. Interpreted in terms of the equilibrium error (3), the implication is that this loglinear specification for the long run equilibrium relationship is not implausible.

Traditionally, the empirical study of these relationships has been cast in terms of estimation of the money demand function. Often the classical proposition of a one-for-one correspondence between money and prices, $\theta_p = 1$ (or, in terms of the equilibrium vector, $\lambda_m = -1/\theta_p = -1$), is adopted as a maintained hypothesis. This is the basis for what Ball (2001, equ. (1)) and McCallum and Nelson (2011, equ. (3)) call the "canonical" or "standard" money demand specification

$$m_t - p_t = \theta_0 + \theta_y y_t + \theta_r r_t. \tag{4}$$

This uses a semilogarithmic behavioral specification for the reciprocal of velocity: $\log k(r_t) = \theta_0 + \theta_r r_t$. (A time trend is also sometimes included, typically motivated as capturing technological or regulatory change in the banking sector.)

As one example of work along these lines, Ball (2001) found that, for U.S. post-World War II annual data through 1996, the income elasticity θ_y is about 0.5 while the interest semi-elasticity θ_r is about -0.05 (for interest rates expressed as percentages). Estimates like these are often compared against the theoretical benchmark of the classic Baumol-Tobin inventory-theoretic square root rule for money holding, which predicts an interest elasticity

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of money demand of -0.5. But, given aggregation issues, there is no particular reason to believe that micro-theoretic results will translate to the aggregate. At historical values for the interest rate, Ball's interest semi-elasticity estimate of -0.05 is not particularly consistent with the square root rule. With respect to the income elasticity, "Depending on the way one interprets the Baumol theory, one may take it as also predicting that the *income* elasticity of money demand is one-half. If this is right, the theory fails badly on U.S. time series evidence." (Lucas 2000, footnote 15.) That is, Ball's income elasticity $\theta_y = 0.5$—which is consistent with the square-root rule—is actually atypical of empirical findings. It is also contrary to plausible assumptions about the long run relationship between prices and income, as we now consider.

1.3 The Homogeneity Restrictions

A special case often considered in textbooks, and adopted by BLNW (2021) as a maintained hypothesis, is a demand for money that is proportional to nominal transactions PY:

$$M_t^{\mathrm{D}} = k(r_t)P_tY_t. \tag{5}$$

This is the special case of homogeneous money demand (2) that imposes a unitary degree of homogeneity, not merely on prices alone as in the canonical money demand function (4), but on both prices and income: $\theta_p = \theta_y = 1$. Equating this money demand specification to exogenous money supply M_t, the equilibrium of the quantity theory is described by

$$\frac{M_t}{P_tY_t} = k(r_t). \tag{6}$$

Given that P_t is the sole endogenous variable of the QTM, in logarithms it is determined in relation to the other variables as

$$p_t = m_t - y_t - \log k(r_t).$$

In terms of the equilibrium error (3), these are the restrictions $\lambda_m = -1$ and $\lambda_y = 1$. I will call these the *homogeneity restrictions.*

The homogeneity restrictions are unlikely to hold in the short run, as is reflected in the specification of the canonical money demand function (4), which does not impose $\theta_y = 1$. But they may hold in the long run, because they correspond to the common-sense propositions that, ceteris paribus, (a) a doubling of the money stock doubles the price level

($\lambda_m = -1$), and (b) a doubling of transactions halves the price level ($\lambda_y = 1$). For example, for U.S. M1 1900–1989, Stock and Watson (1993, p. 811) found that "Overall, the evidence is consistent with there being a single stable long-run demand for money, with an income elasticity near one ..."

Although well-motivated by this common sense, whether the homogeneity restrictions actually hold in practice will depend on the specifics of the empirical implementation, such as the data span and the choice of monetary aggregate. If the homogeneity restrictions are to be employed, they should be subject to test. Let us now consider why it might be of special interest to work with a model where the homogeneity restrictions are compatible with the data.

1.4 The Welfare Cost of Inflation

As was noted in section 1.1, inflation and its welfare cost are inherently long run phenomena. And, as we have just argued, the homogeneity restrictions are sensible—perhaps even compelling—as long run restrictions on the QTM. Combining these ideas, Lucas (2000) showed that closed-form expressions for the welfare cost of inflation can be derived when money demand is of the homogeneity-restricted form (5) and so monetary equilibrium is described by (6). He derived welfare cost formulas for two specifications of $k(r_t)$ that are commonly used in empirical work, a loglinear functional form

$$\log k(r_t) = \alpha - \eta \log r_t = \log(A) - \eta \log r_t \tag{7}$$

and a semi-log one,

$$\log k(r_t) = \beta - \xi r_t = \log(B) - \xi r_t. \tag{8}$$

In the case of the latter, substituting it into the homogeneous money demand function (2) with $\theta_p = 1$ yields the canonical specification (4). Although this canonical semi-log function has most often been used in empirical money demand studies, for annual data over the past century BLNW (2021) favor the loglinear function. The exception is periods of very low interest rates, where BLNW find that a third specification, the Selden-Latané functional form, best fits the data.[1] This specifies velocity as linear in the interest rate, $V_t = C + \delta r_t$, so that

$$\log k(r_t) = -\log(C + \delta r_t). \tag{9}$$

The corresponding welfare cost formula is given in BLNW (2017, equ. (10)). The notation for these three specifications for $\log k(r_t)$ follows Lucas (2000) and BLNW (2017), respectively; the welfare costs require estimates of the intercept parameters A, B, and C, and the slope coefficients η, ξ, and δ. With such estimates these welfare cost formulas can be evaluated at various levels of the nominal interest rate and, by implication, the expected inflation that those nominal rates reflect. Clearly, however, these calculations are only legitimate if the homogeneity restrictions are satisfied so that monetary equilibrium is described by (6).

Lucas and Nicolini (2015) formulate a model that leads to homogeneity-restricted money demand (5) as an appropriate specification. As a monetary aggregate that has a stable relationship with the short term interest rate of the form (6), they propose M1 plus money market deposit account (MMDA) balances—essentially the total of currency and coin in circulation plus all checkable deposits, which they call NewM1. Although conventional demand deposits and MMDA balances are not perfect substitutes to those who hold them, the model predicts that a suitable aggregate can be obtained as a simple-sum aggregation. Lucas and Nicolini show that, in annual time series, NewM1 yields a clean static bivariate relationship between velocity and the short term interest rate over the period 1915–2012 that is well-approximated by the equilibrium condition (6). In turn, BLNW (2017, 2021) employ NewM1 to estimate this equilibrium condition using the loglinear, semilogarithmic, and Selden-Latané functional forms, and calculate (BLNW 2017, p. 26) "...welfare losses at the average short rate that has prevailed over the sample period (expressed as percentage points of GDP). ...For the United States the three specifications tend to produce similar results, with median estimates equal to 0.21 per cent for the Selden-Latané specification, and 0.18 for the other two." That is, at the moderate rates of inflation that have prevailed over the past century, the welfare cost has averaged around one-fifth of one percent of GDP.

Given that Lucas and Nicolini (2015) derived NewM1 to satisfy a stable relationship of the form (6), BLNW (2017,2021) adopt the homogeneity restrictions as a maintained hypothesis. They interpret the equilibrium condition (6) as implying that the reciprocal of velocity, M_t/P_tY_t, and the interest rate r_t are cointegrated, which in turn requires that each be univariate I(1). BLNW devote much of their empirical analysis to investigating these hypotheses. Yet, although this pattern of integration and cointegration is conceivable, it is not the only possibility nor, in the long run, even the most likely. Recall, from the examples given to illustrate the interpretation of the equilibrium error (3), Cochrane's (1991, p. 208)

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compelling argument that in the long run "...interest rates are almost certainly stationary in levels." It would follow that, for data from the past century, the cointegration in (3) is between money, prices, and income. This would render M_t/P_tY_t (or, in terms of the equilibrium error (3), its logarithm) stationary or trend stationary. In this conception, both sides of the equilibrium condition (6) would be stationary or trend stationary, the opposite of BLNW's assumption.

1.5 Implications

These considerations point to the importance of using an empirical methodology that is robust to alternative possibilities for the integration and cointegration properties of the data, which brings us to the PSS tests of the next section. Also evident is that, if Lucas's welfare cost formulas are to be used, the homogeneity restrictions should be testable in order to ensure that the formulas are being evaluated with parameter estimates from models that are actually consistent with the data. These parameters are those of the loglinear (7), semilog (8), and Selden-Latané (9) money demand specifications, whose interest rate coefficients η, ξ, and δ have economic interpretations that can be compared with the literature; for example, the benchmark Baumol-Tobin interest elasticity of -0.5.

Beyond this, the scope for testing and economic interpretation in this framework is limited because the model consists of just a single equation, and the price level is its sole endogenous variable. As has been emphasized, the other variables—money, income, and interest rates—are treated as exogenous. Because they are not endogenous to the QTM, the QTM by itself offers no testable hypotheses about their determination. Specifically, hypotheses that are often thought to go hand-in-hand with the QTM—such as the neutrality or superneutrality of money, or the natural rate hypothesis, however defined—cannot be addressed. To do so requires a more sophisticated multi-equation model with endogenous real variables, as in, for example, Fisher and Seater (1993).

The compensating payoff to this limitation is that a single equation representation for the QTM makes the methodology of PSS available. The use of a single-equation approach harkens back to the now-classic work of David Hendry and his collaborators, notably Hendry and Ericsson (1991) and Baba, Hendry, and Starr (1992) for U.S. data. They emphasized the importance of a systematic approach to the treatment of dynamics, with which they were able to find that (Baba, Hendry, and Starr 1992, p. 56) "...contrary to the prevailing view,

there is good evidence for the existence of a stable, cointegrating money demand function, based on theory, with an error-correction specification." For example, in considering the famous 1970s "case of the missing money" Hendry and Ericsson (1991, p. 873) concluded that in part "...the 'missing money' appears to be due to misspecified dynamics ..."

Like BLNW, Hendry's work was phrased in terms of estimating a money demand function, so that the quantity of money demanded was viewed as the endogenous variable of the model. Although, when money supply is treated as exogenous, the difference may be more a matter of taste than substance, casting these issues explicitly in terms of the quantity theory, where the price level rather than the quantity of money demanded is viewed as the endogenous variable, helps direct attention to the appropriate specifications for the univariate properties of the variables and the legitimacy of the homogeneity restrictions.

2   Modeling Framework

The equilibrium error (3) relates the variables p_t, m_t, y_t, and—via a specification for $\log k(\cdot)$—the interest rate r_t. This equilibrium error must be embedded within a dynamic structure that describes the short run variation in these variables, variation that is driven by the tendency to move toward satisfying long run equilibrium. In particular, this dynamic structure treats the lagged response of prices to changes in money—referred to by McCallum and Nelson (2011) as the "phase shift" between these variables.

Applied econometrics provides essentially two such dynamic structures: a single-equation autoregressive distributed lag/error correction model (ADL/ECM), and vector autoregression/error correction model (VAR/ECM). In a model with several endogenous variables, a VAR system would be the natural choice. But the associated Johansen cointegration testing methodology assumes that all the variables of the system are I(1). In fact, this is the methodology used by BLNW in studying possible cointegration between velocity and the interest rate. But, as I have argued, in testing the quantity theory one wants a methodology that is robust to the integration properties of these variables, not contingent on particular assumptions about them.

Beyond its simplicity, the appeal of the single-equation framework is that PSS tests for a levels relationship become available. This testing methodology is presented most generally in PSS (2001). Rather than attempting to reiterate the general framework, it is more useful to indicate how it applies in the present context.

2.1 The ADL/ECM(2,2,0,1) Model

Given the variables to be studied, in this case $[p_t, m_t, y_t, r_t]$, an ADL is specified by appropriate lag lengths. These lag lengths are, of course, specific to the application and are determined according to the usual testing and model selection criteria. In the empirical work reported in section 4, the chosen lag lengths depend on the monetary aggregate, the specification for $\log k(\cdot)$ and, in one case (a loglinear model for M1), even whether the equilibrium error includes a time trend.

Nevertheless, the essentials of the PSS methodology can be illustrated with the example of an ADL/ECM(2,2,0,1), where the arguments are the lag lengths of the respective variables $[p_t, m_t, y_t, r_t]$. This turns out not to correspond to any of the models I actually end up reporting: some are special cases with shorter lag lengths, others generalizations with longer lag lengths. But the ADL/ECM(2,2,0,1) most economically illustrates the issues that arise.

Because the price level p_t is the sole endogenous variable of the quantity theory, it is the dependent variable of the ADL(2,2,0,1):

$$p_t = \phi_0 + \phi_1 p_{t-1} + \phi_2 p_{t-2} + \omega_0 m_t + \omega_1 m_{t-1} + \omega_2 m_{t-2} + \gamma_0 y_t + \delta_0 r_t + \delta_1 r_{t-1} + \kappa t + \varepsilon_t \quad (10)$$

This states the model in its greatest generality by including a time trend, although I find a time trend to be insignificant in my results. This example also specifies varying lag lengths on the exogenous variables m_t, y_t, and r_t. In particular, exogenous variables with a lag length of zero—in this case y_t, which appears only contemporaneously—turn out to require special handling, as we are about to see.

It is well known that any ADL can be reparameterized in a number of forms, useful for different purposes. The best known of these is the error correction form, which in this case can be written as

$$\Delta p_t = \psi[p_{t-1} + \lambda_m m_{t-1} + \lambda_y y_{t-1} + \beta - \xi r_{t-1} + \mu(t-1)]$$
$$- \phi_2 \Delta p_{t-1} + \omega_0 \Delta m_t - \omega_2 \Delta m_{t-1} + \psi \lambda_y \Delta y_t + \delta_0 \Delta r_t + \varepsilon_t. \quad (11)$$

This reparameterization is the following set of correspondences.

$$\psi = \phi_1 + \phi_2 - 1 \qquad \text{(speed-of-adjustment coeffient)}$$

$$\lambda_m = -\frac{\omega_0 + \omega_1 + \omega_2}{1 - \phi_1 - \phi_2} \qquad \text{(elasticity of prices with respect to money)}$$

$$\lambda_y = -\frac{\gamma_0}{1 - \phi_1 - \phi_2} \qquad \text{(elasticity of prices with respect to income)}$$

$$\beta = -\frac{\phi_0 - \kappa}{1 - \phi_1 - \phi_2} \qquad \text{(intercept)}$$

$$-\xi = -\frac{\delta_0 + \delta_1}{1 - \phi_1 - \phi_2} \qquad \text{(interest semielasticity of money demand)}$$

$$\mu = -\frac{\kappa}{1 - \phi_1 - \phi_2} \qquad \text{(time trend coefficient)}$$

As is standard in this single-equation context, these parameter correspondences are exactly identifying and so are not testable restrictions. For this to be so—that is, for the ECM to place the same structure on the data as the original ADL—exogenous variables that appear in the ADL only contemporaneously, in this case y_t, must appear with a restricted coefficient on their first difference, in this case $\psi\lambda_y$ on Δy_t.

The ECM (11) makes explicit that underlying the ADL (10) is the notion that period-by-period changes in the endogenous variable, Δp_t, are driven by two forces. First, the short term dynamics captured by the first differences on the right hand side of the ECM. And second, the need for p_t to move in a direction driven by its long run equilibrium relationship with the other variables, as described by the quantity theory. This long run equilibrium relationship is the equilibrium error

$$p_{t-1} + \lambda_m m_{t-1} + \lambda_y y_{t-1} + \beta - \xi r_{t-1} + \mu(t-1), \qquad (12)$$

which is the equilibrium error (3) in which $\lambda_k \log k(r_t) = \beta - \xi r_t + \mu t$. That is, this equilibrium error uses the semi-log money demand specification (8) generalized to include a time trend, the latter possibly to capture technical progress in the banking sector. (A time trend in the equilibrium error can also arise from non-zero drifts in the I(1) variables. The role of the time trend term is examined in detail in the appendix.) Within this ECM, the usual questions about money demand can be studied. For example, the canonical money demand specification (4) imposes the homogeneity restriction $\lambda_m = -1$.

In addition to the economic interpretations of the parameters in the equilibrium vector, some of the other parameters introduced by the ADL have important interpretations with respect to dynamics. To ensure convergence of the endogenous variable p_t to a long run

equilibrium, the autoregressive parameters—in this case ϕ_1 and ϕ_2—must satisfy the standard stability conditions. For a second order difference equation these are $\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$, and $|\phi_2| < 1$. In turn, the first of these implies that the speed-of-adjustment parameter $\psi = \phi_1 + \phi_2 - 1$ must be negative: $\psi < 0$. The logic is that, if $\log p_{t-1}$ is above its long run equilibrium value $-[\lambda_m m_{t-1} + \lambda_y y_{t-1} + \beta - \xi r_{t-1} + \mu(t-1)]$ as implied by the quantity theory, so that the equilibrium error (12) is positive, then Δp_t should be negative in order to move p_t toward satisfying long run equilibrium.

This exposition has used semi-log money demand to illustrate the essentials of ADL/ECM models. The alternative of loglinear money demand (7) is straightforward. If the interest rate r_t is to enter logarithmically, then $\log r_t$ simply replaces r_t in the initial specification of the ADL (10). The associated ECM is the same as (11) with $\log r_t$ replacing r_t and, using the notation of loglinear money demand (7), the equilibrium error is notated as

$$p_{t-1} + \lambda_m m_{t-1} + \lambda_y y_{t-1} + \alpha - \eta \log r_{t-1} + \mu(t-1). \tag{13}$$

Whereas the semi-log and loglinear money demand functions are consistent with an equilibrium error that is a linear combination of the variables $[p_t, m_t, y_t, r_t]$ or $[p_t, m_t, y_t, \log r_t]$, respectively, the Selden-Latané function (9) is not. It sets $\lambda_k \log k(r_t) = -\log(C + \delta r_t + \mu t)$ in the equilibrium error (3), which therefore becomes

$$p_t + \lambda_m m_t + \lambda_y y_t - \log(C + \delta r_t + \mu t).$$

Because this is nonlinear in the parameters C and δ, an ECM with this as the equilibrium error cannot be derived from an associated ADL. Even so, this equilibrium error can be embedded within an otherwise linear ECM like (11) and estimated nonlinearly. Such an ECM is, in this respect, ad hoc; nevertheless parameter estimates and welfare costs can be usefully compared with those from the loglinear and semi-log ADL/ECMs. Given that this is a fairly minor variation on the fully linear framework, it seems reasonable to hope that the PSS simulated testing bounds are still approximately correct; nevertheless it must be borne in mind that they no longer strictly hold.

2.2 Testing for an Equilibrium Levels Relationship

Consider the ECM (11) and the question of whether the evolution of the variables is guided by an equilibrium relationship in their levels, as opposed to being influenced by nothing

more than short run interactions among their first differences. The ECM suggests two ways of testing this. One is to test the restriction $\psi = 0$, which implies that short run changes are not driven by long run convergence to equilibrium. The associated pseudo-t statistic is the basis for the PSS t test. The other is to consider another reparameterization of the model, also exactly identifying, which in this example is

$$\Delta p_t = \psi p_{t-1} + \omega m_{t-1} + \gamma y_{t-1} + \kappa_0 + \delta r_{t-1} + \kappa(t-1)$$
$$- \phi_2 \Delta p_{t-1} + \omega_0 \Delta m_t - \omega_2 \Delta m_{t-1} + \gamma \Delta y_t + \delta_0 \Delta r_t + \varepsilon_t, \quad (14)$$

where $\omega = \omega_0 + \omega_1 + \omega_2$, $\gamma = \gamma_0$, and $\delta = \delta_0 + \delta_1$. (Notice once again that, for the relationships to be exactly identifying, a variable appearing only in contemporaneous form in the original ADL (10) appears here with a coefficient restriction: y_{t-1} and Δy_t share the common coefficient γ.) The restrictions $\psi = \omega = \gamma = \delta = \kappa = 0$ are the hypothesis that the variables in their levels do not influence price changes, which are therefore related to the other variables only via their short term changes. This is the PSS F test; the statistic is a transformation of a Wald statistic. (Incidentally, notice that the intercept κ_0 is not among these restrictions. It is a reparameterization of the intercept ϕ_0 in the ADL or, in the ECM's, a reparameterization of the intercept in the equilibrium error. Even in the absence of a long run equilibrium relationship, the short run relationship between the first-differences could involve an intercept, and it would be inappropriate to include $\kappa_0 = 0$ in the null hypothesis.)

Although these t and F statistics are standard, their distributions are not. Consider the special case of $\kappa = 0$, so that there is no time trend in the model, the implication being, for example, that any nonzero drifts in the I(1) variables cancel and/or there is no trend in velocity. This is PSS Case III, which will prove to be of special relevance because it turns out to be supported in my results. PSS (2001) tabulate critical value bounds on each of the t and F statistics for the polar extremes of all I(0) variables versus all I(1) variables; the null hypothesis is of an absence of a levels relationship. They recommend that the two statistics be interpreted sequentially: "...we suggest the following procedure for ascertaining the existence of a level relationship ...: test H_0 ... using the bounds procedure based on the Wald or F-statistic ...: (a) if H_0 is not rejected, proceed no further; (b) if H_0 is rejected, test ... using the bounds procedure based on the t-statistic ..." (PSS 2001, p. 304). Intuitively, begin by testing whether the variables belong in the model in their levels;

if they do, then test whether a long run equilibrium relationship among those levels drives the short run fluctuations.

Although PSS Case III turns out to be of principal empirical relevance in my analysis, consider the more general possibility that, if there is a long run equilibrium relationship between the variables, it includes a time trend, perhaps because nonzero drifts of I(1) variables do not cancel, or velocity trends. In this case a time trend belongs in the equilibrium error: $\kappa \neq 0$ in (14) or, equivalently, $\mu \neq 0$ in (12) or (13). Now the hypothesis that variables in their levels should not enter the PSS regression (14) adds the further restriction $\kappa = 0$ to the Case III restrictions $\psi = \omega = \gamma = \delta = 0$. This is PSS Case IV. Whereas, like Case III, the F-statistic can be bounded under this null hypothesis (PSS 2001, Corollaries 3.1 and 3.2), this turns out not to be true of the t-statistic (PSS 2001, Corollaries 3.3 and 3.4). Hence only the bounds F-test is available in this instance.[2]

3 Data and Preliminary Evidence

The natural data sources for this study are those of Lucas and Nicolini (2015) and BLNW. Officer and Williamson (www.measuringworth.com) have constructed consistent historical GDP data (nominal, real, and the associated deflator) that they continue to update. FRED has consistent series for M1 and M2 back to 1915 that continue to be updated.[3] MMDA is obtained as described in Lucas and Nicolini, and NewM1 constructed as the simple sum of M1 and MMDA. The three month treasury bill rate, obtained from the Lucas-Nicolini documentation files and updated from the *Economic Report of the President,* is used for the short-term interest rate.[4]

3.1 Descriptive Behavior

Figure 1a shows the evolution over time of the logarithms of real GDP $y_t = \log Y_t$, the deflator $p_t = \log P_t$, and the three monetary aggregates. (M1 and NewM1 are coincident until 1982, when non-zero MMDA balances begin.) These have grown over the past century in the way familiar to all macroeconomists, and so are plainly nonstationary.

The growth in these aggregates is not unrelated, as is revealed by an inspection of the corresponding velocities. Consider V1, NewV1, and V2 (constructed as $V = PY/M$ with M set alternatively to M1, NewM1, and M2) shown in Figure 1b along with the interest rate. The stability of velocity is directly related to how broadly its monetary aggregate is

defined: M2 velocity is the most stable, M1 velocity the least. The most obvious regularity in V1 and NewV1 is the extent to which their historical variation is related to interest rates. Indeed, the historical peak of NewV1 corresponds virtually exactly with the peak in the T-bill rate around 1980. Just as the reasoning behind the interest elasticity of money demand suggests, money circulates more rapidly in the economy the higher the opportunity cost of holding it. This is especially evident in NewM1, the monetary aggregate advocated by Lucas and Nicolini (2015).

Another obvious regularity in these velocities is that they share an important feature. All three have returned to values that today are roughly what they were a century ago, suggesting mean reversion. Perhaps surprisingly, technological advance in the banking and financial sector has not systematically increased velocity (although one could easily have been misled into believing the contrary had one focused solely on M1 prior to around 2005). It follows that, especially for NewM1 and M2, these velocities are unlikely to be integrated processes, which have infinite variance and must—even with zero drift—ultimately wander arbitrarily far from their starting values.

However if, in studying the limited sample period of the past century, integration is to be considered as a candidate behavior, the absence of any apparent long run trend suggests a zero-drift specification. The implication is that, for purposes of testing, a researcher would be inclined to believe a "constant, no trend" unit root test before a "constant plus trend" version.

The other variable exhibiting mean reversion is the interest rate, which, from its high around 1980, has returned to levels of the 1930s. This is, of course, consistent with Cochranes's (1991) view, cited in section 1.2, that in the long run "...interest rates are almost certainly stationary ...".

These remarks suggest that the natural default interpretation of the descriptive evidence in Figure 1 is that, in the equilibrium error (3), the cointegrating relationship is between prices p_t, money m_t, and transactions y_t. The reciprocal-of-velocity term $\log k(r_t)$ should probably be treated as stationary, but nevertheless appears in the equilibrium error because the interest rate plays a role in the equilibration of the price level. That role is testable via λ_k or, equivalently, the coefficients that appear in any parameterization of the $\lambda_k \log k(r_t)$ term, such as η, ξ, or δ in the loglinear, semilog, or Selden-Latané specifications (7), (8), and (9). These parameters should be statistically significant: empirically, money demand is

well known to be interest elastic. At the descriptive level of Figure 1b, this elasticity is most clearly revealed in the relationship that we have noted between the interest rate and the velocity of NewM1. Hence, even if stationary, the $\log k(r_t)$ term belongs in the equilibrium error.

3.2 Univariate Tests

But is this default interpretation supported by more rigorous statistical analysis? To treat p_t, m_t, and y_t as cointegrated they must be unit root processes. Table 1 contrasts, for the key time series, the best known of two types of unit root tests: augmented Dickey-Fuller (ADF) tests, for which the null hypothesis is the presence of a unit root (i.e. difference stationarity), and Kwiatkowsky-Phillips-Schmidt-Shin (KPSS) tests, for which the null is the absence of a unit root (i.e. trend stationarity). Of course, there are many other unit root tests that could be applied, but the key points can be illustrated with these most common of tests.

The upper portion of Table 1 includes results for the variables shown in Figure 1a: p_t, y_t, and the logs of M1, NewM1, and M2. Because all trend upward, a "constant plus trend" test specification is the natural choice, although the test decisions are not sensitive to it. These decisions are that the ADF tests do not reject the null of a unit root, while the KPSS tests reject the null of trend stationary. Hence, as most empirical macroeconomists would expect, the evidence is compelling that p_t, y_t, and m_t should be modeled as drifting I(1), and this behavior of m_t does not depend on the aggregate used to measure it.

How about the interest rate r_t? Here the substantive conclusions are insensitive to the choice of a "constant" or "constant plus trend" specification and, reassuringly, whether the interest rate is untransformed or studied in its logarithm. For researchers who accept that, on the basis of the descriptive evidence of Figure 1b and Cochrane's reasoning, the natural null is of stationarity, the KPSS tests do not reject that null at conventional significance levels (although they come close). But researchers who instead believe that the null should be of difference stationarity will not find evidence in the ADF tests to reject that view. Neither school of thought will be persuaded that its a priori belief is false. As much as anything, the rate of interest illustrates the notorious weak power of unit root tests.

This is not quite so true of velocity, where the results vary somewhat with the monetary aggregate in ways that make sense in relation to the patterns in Figure 1b. The strongest

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evidence for integrated behavior is M1 velocity, where the ADF tests fail to reject while the KPSS tests do. Velocity based on NewM1 similarly yields ADF tests that do not reject, but a KPSS test that does not under what I have argued to be the natural default of a "constant, no trend" specification. The strongest evidence for non-integrated velocity comes from M2 where, for "constant, no trend" specifications, an ADF test rejects unit root behavior at 10% significance while a KPSS test does not reject stationarity.

The bottom portion of Table 1 shows that these general patterns in the velocity series are, as one would expect, little altered when studied in their logarithms. As casual inspection of Figure 1b suggests is likely, the evidence for integrated behavior weakens as the monetary aggregate broadens.

That, when the researcher adopts the null of unit root behavior, the evidence in the data is generally not strong enough to reject that null—especially for velocities based on narrower monetary aggregates—is consistent with the findings of BLNW (2021). On the basis of bootstrapped Elliot-Rothenberg-Stock tests (which, like ADF tests, have a unit root null) they do not reject unit root behavior, and so conclude that "Evidence of a unit root in M1 velocity and the short rate is typically strong ..." (BLNW Online Appendix, Sec. C.1).

That this conclusion may be sensitive not only to the choice of null but also to the monetary aggregate serves to direct attention to the legitimacy of the homogeneity restrictions. The BLNW view is that, during the past century, the equilibrium error (3) is best treated as a cointegrating relationship between $p_t - m_t + y_t$ and r_t (as the latter enters via the empirical specification for $\lambda_k k(r_t)$), each of which is I(1). But this view has as its premise that the homogeneity restrictions are satisfied, which BLNW take as a maintained hypothesis.

To anticipate the results of the next section, I find that the homogeneity restrictions are best satisfied by M2, and in fact are quite strongly rejected for M1 and NewM1. Given the ambivalence that we have seen in the univariate unit root tests on M2 velocity, this motivates interest in an approach to cointegration testing that is robust to assumptions about the integration properties of p_t, m_t, y_t, and r_t individually. This in turn brings us to PSS testing.

4   Estimation Results

Estimation results for ADL/ECM's based on the three money demand specifications and three monetary aggregates are presented in Tables 2–10. Lag lengths were determined using a combination of hypothesis testing, model selection criteria, and diagnostic tests for residual autocorrelation. With respect to model selection criteria, it is well known that Schwarz's Bayesian information criterion (BIC) is consistent, but in finite samples can sometimes lead to overly parsimonious lag lengths. Akaike's information criterion (AIC) is typically less parsimonious. I therefore considered both criteria, eliminating statistically insignificant lags from the specification yielded by the AIC while ensuring an absence of significant residual autocorrelation, but not below the lag lengths recommended by the BIC.

The resulting lag lengths varied somewhat depending on the money demand specification and the monetary aggregate. But given those, with the sole exception of the loglinear model for M1 (Table 2), the preferred lag lengths were not sensitive to the inclusion of a time trend. In any event, a time trend is insignificant for all models and aggregates. Therefore, although results for the models with a time trend (Case IV in the PSS taxonomy) are included for comparison, the following discussion focuses on the versions of the models that omit a time trend (PSS Case III).

4.1   Parameters and Restrictions

Reassuringly, the estimation results show a fair degree of consistency in the economic behavior described.

First, for all models and aggregates, the estimated speed-of-adjustment coefficient ψ and autoregressive parameters ϕ_j satisfy the conditions for dynamic stability and convergence to long run equilibrium. Given the use of annual data, only short lag lengths are needed to obtain models satisfying this fundamental requirement, as well as exhibiting an absence of residual autocorrelation.

Second, narrower definitions of money tend to be more interest sensitive. For example, in the case of the loglinear model the interest elasticities $-\eta$ are -0.1438, -0.1282, and -0.0643 for M1, NewM1, and M2, respectively (Tables 2, 3, and 4). For the semilog model the corresponding semielasticities $-\xi$ are -11.7857, -7.4449, and -4.9746 (Tables 5, 6, and 7). Consistent with standard intuition, the more an aggregate omits interest-bearing categories of deposits, the greater the tendency to avoid holding it when rates increase.

(The pattern is less evident in the Selden-Latané model (Tables 8, 9, and 10), where the nonlinear functional form results in this parameter being estimated imprecisely.)

These interest elasticity and semielasticity estimates are all statistically significant and broadly consistent with conventional values in the literature. For example, Lucas (2000) used a calibrated semielasticity of $\xi = 7$, which is in the middle of my range of estimates. Ball (2001, Table 2) reports M1 interest semielasticities in the range -0.0277 to -0.1340 (depending on the sample period and choice of interest rate) when the T-bill rate is expressed as a percent, which translates into $-\xi$ estimates of -2.77 through -13.40. Similarly, Stock and Watson (1993, p. 811) concluded that the interest semielasticity of M1 is "near -0.10", so $-\xi \approx -10$.

The loglinear model has been used less often by researchers. For quarterly M1, 1980–2006, Ireland (2009, Tables 2 and 3) reports interest elasticities $-\eta$ of -0.0572 when the homogeneity restrictions are not imposed, -0.0873 when they are. Like mine, these estimates of η are well below the Baumol-Tobin square root rule of $\eta = 0.5$ used by Lucas (2000), a common empirical finding.

Third, under all three money demand specifications, for only the broadest aggregate M2 is the income elasticity λ_y statistically significant. People's willingness to hold the narrower definitions of money, M1 and NewM1, is evidently not substantially determined by their incomes, at least in the broader sweep of history represented in these data. For M2, the estimates of λ_y are 0.6955, 1.5901, and 1.3696 for the loglinear, semi-log, and Selden-Latané specifications, respectively (Tables 4, 7, and 10), and a two-standard-error confidence bound easily includes the homogeneity restriction $\lambda_y = 1$ in all three cases.

Fourth, the strength of the relationship between money and prices tends to be directly related to the broadness of the monetary aggregate, both in terms of coefficient magnitudes and statistical significance. For example, in the case of the loglinear model the elasticities $-\lambda_m$ are 0.2704, 0.4793, and 0.8515 for M1, NewM1, and M2, respectively (Tables 2, 3, and 4). For M2 in all three models (Tables 4, 7, and 10), a two-standard-error confidence bound easily includes the homogeneity restriction $-\lambda_m = 1$. Hence it is the broadest of the aggregates that most reliably has a one-for-one long run association with prices.

Finally, Tables 2–10 report Wald tests for the homogeneity restrictions. Across models, $\lambda_y = 1$ and $\lambda_m = -1$ are generally rejected, individually or jointly, for M1 and NewM1, but not for M2. If these restrictions are to be imposed on the quantity theory, one is most

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comfortable doing so for M2.

4.2 PSS Tests of the Quantity Theory

Having considered the individual parameters and restrictions of these models, let us turn to a more general question. To what extent are these alternative aggregates and money demand specifications consistent with the quantity theory—that is, a long run equilibrium relationship between money, prices, income, and interest rates?

Recall from section 2.2 that PSS (2001) tabulate critical value bounds for the non-standard distributions of their t and F statistics; the null hypothesis is of no levels relationship. The bounds are for the extremes of an equilibrium error that includes all I(0) versus all I(1) variables. For example, for an equilibrium error without a time trend (Case III in the PSS taxonomy), the 10% critical bounds for the F statistic are 2.72 if all variables are I(0) and 3.77 if all are I(1) (PSS Table CI(iii)). A researcher who believes that, along with the other variables, interest rates are best treated as I(1) would insist on evidence as strong as $F > 3.77$ to reject the null at a 10% significance level, finding in favour of the existence of a long run equilibrium relationship. One who believes that interest rates are I(0) would accept somewhat weaker evidence, but would still require something stronger than $F > 2.72$ given that the other variables—money, prices, and income—are naturally regarded as I(1).

Similar bounds hold for the PSS t statistic. Recall that PSS recommend a two-stage testing procedure that begins by checking the F statistic and, if it rejects, then checks the t. The strongest evidence for an equilibrium levels relationship is if both reject.

The PSS critical values are for linear models, and cannot necessarily be assumed to apply for the Selden-Latané specification. Hence, although Tables 8–10 provide mechanically-computed PSS statistics for comparison with the other money demand functions, it is best to focus on the results for the loglinear and semilog models. The strongest evidence favoring the quantity theory comes from the loglinear specification based on M1 and M2 (Tables 2 and 4); these aggregates yield F statistics of 6.5938 and 6.7614, strongly rejecting the null of an absence of an equilibrium relationship, regardless of whether $\log r_t$ is I(0) or I(1). The other aggregate, NewM1, yields $F = 3.3000$ (Table 3), which is in the intermediate range. At the second stage, the t statistics either reject (M2) or fall within the bounds (M1 and NewM1).

The evidence provided by the semilog model is weaker. The F statistic rejects unambiguously for M1 ($F = 3.8985$; Table 5) but is in the intermediate range for NewM1 and M2 (Tables 6 and 7). The t statistic fails to reject for M1 and M2, and is in the intermediate range for NewM1.

On balance, then, PSS tests provide substantial evidence supporting the quantity theory, but the strength of that evidence varies with the monetary aggregate and money demand specification. The strongest evidence comes from the loglinear model based on M2.

5 Application: The Welfare Cost of Inflation

There are several reasons why inflation is costly. It harms people on fixed incomes, and unanticipated inflation has redistributive effects between lenders and borrowers. But, important as these distributional effects may be to those affected, they cancel in the aggregate.

For the aggregate economy the costs of inflation derive principally from two sources. First, inflation interferes with the price system: market participants seeking to respond to relative price movements find themselves having to disentangle observed price changes into their relative and aggregate components, and can do so only imperfectly. Second, inflation leads people to economize on their money holdings, causing them to expend effort to economize on something that is costless to create.

This was all seen clearly by Bailey (1956, pp. 93–4):

> ... the reasons why open inflation is not an advisable method of government finance have not been worked out with theoretical precision; the arguments against it have concentrated on the redistributive and disruptive aspects of inflation—the hardship involved for people whose income and wealth are fixed in money terms and the misallocations of resources that may result from the heightened uncertainties concerning future relative and absolute prices. Such arguments have, by and large, overlooked another aspect of the effects of inflationary finance, an aspect which in a certain sense is more fundamental because it cannot be avoided by sliding-scale arrangements or by precise foreknowledge of the courses of individual prices. This aspect is a welfare cost of open inflation, which, in effect, is a tax on the holding of cash balances, a cost which is fully analogous to the welfare cost (or "excess burden") of an excise tax on a commodity or productive service.
>
> The theoretical analysis of the welfare cost of open inflation is, in fact, a reasonably straightforward extension of the theory of the welfare cost of an excise tax and, indeed, is in some respects much simpler. ... this welfare cost is measured by the area under an appropriately defined liquidity preference curve ...

At high rates of inflation all these costs are large to an extent that probably defies quantification, as evidenced by the abandonment of money in favor of barter in the final stages of hyperinflation. But how about the more moderate rates of inflation that have typically characterized most developed nations during the past century since the demise of the classical gold standard? The literature quantifying this has focused on Bailey's "more fundamental" excess burden of the implicit tax on money holding. The relevant "...area under an appropriately defined liquidity preference curve ..." depends on the form of that curve. For the loglinear and semi-log money demand functions (7) and (8) Lucas (2000) derived closed-form expressions for the excess burden as a function of the interest rate r:

$$w(r) = A\frac{\eta}{1-\eta}r^{1-\eta} \qquad \text{and} \qquad w(r) = \frac{B}{\xi}[1 - (1 + \xi r)e^{-\xi r}],$$

Here "...$w(r)$ has the interpretation ...as the fraction of income people would require as compensation in order to make them indifferent between living in a steady state with an interest rate constant at r and an otherwise identical steady state with an interest rate of (or near) zero." (Lucas 2000, pp. 250–251.) The analogous formula for Selden-Latané money demand is (BLNW 2017, equ. (10))

$$w(r) = \frac{\log(C + \delta r) - \log C}{\delta} - \frac{r}{C + \delta r}.$$

These formulas apply under two conditions. First, when the quantity theory holds as a good approximation to the long run interaction of money, prices, income, and interest rates (or, in the Lucas-Nicolini phrasing, when there is a stable long run money demand function). Second, when this long run relationship satisfies the homogeneity restrictions $-\lambda_m = \lambda_y = 1$. The estimation results of the preceding section suggest that these two conditions are most likely to hold for the loglinear model based on M2.

Furthermore, in the Bailey-Lucas conception of its welfare cost, inflation has its effect via the interest sensitivity of money demand. Higher expected inflation is incorporated into interest rates (the Fisher effect), in response to which people incur the cost of economizing on their money holdings. Evaluating the welfare cost of inflation therefore hinges on the interest sensitivity of money demand, and here too the evidence indicates that loglinear money demand based on M2 serves us well. As we have seen, although for both the semilog and loglinear models the interest sensitivity declines as the monetary aggregate broadens, it is still strongly statistically significant in the loglinear M2 model (Table 4). This is less true in the semi-log M2 model (Table 7).

On balance, then, a researcher coming from the ADL/ECM framework and wishing to use Lucas's formulas to evaluate the welfare cost of inflation would be most inclined to believe the results from the loglinear model based on M2 (Table 4). The homogeneity restrictions $-\lambda_m = \lambda_y = 1$ are not rejected and the interest elasticity $-\hat{\eta} = 0.0643$ is strongly statistically significant, something that continues to be true after the restrictions are imposed (Table 13), where the interest elasticity changes only slightly to $-\hat{\eta} = -0.0602$. Interestingly, a log-log model is also the preferred specification of Lucas (2000) and BLNW (2021), except for very low interest rates where BLNW find that the Selden-Latané functional form fits the data better.

A second choice would be the semi-log model based on M2 (Table 7): the homogeneity restrictions are compatible with the data and the interest semi-elasticity $-\hat{\xi} = -4.9746$ is marginally significant, as continues to be true after the restrictions are imposed and the semielasticity becomes $-\hat{\xi} = -6.4891$ (Table 16). The Selden-Latané model based on M2 may also have something to offer: the homogeneity restrictions are not rejected and, when imposed (Table 19), contribute enough additional structure that the precision with which the interest parameter δ is estimated is substantially improved.

Despite this preference for the models based on M2, for completeness Tables 11–19 present welfare calculations over the full range of aggregates and money demand specifications. For purposes of comparison the results for the "canonical" semi-log functional form using M1 or NewM1 may be of interest, given its widespread use in the empirical literature. So too may be the results for the other money demand functions based on NewM1, given Lucas and Nicolini's advocacy of it as their preferred aggregate and its use by BLNW (2017, 2021). To be sure, we have seen that our results are less supportive of these alternatives, at least for the purpose of implementing Lucas's formulas. In all three functional forms the use of either M1 or NewM1 leads to rejection of the homogeneity restrictions $-\lambda_m = \lambda_y = 1$ (Tables 2, 3, 5, 6, 8, and 9) on which the applicability of Lucas's formulas is predicated. Imposing these restrictions on models where they are rejected can lead to implausible estimates of the interest rate parameter, as occurs in, for example, the semilog model based on M1 (Table 14).

Notwithstanding these qualifications, Tables 11–19 re-estimate all the models of Tables 2–10 imposing the homogeneity restrictions $-\lambda_m = \lambda_y = 1$.[5] The estimates of the money demand parameters are then used in Lucas's formulas to evaluate the welfare cost of inflation

$w(r)$ at some illustrative interest rates.

Based on the arguments just made, Table 13 for the loglinear model using M2 should probably be viewed as the benchmark results. It indicates that, in periods of low interest rates (say 3 percent) and, by implication, low expected inflation, the welfare cost of inflation is estimated to be on the order of one-tenth of one percent of income: $w(0.03) = 0.112$. In periods of higher inflation and interest rates (say 14 percent), such as prevailed in the late 1970s and early '80s, the welfare cost is closer to one half of one percent: $w(0.14) = 0.476$. These values are for the model without a time trend (Case III in the PSS taxonomy), given the insignificance of a time trend term, but are little affected by the inclusion of a time trend (PSS Case IV).

To be sure, these estimates are sensitive to plausible variations on the model and aggregate. Consider the seemingly minor variation of the semi-log model, retaining M2 as the aggregate (Table 16). At $r = 0.03$ the welfare cost is estimated to be 0.197 percent of income, while $r = 0.14$ yields $w(0.14) = 2.729$ percent.

In fact, for $r = 0.14$ these values $w(r) = 0.476$ and $w(r) = 2.729$ yielded by the log-linear and semi-log models, respectively, encompass the full range of cost estimates across the models and aggregates, even implausible ones where the homogeneity restrictions are rejected or the PSS tests are not particularly supportive of cointegration. Hence, even at the highest interest rates that the U.S. has ever experienced, the most extreme estimate of welfare cost is less than three percent of income. This is large—three percent of income is a lot—but not entirely implausible. And the estimate yielded by the model that the data most strongly support, loglinear money demand based on M2, is the much lower $w(0.14) = 0.476$.

At the other end of the spectrum of historical interest rates, $r = 0.03$, the full variation of cost estimates $w(0.03)$ is in the range 0.112–0.784, or less than one percent of income.

Having considered these variations in the welfare cost estimates, it is evident that the estimates $w(0.03) = 0.112$ and $w(0.14) = 0.476$ yielded by the benchmark model, loglinear money demand based on M2, are at the bottom end of the ranges for those interest rates. Comparing Table 13 with the other tables, this turns out to be true for *all* of the illustrative interest rates used.

That is, the model that the data most strongly support yields fairly conservative welfare cost estimates at all levels of historical interest rates, relative to my other estimates. However these estimates are not necessarily conservative relative to the broader literature. Table

20 collects my welfare cost estimates for the models based on M2 (from Tables 13, 16, and 19) and compares them with prominent contributions from the literature. My benchmark estimates are lower than Lucas's (2000) for his loglinear model, but not relative to Ireland's (2009) based on a semi-log model. Both used M1. More recently, BLNW (2017, p. 26) use NewM1, 1915–2016, and find "...welfare losses at the average short rate that has prevailed over the sample period ...equal to 0.21 per cent for the Selden-Latané specification, and 0.18 for the other two."

In conclusion, at interest rates (and the embedded expectations of inflation they reflect) that the U.S. has experienced over the past century, my benchmark results based on loglinear money demand predict welfare costs in the range of one-tenth to one-half of one percent of income. This is remarkably similar to BLNW's (2017, p. 26) median of 0.18 percent from a loglinear model. In the context of the broader literature, my estimates are roughly middle-of-the-road.

6 Structural Change

The century of data over which these models are estimated saw momentous events, most obviously World War I followed by the Spanish flu epidemic, the Great Depression, World War II, the 1951 Treasury-Fed accord, the inflation and high nominal interest rates of the 1970s and '80s, and—following a prematurely-named "great moderation"—the housing bubble, financial crisis, and great recession of the new century. It might seem implausible that models as simple as these can adequately describe the dynamics of money and prices over the full course of this historical tumult.

There are several approaches to investigating possible structural change, including well-known ones dating back many years. However arguably the most systematic and comprehensive is the indicator saturation (IS) methodology developed by David Hendry and collaborators as an application of general-to-specific (GETS) specification testing. This permits the testing of indicator variables (i.e. dummy shifts) at each observation of the sample by using block estimation and general-to-specific algorithms. The distributional results for impulse indicators were originally worked out by Hendry, Johansen, and Santos (2008) and later extended to step and trend indicators by Castle, Doornik, Hendry, and Pretis (2015).[6]

Given that the IS methodology is just one of numerous approaches to studying structural change, and itself depends on various choices of implementation settings, how best

to introduce structural change across the full range of my aggregates and money demand specifications could be debated almost endlessly. Rather than attempt such a comprehensive treatment, which is probably illusory in any event, it seems more useful to concentrate on the sensitivity of the findings for my preferred model, the ADL(1,4,0,3) for loglinear M2. Table 21 reports the results of applying the IS methodology to this model, and so should be compared with the earlier Tables 4 and 13. Given the universal finding, across the models and aggregates of Tables 2–10, of the absence of a time trend in the equilibrium error (i.e. $\mu = 0$), Table 21 focuses on impulse and step indicators (as opposed to trend indicators).

Under default settings the GETS algorithm yields dates of structural change that are highly intuitive, as shown in the first column of Table 21. Following World War I there was, ceteris paribus, an upward shift in prices in 1920 followed by an almost exactly offsetting shift the next year. The Great Depression saw a negative shift in prices in 1931 that was more than offset in 1933. And, in the post-World War II transition to a peacetime economy, there was an upward shift in prices in 1946 that was offset in 1948. That these shifts tend to appear in offsetting pairs is consistent with the notion that in the long run the price level is determined, not by the specifics of discrete historical events, however momentous, but by their long run equilibrium relationship with the money supply as governed by the QTM. Furthermore the pattern is robust to the imposition of the homogeneity restrictions (second column of Table 21).

Even allowing for these structural shifts, the results for the model without homogeneity imposed (first column of Table 21) are qualitatively similar to the earlier Table 4: dynamics are convergent, the homogeneity restrictions are compatible with the data, and the interest elasticity of money demand is strongly statistically significant. The main difference is that this interest sensitivity is greater than before ($-\eta = -0.1295$ instead of the earlier -0.0693 of Table 4), even more so when the homogeneity restrictions are imposed ($-\eta = -0.1572$ instead of the earlier -0.0602 of Table 13).

Given that the welfare cost of inflation works through the interest elasticity, the result is larger welfare costs $w(r)$. Whereas previously (in Tables 13 and 20) these range between $w(0.03) = 0.112$ and $w(0.14) = 0.476$ percent of income, now (in the second column of Table 21) the range is 0.362–1.326 percent, about three times as high. This puts the welfare costs closer to those of the other models summarized in Table 20 and suggests that, once structural change is taken into account, the BLNW (2017, p. 26) median value of 0.18

percent may be an underestimate.

7 Summary and Conclusions

The analysis of this paper is simple. Being a theory of price level determination, the QTM has just a single endogenous variable. It can therefore be studied with the simplest of dynamic time series econometric tools, a single-equation ADL/ECM in which the quantity theory specifies the equilibrium error. Because the QTM is a theory of price level determination in the long run, I have estimated the model with annual data over the past century. The use of annual data means that dynamics are adequately treated with fairly short lag lengths, so that the model can be modestly parameterized relative to the data. There is therefore some hope of obtaining useful inferences about the parameters.

I have used standard money demand specifications and three monetary aggregates: M1, NewM1, and M2. In principle the quantity theory is not inherently linked to cointegration; the QTM is a theory of the price level even if all series vary in a manner that is stationary. But in practice, in most countries and time periods, money and prices trend, as does national income; this is certainly so for the U.S. since the demise of the international gold standard following World War I. If this nonstationarity is integrated, as tests suggest, the quantity theory translates into the prediction of cointegration between these variables. For the purpose of studying this prediction, the single equation ADL/ECM framework makes available the bounds testing methodology of PSS. The great advantage of PSS tests is that they are robust to the integration properties of the variables, most importantly the interest rate, the fourth variable that must enter into any specification of the QTM. Whereas the treatment of money, prices, and income as integrated processes is compelling, interest rates are a more open question. Over shorter sample periods their time series properties may well be best approximated as integrated; but in the long run—which is what the QTM is all about—casual observation indicates that they exhibit mean reversion.

PSS tests yield credible rejections of the null of the absence of an equilibrium relationship, for various combinations of aggregates and money demand specifications. Hence there is much in these results that is supportive of the QTM. The evidence is strongest for loglinear money demand (that is, where the interest rate appears as $\log r_t$) based on M1 or M2.

Following Bailey (1956), a classic application of estimated money demand parameters is to the welfare cost of inflation. Lucas (2000) derived simple formulas for this welfare cost for

loglinear and semi-log money demand; BLNW (2017) provide an analogous formula for the Selden-Latané functional form. These formulas require that the quantity theory satisfies the homogeneity restrictions, which are merely the requirements that the price level be related to both the money stock and income by unitary elasticities. There is no particular reason to believe that these restrictions should hold in the short run, because the QTM is unlikely to explain the short term dynamics of the variables. But it is entirely plausible that they should hold in the long run, being nothing more than the classic propositions that, ceteris paribus, a doubling of the quantity of money in circulation doubles the price level, or a doubling of economic activity halves the price level.

Inevitably, the extent to which these classic propositions are actually manifested in the data will depend on the definition of the monetary aggregate. The literature (e.g. Lucas (2000), Ireland (2009)) has tended to adopt the homogeneity restrictions as a maintained hypothesis and use M1, although BLNW (2017, 2021) use the aggregate NewM1 advocated by Lucas and Nicolini (2015). Indeed, it is perhaps not too much of a caricature to say that Lucas and Nicolini's contribution was to design a monetary aggregate that has a stable contemporaneous long run relationship with velocity, hence satisfying the homogeneity restrictions—although they did not test those restrictions inferentially.

In contrast I find that, regardless of the form of the money demand function, the homogeneity restrictions are *always* rejected jointly (although not always individually) for M1 and NewM1 (Tables 2, 3, 5, 6, 8, 9). Yet they are *never* rejected (either individually or jointly) for M2 (Tables 4, 7, 10). This suggests that M2 is the preferred aggregate for implementing Lucas's welfare cost formulas.

Loglinear money demand based on M2 exhibits the appealing combination of cointegration, non-rejection of the homogeneity restrictions, and a statistically significant interest elasticity. With structural shifts treated by indicator saturation, this benchmark model yields welfare costs in the range of 0.362–1.326 percent of income over the interest rates that have prevailed in the past century.

The simplicity of this analysis belies other recent approaches to these issues. BLNW (2017, 2021) adopt the homogeneity restrictions as a maintained hypothesis and interpret the QTM as predicting cointegration between velocity and the interest rate. This in turn requires that velocity and interest rates be, in their univariate behavior, integrated processes. In contrast, I have argued that nothing in the quantity theory requires or implies

that velocity or interest rates be individually integrated, or cointegrated in their joint behavior. Indeed, if (the log of) velocity is I(1) when interest rates are stationary, this would imply non-cointegration of money, prices, and income, the opposite of the quantity theory.

Perhaps the defining feature of a simple analysis is that it can potentially be complicated in myriad ways. In the present case, many possible elaborations and extensions are possible. Some have already been noted, such as a more extensive investigation of structural change across models and aggregates, or the possibility of embedding the quantity theory in a multi-equation model with a larger set of endogenous variables that studies a more comprehensive set of classical propositions, in the spirit of Fisher and Seater (1993). The point of the present analysis is not that these extensions would be uninteresting—far from it—but that the quantity theory emerges so clearly from a simple empirical model that largely abstracts from them.

Appendix

The Source of the Time Trend in the Equilibrium Error

A time trend in the equilibrium error can arise when components are either trend stationary and/or integrated with drifts. The possibilities can be illustrated with two examples based on the quantity theory.

An Example

Consider the four variables of the quantity theory, p_t, m_t, y_t, and r_t. Suppose that the first three are I(1) and cointegrated. Univariate I(1) behavior means that each of p_t, m_t, and y_t evolves as

$$\Delta p_t = \mu_p + u_{pt} \qquad \Leftrightarrow \qquad p_t = \mu_p t + v_{pt} \tag{15a}$$

$$\Delta m_t = \mu_m + u_{mt} \qquad \Leftrightarrow \qquad m_t = \mu_m t + v_{mt} \tag{15b}$$

$$\Delta y_t = \mu_y + u_{yt} \qquad \Leftrightarrow \qquad y_t = \mu_y t + v_{yt}. \tag{15c}$$

The left hand side expressions state these processes in their conventional I(1) form, where the μ_i are the drifts and the u_{it} are stationary disturbances. Given that p_t, m_t, and y_t are defined to be in log form, the drifts μ_i are their steady state growth rates: $\mathrm{E}(\Delta p_t) = \mu_p$ is the steady state inflation rate, $\mathrm{E}(\Delta m_t) = \mu_m$ the steady state money supply growth rate, and $\mathrm{E}(\Delta y_t) = \mu_y$ the steady state growth rate of real GDP. The right hand side expressions of (15) restate these I(1) specifications in their "accumulated" forms as I(1) disturbances $v_{it} = \sum_{j=0}^{\infty} u_{i,t-j}$ superimposed on linear trends in which the drifts μ_i appear as slope coefficients. (That is, any drifting I(1) variable can always be re-stated as a zero-mean I(1) disturbance superimposed on a linear trend. By the Beveridge-Nelson decomposition, these I(1) disturbances are each the total of a random walk stochastic trend and a transitory component. The I(1) variable as a whole is therefore the total of a deterministic trend, a stochastic trend, and stationary transitory variation.)

Cointegration between p_t, m_t, and y_t means that a linear combination of the I(1) disturbances v_{it} is stationary, so that they share a common stochastic trend. Denoting the

normalized cointegrating vector by $[1, \lambda_m, \lambda_y]$, this linear combination is

$$
\begin{aligned}
v_{pt} + \lambda_m v_{mt} + \lambda_y v_{yt} &= p_t - \mu_p t + \lambda_m (m_t - \mu_m t) + \lambda_y (y_t - \mu_y t) \\
&= p_t + \lambda_m m_t + \lambda_y y_t - (\mu_p + \lambda_m \mu_m + \lambda_y \mu_y)t \qquad (16) \\
&\sim \text{stationary.}
\end{aligned}
$$

This linear combination is stationary because the time trend term cancels with the drifts embedded within the observables p_t, m_t, and y_t. Hence a cointegrating relationship can include a time trend: in the long run variables can evolve together stochastically via their common stochastic trend(s), yet move apart deterministically.

Turning to the fourth variable of the quantity theory, the interest rate r_t, suppose that, unlike the others, it is stationary and that the reciprocal-of-velocity $k(r_t)$ evolves according to

$$
\log k(r_t) = \beta + \mu_k t - \xi r_t + u_{kt},
$$

where u_{kt} denotes a stationary disturbance so that $\log k(r_t)$ is trend stationary. The time trend term $\mu_k t$ might be due to, say, technical change in the banking system.

Consider introducing this as an additional term into the linear combination (16).

$$
\begin{aligned}
p_t + \lambda_m m_t &+ \lambda_y y_t - (\mu_p + \lambda_m \mu_m + \lambda_y \mu_y)t + \log k(r_t) \\
&= p_t + \lambda_m m_t + \lambda_y y_t - (\mu_p + \lambda_m \mu_m + \lambda_y \mu_y)t + \beta + \mu_k t - \xi r_t + u_{kt}
\end{aligned}
$$

In view of the assumed stationarity of r_t, u_{kt}, and (16), the following linear combination must be trend stationary.

$$
\{p_t + \lambda_m m_t + \lambda_y y_t - (\mu_p + \lambda_m \mu_m + \lambda_y \mu_y)t\} + \beta + \mu_k t - \xi r_t \sim \text{trend stationary} \qquad (17)
$$

This is the equilibrium error (12) in which $\mu = -(\mu_p + \lambda_m \mu_m + \lambda_y \mu_y) + \mu_k$. Whereas the expression in braces is a cointegrating relationship (between p_t, m_t, and y_t), the expression as a whole is not, because it is nonstationary. However this nonstationarity is of a particular form—trend stationarity. The expression (17) as a whole therefore constitutes, not a cointegrating relationship, but nevertheless an equilibrium relationship between the levels of the four variables.

In some special cases this equilibrium relationship may not include a time trend. Suppose that velocity is stationary, so $\mu_k = 0$, perhaps because there has been no technical change

in the banking system. Then the time trend as a whole disappears from the equilibrium relationship if

$$\mu_p + \lambda_m \mu_m + \lambda_y \mu_y = 0. \tag{18}$$

Recall that the μ_i are steady state growth rates of prices, money, and income. The restriction (18) therefore holds if these growth rates are related by

$$\mathrm{E}(\Delta p_t) + \lambda_m \, \mathrm{E}(\Delta m_t) + \lambda_y \, \mathrm{E}(\Delta y_t) = 0. \tag{19}$$

To illustrate with one circumstance in which this could be true, consider the situation in which money demand (2) is homogenous of degree one in nominal transactions, so that the homogeneity restrictions hold: $-\lambda_m = \lambda_y = 1$. Then (19) reduces to

$$\mathrm{E}(\Delta p_t) - \mathrm{E}(\Delta m_t) + \mathrm{E}(\Delta y_t) = 0,$$

so the drifts are related by

$$\mu_p = \mu_m - \mu_y. \tag{20}$$

This is the familiar textbook equality that, in the long run, the inflation rate equals the growth rate of the money stock less the growth rate in real economic activity. In the further special case of a stagnant economy, $\mu_y = 0$, the inflation rate equals the money supply growth rate.

This example is of the case in which the variables p_t, m_t, and y_t are assumed to be I(1) and cointegrated while r_t is stationary and $\log k(r_t)$ is trend stationary. It shows that a time trend in the equilibrium error can arise from either or both of two sources: a cointegating relationship in which the drifts do not cancel, or a trend stationary component.

A Second Example

While the presence of a trend stationary component in the equilibrium error—in the above example, $\log k(r_t)$—is sufficient for a trend term to appear, it is not necessary. To emphasize this, consider the possibility that BLNW are correct and that velocity and the interest rate are best treated as I(1). Like the other variables in (15), $\log k(r_t)$ has an I(1) specification:

$$\Delta \log k(r_t) = \mu_k + u_{kt} \quad \Leftrightarrow \quad \log k(r_t) = \mu_k t + v_{kt}.$$

As before, u_{kt} is a stationary disturbance while its accumulation v_{kt} is a zero-mean I(1) disturbance.

Generalizing from the previous cointegrating relationship (16), the cointegrating relationship among the four variables of the quantity theory is now of the form

$$v_{pt} + \lambda_m v_{mt} + \lambda_y v_{yt} + \lambda_k v_{kt} = p_t - \mu_p t + \lambda_m (m_t - \mu_m t) + \lambda_y (y_t - \mu_y t) + \lambda_k (\log k(r_t) - \mu_k t)$$

$$= p_t + \lambda_m m_t + \lambda_y y_t + \lambda_k \log k(r_t) - (\mu_p + \lambda_m \mu_m + \lambda_y \mu_y + \lambda_k \mu_k) t$$

$$\sim \text{stationary}$$

where $[1, \lambda_m, \lambda_y, \lambda_k]$ is the cointegrating vector. In the case of, say, a semi-log money demand specification $\lambda_k \log k(r_t) = \beta - \xi r_t$, this is the equilibrium error (12) in which $\mu = -(\mu_p + \mu_m \lambda_m + \mu_y \lambda_y + \mu_k \lambda_k)$:

$$p_t + \lambda_m m_t + \lambda_y y_t + \beta - \xi r_t - (\mu_p + \lambda_m \mu_m + \lambda_y \mu_y + \lambda_k \mu_k) t. \tag{21}$$

The cointegrating vector is reinterpreted as $[1, \lambda_m, \lambda_y, -\xi]$. Hence the trend term in this equilibrium error arises, not from any trend stationary component within the quantity theory, but entirely from the nonzero drifts of the variables.

Depending on these drifts, once again the trend term may disappear from the equilibrium error, namely when $\mu_p + \lambda_m \mu_m + \lambda_y \mu_y + \lambda_k \mu_k = 0$ and, as a generalization of (19), the steady state growth rates are related by

$$\text{E}(\Delta p_t) + \lambda_m \text{E}(\Delta m_t) + \lambda_y \text{E}(\Delta y_t) + \lambda_k \text{E}(\Delta \log k(r_t)) = 0. \tag{22}$$

One circumstance in which this could be true is if the homogeneity restrictions $-\lambda_m = \lambda_y = 1$ hold, in which case the condition simplifies to

$$\text{E}(\Delta p_t) - \text{E}(\Delta m_t) + \text{E}(\Delta y_t) + \lambda_k \text{E}(\Delta \log k(r_t)) = 0,$$

so the drifts are related by

$$\mu_p = \mu_m - \mu_y + \lambda_k \mu_k. \tag{23}$$

So, as a generalization of the simpler relationship (20), now the long run inflation rate depends on money supply growth less the growth in real economic activity, plus a term related to growth in velocity. If this holds joint with the homogeneity restrictions then the time trend term disappears from the equilibrium error (21).

The velocity term depends on the specification of $\lambda_k \log k(r_t)$. In the case of semi-log money demand, $\lambda_k \log k(r_t) = \beta - \xi r_t$, we have $\lambda_k \Delta \log k(r_t) = -\xi \Delta r_t$ and

$$\text{E}(\Delta p_t) = \text{E}(\Delta m_t) - \text{E}(\Delta y_t) - \xi \text{E}(\Delta r_t). \tag{24}$$

Steady state inflation depends on money growth, income growth, and the interest rate drift modified by the interest semi-elasticity.

Consider the further special case in which r_t, although I(1), has zero drift. Then $E(\Delta r_t) = 0$ and the steady state relationship (24) reduces to the textbook special case (20) in which the inflation rate is determined by the extent to which money growth exceeds growth in real economic activity.

Like the first example, this one has been illustrated with a semi-log money demand specification, but clearly the substance does not hinge on that choice. Under the alternative of loglinear money demand, in the equilibrium error (17) $-\eta \log r_t$ would replace $-\xi r_t$, while in (24) $-\eta \Delta \log r_t$ would replace $-\xi \Delta r_t$.

Conclusions

We have considered two examples of how a time trend can arise within the equilibrium error of the ECM, and the special cases in which it will disappear.

My preferred conception of the world is rather like the first example, where velocity and the interest rate are each stationary, and the cointegrating relationship is between p_t, m_t, and y_t. With respect to the time trend in the equilibrium error, both conditions for its omission are plausibly satisfied in the long run. First, in line with Cochrane's logic, the interest rate does not trend, so $\mu = 0$. And second, the homogeneity restrictions hold and the steady state growth rates of money, prices, and income satisfy the textbook equality (20).

BLNW's conception of the world is more like the second example, where velocity and the interest rate are each I(1) and the cointegrating relationship is between them, not between p_t, m_t, and y_t. If a time trend disappears from the cointegrating relationship, it is because the relationship (22) holds, perhaps because the homogeneity restrictions are satisfied and (23) obtains.

In either example, the conditions under which the time trend will disappear are unlikely to hold in the short or even medium term. They may, however, plausibly hold in the long run to which the QTM is intended to apply. In such applications, it would therefore not be too surprising to find that a time trend in the equilibrium error is statistically insignificant, as indeed is the case in my findings across all models and aggregates.

Notes

[1]Another example of recent work that uses these three money demand specifications is Benati (2020).

[2]If the model includes a time trend but the time trend is not part of the equilibrium error, so that the hypothesis of an absence of the levels from the model does not include the restriction $\kappa = 0$, then the t-statistic can be bounded. This is Case V (PSS 2001, Corollaries 3.3 and 3.4). However this would not appear to be the relevant specification in the quantity theory application where, if a time trend exists in the model, it arises from within the equilibrium levels relationship. In the empirical application that they used to illustrate their methodology, PSS similarly reasoned that "...the statistic F_{IV} which sets the trend coefficient to zero under the null hypothesis of no level relationship, Case IV ..., is more appropriate than F_V, Case V ..., which ignores this constraint." In my application to the QTM the distinction between Cases IV and V not of great significance given that, across all models and aggregates, $\mu = 0$ is not rejected. This suggests that a time trend does not belong in the model, whether within the equilibrium error or outside it. Hence my emphasis on Case III as, in my application, the empirically relevant one.

[3]The Federal Reserve Economic Data acronyms are M1SL and M2SL, which are seasonally adjusted monthly series. I follow BLNW in using the December 1 value as the annual value.

[4]Alternatives for the short term interest rate are the three month commercial paper rate (used, for example, by Ireland (2009)) or the "consistent series for the short-term interst rate for ordinary funds" from www.measuringworth.com. However in my exploratory work I found the substantive results to be insensitive to the interest rate choice.

[5]The only case in which the estimation of these restricted models turned out to be problematic was for the loglinear model based on M1 (Table 11). For the version of this model that omits a time trend (Case III in the PSS taxonomy), the restrictions $-\lambda_m = \lambda_y = 1$ are so strongly rejected by the data (Table 2) that attempting to impose them yields estimates of the remaining parameters that are imprecise to the point of being meaningless. This finding was insensitive to exploratory variations on the lag lengths.

[6]The IS methodology is implemented and comprehensively documented in the R package `gets` (for "general-to-specific" testing). It is also implemented in EViews version 12, from which the results of Table 21 are obtained. Useful guides to the application of the methodology, with reference to the R implementation, are given by Pretis, Reade, and Sucarrat (2018) and Sucarrat (2020).

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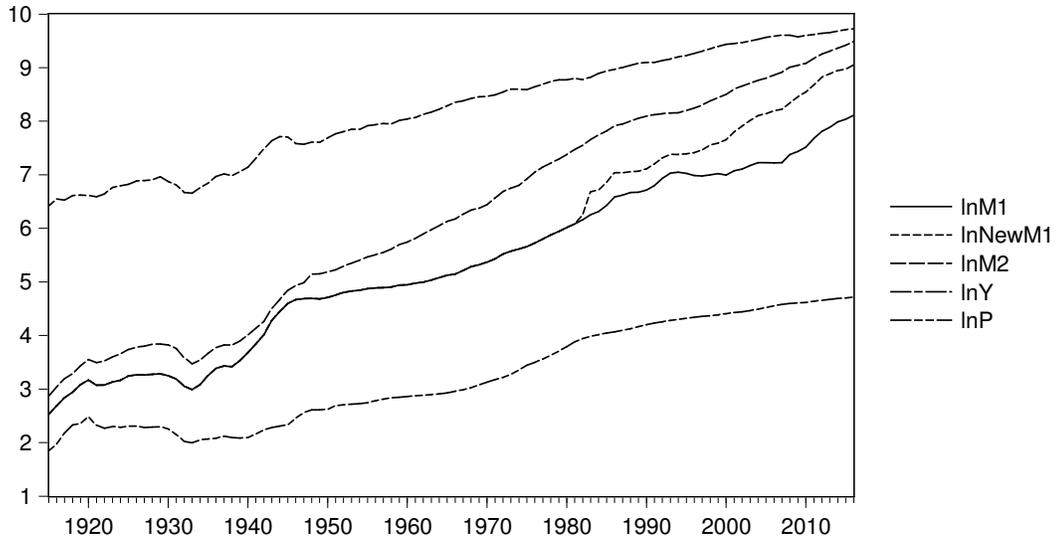
(a) Money, income, and prices
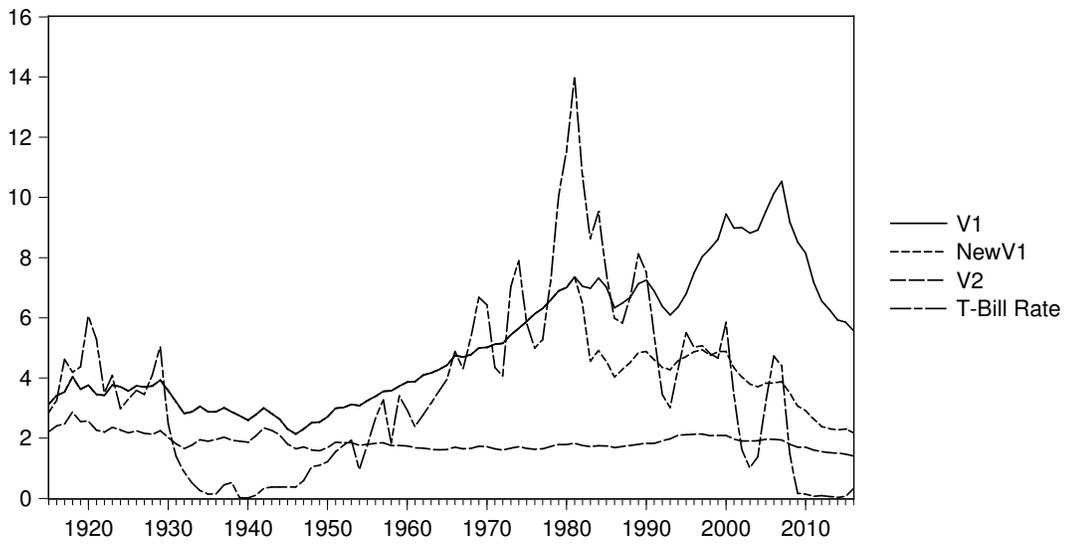
(b) Velocities and the T-bill rate

Figure 1: The Aggregates of the Quantity Theory

Table 1: Univariate Unit Root Tests

Variable	ADF tests (p-values)[a]		KPSS test statistics[b]	
	constant	constant+trend	constant[c]	constant+trend[d]
log GDP deflator, p_t	0.988	0.191	5.880***	0.284***
log real GDP, y_t	0.760	0.177	0.841***	0.133*
interest rate, r_t	0.411	0.753	0.132	0.101
$\log r_t$	0.509	0.824	0.110	0.109
Money stock, m_t:				
log M1	0.976	0.131	3.845***	0.314***
log NewM1	0.995	0.394	1.050***	0.211**
log M2	0.969	0.272	2.109***	0.224***
Velocity, $V_t = P_t Y_t / M_t$:				
V1	0.637	0.691	0.418*	0.338***
NewV1	0.643	0.945	0.265	0.398***
V2	0.088	0.155	0.318	0.130*
Log velocity,[e] $\log k_t = -\log V_t$:				
log K1	0.731	0.787	0.457*	0.363***
log NewK1	0.771	0.981	0.436*	1.039***
log K2	0.173	0.262	0.312	0.124*

[a] ADF tests use two augmenting lags, which was typically the preferred lag length selected by Schwarz's Bayesian information criterion.
[b] KPSS tests use a Bartlett kernel with an Andrews-selected bandwidth.
[c] Critical values are 0.347 (10%), 0.463 (5%), 0.739 (1%).
[d] Critical values are 0.119 (10%), 0.146 (5%), 0.216 (1%).
[e] Tests are invariant to the negative sign relating $\log k_t$ and $\log V_t$.

Table 2: ADL/ECM for M1 under Loglinear Money Demand

| | ADL/ECM($\cdot,\cdot,\cdot,\cdot$) | |
Equilibrium error: $p_t + \lambda_m m_t + \lambda_y y_t + \alpha - \eta \log r_t + \mu t$	(2,4,2,1) Case III $\mu = 0$	(2,4,0,3) Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient[a], $\psi = \phi_1 + \phi_2 - 1$	-0.0554	-0.0821
	(0.0167)	(0.0216)
interest elasticity[b], $-\eta$	-0.1438	-0.1749
	(0.0506)	(0.0399)
time trend, μ		-0.0260
		(0.0185)
transactions elasticity, λ_y	-0.5693	0.4967
	(0.6359)	(0.5834)
$-\lambda_m$	0.2704	0.4554
	(0.4036)	(0.2431)
Wald tests of homogeneity restrictions (p-values):		
(1) Unitary money elasticity: $\lambda_m = -1$	0.071	0.025
(2) Unitary income elasticity: $\lambda_y = 1$	0.014	0.388
(3) y and $-m$ share common coefficient: $-\lambda_m = \lambda_y$	0.000	0.937
(4) Given (3), common coefficient is 1	0.006	0.025
(5) Joint test of (2) and (3): $-\lambda_m = \lambda_y = 1$	0.000	0.081
Autoregressive coefficients of ADL parameterization:[c]		
ϕ_1	1.0821	1.0831
	(0.0922)	(0.0955)
ϕ_2	-0.1375	-0.1651
	(0.0876)	(0.0904)
LM test for residual autocorrelation[d] (p-value)	0.597	0.815
PSS tests of null of no levels relationship:		
F statistic	6.5938[e]	5.8853[f]
t statistic	-3.3077[g]	

Notes: Standard errors are in parentheses.

[a] Convergence to equilibrium requires $-1 < \psi < 0$.

[b] For comparison, the Baumol-Tobin square-root rule for money holding predicts an interest elasticity of -0.5.

[c] The dynamic stability conditions $\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$, and $|\phi_2| < 1$ are satisfied.

[d] Reported tests are based on 3 lags, but the test decisions (of no rejection of the null of absence of residual autocorrelation) are generally robust to variations on the lag length.

[e] 10% critical values for PSS Case III are: 2.72 if all variables are I(0), 3.77 if all variables are I(1). (PSS Table CI(iii).)

[f] 10% critical values for PSS Case IV are: 2.97 if all variables are I(0), 3.74 if all variables are I(1). (PSS Table CI(iv).)

[g] 10% critical values are PSS Case III: -2.57 if all variables are I(0), -3.46 if all variables are I(1). (PSS Table CII(iii).)

Table 3: ADL/ECM for NewM1 under Loglinear Money Demand

	ADL/ECM(2,1,0,0)	
Equilibrium error: $p_t + \lambda_m m_t + \lambda_y y_t + \alpha - \eta \log r_t + \mu t$	Case III $\mu = 0$	Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient,[a] $\psi = \phi_1 + \phi_2 - 1$	-0.0820	-0.0939
	(0.0287)	(0.0347)
interest elasticity,[b] $-\eta$	-0.1282	-0.1369
	(0.0365)	(0.0348)
time trend, μ		-0.0123
		(0.0180)
transactions elasticity, λ_y	-0.0078	0.3145
	(0.3056)	(0.5169)
$-\lambda_m$	0.4793	0.4613
	(0.1662)	(0.1507)
Wald tests of homogeneity restrictions (p-values):		
(1) Unitary money elasticity: $\lambda_m = -1$	0.002	0.000
(2) Unitary income elasticity: $\lambda_y = 1$	0.001	0.185
(3) y and $-m$ share common coefficient: $-\lambda_m = \lambda_y$	0.001	0.769
(4) Given (3), common coefficient is 1	0.790	0.000
(5) Joint test of (2) and (3): $-\lambda_m = \lambda_y = 1$	0.003	0.002
Autoregressive coefficients of ADL parameterization:[c]		
ϕ_1	1.2703	1.2714
	(0.0933)	(0.0937)
ϕ_2	-0.3523	-0.3652
	(0.0850)	(0.0878)
LM test for residual autocorrelation[d] (p-value)	0.786	0.725
PSS tests of null of no levels relationship:		
F statistic	3.3000[e]	2.6971[f]
t statistic	-2.8596[g]	

Notes: Standard errors are in parentheses.

[a] Convergence to equilibrium requires $-1 < \psi < 0$.

[b] For comparison, the Baumol-Tobin square-root rule for money holding predicts an interest elasticity of -0.5.

[c] The dynamic stability conditions $\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$, and $|\phi_2| < 1$ are satisfied.

[d] Reported tests are based on 3 lags, but the test decisions (of no rejection of the null of absence of residual autocorrelation) are generally robust to variations on the lag length.

[e] 10% critical values for PSS Case III are: 2.72 if all variables are I(0), 3.77 if all variables are I(1). (PSS Table CI(iii).)

[f] 10% critical values for PSS Case IV are: 2.97 if all variables are I(0), 3.74 if all variables are I(1). (PSS Table CI(iv).)

[g] 10% critical values are PSS Case III: -2.57 if all variables are I(0), -3.46 if all variables are I(1). (PSS Table CII(iii).)

Table 4: ADL/ECM for M2 under Loglinear Money Demand

	ADL/ECM(1,4,0,3)	
Equilibrium error: $p_t + \lambda_m m_t + \lambda_y y_t + \alpha - \eta \log r_t + \mu t$	Case III $\mu = 0$	Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient,[a] $\psi = \phi_1 - 1$	-0.0899	-0.0881
	(0.0250)	(0.0252)
interest elasticity,[b] $-\eta$	-0.0643	-0.0864
	(0.0271)	(0.0486)
time trend, μ		-0.0147
		(0.0251)
transactions elasticity, λ_y	0.6955	0.9027
	(0.3700)	(0.5010)
$-\lambda_m$	0.8515	0.7411
	(0.1853)	(0.2747)
Wald tests of homogeneity restrictions (p-values):		
(1) Unitary money elasticity: $\lambda_m = -1$	0.423	0.346
(2) Unitary income elasticity: $\lambda_y = 1$	0.410	0.846
(3) y and $-m$ share common coefficient: $-\lambda_m = \lambda_y$	0.403	0.773
(4) Given (3), common coefficient is 1	0.853	0.353
(5) Joint test of (2) and (3): $-\lambda_m = \lambda_y = 1$	0.703	0.634
Autoregressive coefficient of ADL parameterization:		
ϕ_1	0.9101	0.9119
	(0.0250)	(0.0252)
LM test for residual autocorrelation[c] (p-value)	0.114	0.123
PSS tests of null of no levels relationship:		
F statistic	6.7614[d]	5.4431[e]
t statistic	-3.5976[f]	

Notes: Standard errors are in parentheses.

[a] Convergence to equilibrium requires $-1 < \psi < 0$ or, equivalently, $|\phi_1| < 1$.

[b] For comparison, the Baumol-Tobin square-root rule for money holding predicts an interest elasticity of -0.5.

[c] Reported tests are based on 3 lags, but the test decisions (of no rejection of the null of absence of residual autocorrelation) are generally robust to variations on the lag length.

[d] 10% critical values for PSS Case III are: 2.72 if all variables are I(0), 3.77 if all variables are I(1). (PSS Table CI(iii).)

[e] 10% critical values for PSS Case IV are: 2.97 if all variables are I(0), 3.74 if all variables are I(1). (PSS Table CI(iv).)

[f] 10% critical values are PSS Case III: -2.57 if all variables are I(0), -3.46 if all variables are I(1). (PSS Table CII(iii).)

Table 5: ADL/ECM for M1 under Semi-log Money Demand

	ADL/ECM(1,1,2,2)	
Equilibrium error: $p_t + \lambda_m m_t + \lambda_y y_t + \beta - \xi r_t + \mu t$	Case III $\mu = 0$	Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient,[a] $\psi = \phi_1 - 1$	−0.0466 (0.0194)	−0.0271 (0.0222)
interest semielasticity,[b] $-\xi$	−11.7857 (4.3447)	−12.7971 (7.9728)
time trend, μ		0.0920 (0.1115)
transactions elasticity, λ_y	0.1446 (0.6875)	−2.1742 (3.1687)
$-\lambda_m$	0.6652 (0.4457)	0.8824 (0.7729)
Wald tests of homogeneity restrictions (p-values):		
(1) Unitary money elasticity: $\lambda_m = -1$	0.452	0.879
(2) Unitary income elasticity: $\lambda_y = 1$	0.213	0.316
(3) y and $-m$ share common coefficient: $-\lambda_m = \lambda_y$	0.037	0.333
(4) Given (3), common coefficient is 1	0.016	0.867
(5) Joint test of (2) and (3): $-\lambda_m = \lambda_y = 1$	0.000	0.605
Autoregressive coefficients of ADL parameterization:		
ϕ_1	0.9534 (0.0194)	0.9729 (0.0222)
LM test for residual autocorrelation[c] (p-value)	0.167	0.262
PSS tests of null of no levels relationship:		
F statistic	3.8985[d]	3.7966[e]
t statistic	−2.4083[f]	

Notes: Standard errors are in parentheses.

[a] Convergence to equilibrium requires $-1 < \psi < 0$ or, equivalently, $|\phi_1| < 1$.

[b] For comparison, Ball (2001) reports a U.S. interest semielasticity of approximately −0.05 when the T-bill rate is expressed as a percent, which translates into $-\xi = -5$.

[c] Reported tests are based on 3 lags, but the test decisions (of no rejection of the null of absence of residual autocorrelation) are robust to variations on the lag length.

[d] 10% critical values for PSS Case III are: 2.72 if all variables are I(0), 3.77 if all variables are I(1). (PSS Table CI(iii).)

[e] 10% critical values for PSS Case IV are: 2.97 if all variables are I(0), 3.74 if all variables are I(1). (PSS Table CI(iv).)

[f] 10% critical values are PSS Case III: −2.57 if all variables are I(0), −3.46 if all variables are I(1). (PSS Table CII(iii).)

Table 6: ADL/ECM for NewM1 under Semi-log Money Demand

| | ADL/ECM(2,1,0,1) | |
Equilibrium error: $p_t + \lambda_m m_t + \lambda_y y_t + \beta - \xi r_t + \mu t$	Case III $\mu = 0$	Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient[a], $\psi = \phi_1 + \phi_2 - 1$	−0.0952	−0.0935
	(0.0331)	(0.0340)
interest semielasticity[b], $-\xi$	−7.4449	−7.4194
	(1.5486)	(1.5852)
time trend, μ		−0.0035
		(0.0149)
transactions elasticity, λ_y	0.1936	0.1210
	(0.2288)	(0.3911)
$-\lambda_m$	0.5767	0.5925
	(0.1233)	(0.1410)
Wald tests of homogeneity restrictions (p-values):		
(1) Unitary money elasticity: $\lambda_m = -1$	0.001	0.004
(2) Unitary income elasticity: $\lambda_y = 1$	0.000	0.025
(3) y and $-m$ share common coefficient: $-\lambda_m = \lambda_y$	0.000	0.230
(4) Given (3), common coefficient is 1	0.680	0.001
(5) Joint test of (2) and (3): $-\lambda_m = \lambda_y = 1$	0.002	0.003
Autoregressive coefficients of ADL parameterization:[c]		
ϕ_1	1.2081	1.2040
	(0.0920)	(0.0940)
ϕ_2	−0.3033	−0.2976
	(0.0802)	(0.0840)
LM test for residual autocorrelation[d] (p-value)	0.607	0.593
PSS tests of null of no levels relationship:		
F statistic	2.7953[e]	2.2251[f]
t statistic	−2.8715[g]	

Notes: Standard errors are in parentheses.

[a] Convergence to equilibrium requires $-1 < \psi < 0$.

[b] For comparison, Ball (2001) reports a U.S. interest semielasticity of approximately -0.05 when the T-bill rate is expressed as a percent, which translates into $-\xi = -5$.

[c] Dynamic stability requires $\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$, and $|\phi_2| < 1$. These restrictions are satisfied in all cases.

[d] Reported tests are based on 3 lags, but the test decisions (of no rejection of the null of absence of residual autocorrelation) are generally robust to variations on the lag length.

[e] 10% critical values for PSS Case III are: 2.72 if all variables are I(0), 3.77 if all variables are I(1). (PSS Table CI(iii).)

[f] 10% critical values for PSS Case IV are: 2.97 if all variables are I(0), 3.74 if all variables are I(1). (PSS Table CI(iv).)

[g] 10% critical values are PSS Case III: -2.57 if all variables are I(0), -3.46 if all variables are I(1). (PSS Table CII(iii).)

Table 7: ADL/ECM for M2 under Semi-log Money Demand

	ADL/ECM(1,1,0,1)	
Equilibrium error: $p_t + \lambda_m m_t + \lambda_y y_t + \beta - \xi r_t + \mu t$	Case III $\mu = 0$	Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient,[a] $\psi = \phi_1 - 1$	-0.0686	-0.0707
	(0.0289)	(0.0294)
interest semielasticity,[b] $-\xi$	-4.9746	-4.1873
	(2.6320)	(2.9044)
time trend, μ		0.0100
		(0.0221)
transactions elasticity, λ_y	1.5901	1.4639
	(0.5387)	(0.5797)
$-\lambda_m$	1.2753	1.3584
	(0.2791)	(0.3386)
Wald tests of homogeneity restrictions (p-values):		
(1) Unitary money elasticity: $\lambda_m = -1$	0.324	0.290
(2) Unitary income elasticity: $\lambda_y = 1$	0.273	0.424
(3) y and $-m$ share common coefficient: $-\lambda_m = \lambda_y$	0.230	0.838
(4) Given (3), common coefficient is 1	0.406	0.267
(5) Joint test of (2) and (3): $-\lambda_m = \lambda_y = 1$	0.285	0.538
Autoregressive coefficient of ADL parameterization:		
ϕ_1	0.9314	0.9293
	(0.0289)	(0.0294)
LM test for residual autocorrelation[c] (p-value)	0.114	0.114
PSS tests of null of no levels relationship:		
F statistic	3.7110[d]	2.9831[e]
t statistic	-2.3709[f]	

Notes: Standard errors are in parentheses.

[a] Convergence to equilibrium requires $-1 < \psi < 0$ or, equivalently, $|\phi_1| < 1$.

[b] For comparison, Ball (2001) reports a U.S. interest semielasticity of approximately -0.05 when the T-bill rate is expressed as a percent, which translates into $-\xi = -5$.

[c] Reported tests are based on 3 lags, but the test decisions (of no rejection of the null of absence of residual autocorrelation) are robust to variations on the lag length.

[d] 10% critical values for PSS Case III are: 2.72 if all variables are I(0), 3.77 if all variables are I(1). (PSS Table CI(iii).)

[e] 10% critical values for PSS Case IV are: 2.97 if all variables are I(0), 3.74 if all variables are I(1). (PSS Table CI(iv).)

[f] 10% critical values are PSS Case III: -2.57 if all variables are I(0), -3.46 if all variables are I(1). (PSS Table CII(iii).)

Table 8: ADL/ECM for M1 under Selden-Latané Money Demand

Equilibrium error: $p_t + \lambda_m m_t + \lambda_y y_t - \log(C + \delta r_t + \mu t)$	ADL/ECM(1,1,2,2)	
	Case III $\mu = 0$	Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient,[a] $\psi = \phi_1 - 1$	−0.0482	−0.0283
	(0.0188)	(0.0143)
interest parameter, δ	1.9130	8.5669
	(6.2936)	(53.7848)
time trend, μ		−0.0034
		(0.0202)
transactions elasticity, λ_y	0.4602	0.9445
	(0.6659)	(1.2810)
$-\lambda_m$	0.8689	1.4113
	(0.4328)	(0.8760)
Wald tests of homogeneity restrictions (p-values):		
(1) Unitary money elasticity: $\lambda_m = -1$	0.762	0.639
(2) Unitary income elasticity: $\lambda_y = 1$	0.418	0.965
(3) y and $-m$ share common coefficient: $-\lambda_m = \lambda_y$	0.089	0.303
(4) Given (3), common coefficient is 1	0.006	0.010
(5) Joint test of (2) and (3): $-\lambda_m = \lambda_y = 1$	0.000	0.029
Autoregressive coefficients of ADL parameterization:		
ϕ_1	0.9518	0.9717
	(0.0188)	(0.0143)
LM test for residual autocorrelation[b] (p-value)	0.223	0.270
PSS tests of null of no levels relationship:[c]		
F statistic	21.3241	16.8004
t statistic	−2.5687	

Notes: Standard errors are in parentheses.

[a] Convergence to equilibrium requires $-1 < \psi < 0$ or, equivalently, $|\phi_1| < 1$.

[b] Reported tests are based on 3 lags, but the test decisions (of no rejection of the null of absence of residual autocorrelation) are generally robust to variations on the lag length.

[c] Reported for comparison with the other money demand specifications. Because the Selden-Latané ECM is estimated nonlinearly, these test statistics may not have the distributions tabulated by PSS.

Table 9: ADL/ECM for NewM1 under Selden-Latané Money Demand

| | ADL/ECM(2,1,2,2) | |
Equilibrium error: $p_t + \lambda_m m_t + \lambda_y y_t - \log(C + \delta r_t + \mu t)$	Case III $\mu = 0$	Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient,[a] $\psi = \phi_1 + \phi_2 - 1$	-0.1056	-0.1013
	(0.0390)	(0.0426)
interest parameter, δ	0.5563	0.4112
	(0.7227)	(0.7182)
time trend, μ		-0.0001
		(0.0003)
transactions elasticity, λ_y	0.2392	0.2170
	(0.2289)	(0.2544)
$-\lambda_m$	0.5992	0.6196
	(0.1247)	(0.1484)
Wald tests of homogeneity restrictions (p-values):		
(1) Unitary money elasticity: $\lambda_m = -1$	0.001	0.010
(2) Unitary income elasticity: $\lambda_y = 1$	0.001	0.002
(3) y and $-m$ share common coefficient: $-\lambda_m = \lambda_y$	0.001	0.031
(4) Given (3), common coefficient is 1	0.707	0.016
(5) Joint test of (2) and (3): $-\lambda_m = \lambda_y = 1$	0.003	0.007
Autoregressive coefficients of ADL parameterization:[b]		
ϕ_1	1.1758	1.1794
	(0.0956)	(0.0972)
ϕ_2	-0.2814	-0.2807
	(0.0834)	(0.0837)
LM test for residual autocorrelation[c] (p-value)	0.410	0.346
PSS tests of null of no levels relationship:[d]		
F statistic	14.5217	11.8452
t statistic	-2.7107	

Notes: Standard errors are in parentheses.

[a] Convergence to equilibrium requires $-1 < \psi < 0$.

[b] The dynamic stability conditions $\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$, and $|\phi_2| < 1$ are satisfied.

[c] Reported tests are based on 3 lags, but the test decisions (of no rejection of the null of absence of residual autocorrelation) are generally robust to variations on the lag length.

[d] Reported for comparison with the other money demand specifications. Because the Selden-Latané ECM is estimated nonlinearly, these test statistics may not have the distributions tabulated by PSS.

Table 10: ADL/ECM for M2 under Selden-Latané Money Demand

| | ADL/ECM(2,1,1,1) | |
Equilibrium error: $p_t + \lambda_m m_t + \lambda_y y_t - \log(C + \delta r_t + \mu t)$	Case III $\mu = 0$	Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient,[a] $\psi = \phi_1 + \phi_2 - 1$	-0.0677	-0.0644
	(0.0292)	(0.0309)
interest parameter, δ	45.0618	35.1352
	(118.6980)	(100.1050)
time trend, μ		-0.0453
		(0.2126)
transactions elasticity, λ_y	1.3696	1.1491
	(0.5189)	(0.6689)
$-\lambda_m$	1.1669	1.2620
	(0.2681)	(0.5263)
Wald tests of homogeneity restrictions (p-values):		
(1) Unitary money elasticity: $\lambda_m = -1$	0.534	0.619
(2) Unitary income elasticity: $\lambda_y = 1$	0.476	0.531
(3) y and $-m$ share common coefficient: $-\lambda_m = \lambda_y$	0.424	0.567
(4) Given (3), common coefficient is 1	0.480	0.792
(5) Joint test of (2) and (3): $-\lambda_m = \lambda_y = 1$	0.516	0.804
Autoregressive coefficients of ADL parameterization:[b]		
ϕ_1	1.1436	1.1418
	(0.0882)	(0.0887)
ϕ_2	-0.2113	-0.2062
	(0.0852)	(0.0854)
LM test for residual autocorrelation[c] (p-value)	0.381	0.301
PSS tests of null of no levels relationship:[d]		
F statistic	4.5096	4.1520
t statistic	-2.3204	

Notes: Standard errors are in parentheses.

[a] Convergence to equilibrium requires $-1 < \psi < 0$.

[b] The dynamic stability conditions $\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$, and $|\phi_2| < 1$ are satisfied.

[c] Reported tests are based on 3 lags, but the test decisions (of no rejection of the null of absence of residual autocorrelation) are generally robust to variations on the lag length.

[d] Reported for comparison with the other money demand specifications. Because the Selden-Latané ECM is estimated nonlinearly, these test statistics may not have the distributions tabulated by PSS.

Table 11: The Welfare Cost of Inflation: M1 under Loglinear Money Demand

	ADL/ECM(2,4,0,3)
Restricted equilibrium error imposes $-\lambda_m = \lambda_y = 1$:	Case IV
$p_t - m_t + y_t + \log A - \eta \log r_t + \mu t$	$\mu \neq 0$
Parameters of the error correction term:	
speed-of-adjustment coefficient, $\psi = \phi_1 + \phi_2 - 1$	-0.0710
	(0.0193)
interest elasticity,[a] $-\eta$	-0.1990
	(0.0417)
intercept parameter,[b] A	0.2076
	(0.0349)
time trend, μ	-0.0149
	(0.0017)
Welfare cost $w(r)$ (percent of income)	
at $r = 0.03$	0.311
at $r = 0.05$	0.468
at $r = 0.06$	0.542
at $r = 0.13$	1.006
at $r = 0.14$	1.068

Notes: Standard errors are in parentheses. No results are reported for the no trend ($\mu = 0$) Case III because the imposition of $-\lambda_m = \lambda_y = 1$ yields estimates of the remaining parameters that are imprecise to the point of being meaningless. This finding is insensitive to alternative lag length specifications.

[a] For comparison, the Baumol-Tobin square-root rule for money holding predicts an interest elasticity of -0.5. Lucas (2000, Fig. 2) found that this value provided a good fit to U.S. M1, 1900–1994, for this money demand specification.

[b] Obtained by nonlinear least squares estimation of the ECM. For comparison, Lucas (2000) used the calibrated value $A = 0.0488$ and (in his Sections 4 and 5) a theoretical benchmark of $A = 0.05$.

Table 12: The Welfare Cost of Inflation: NewM1 under Loglinear Money Demand

	ADL/ECM(2,1,0,0)	
Restricted equilibrium error imposes $-\lambda_m = \lambda_y = 1$: $p_t - m_t + y_t + \log A - \eta \log r_t + \mu t$	Case III $\mu = 0$	Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient, $\psi = \phi_1 + \phi_2 - 1$	-0.0286	-0.0326
	(0.0229)	(0.0247)
interest elasticity,[a] $-\eta$	-0.2883	-0.2698
	(0.1412)	(0.1208)
intercept parameter,[b] A	0.0791	0.0943
	(0.0512)	(0.0562)
time trend, μ		-0.0019
		(0.0041)
Welfare cost $w(r)$ (percent of income)[c]		
at $r = 0.03$	0.264	0.269
at $r = 0.05$	0.380	0.391
at $r = 0.06$	0.433	0.447
at $r = 0.13$	0.750	0.786
at $r = 0.14$	0.791	0.829

Notes: Standard errors are in parentheses.

[a] For comparison, the Baumol-Tobin square-root rule for money holding predicts an interest elasticity of -0.5. Lucas (2000, Fig. 2) found that this value provided a good fit to U.S. M1, 1900–1994, for this money demand specification.

[b] Obtained by nonlinear least squares estimation of the ECM. For comparison, Lucas (2000) used the calibrated value $A = 0.0488$ and (in his Sections 4 and 5) a theoretical benchmark of $A = 0.05$.

[c] For comparison BLNW (2017, p. 26) find, for US welfare costs based on NewM1 and loglinear money demand, a median estimate of 0.18 percent.

Table 13: The Welfare Cost of Inflation: M2 under Loglinear Money Demand

	ADL/ECM(1,4,0,3)	
Restricted equilibrium error imposes $-\lambda_m = \lambda_y = 1$:	Case III	Case IV
$p_t - m_t + y_t + \log A - \eta \log r_t + \mu t$	$\mu = 0$	$\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient, $\psi = \phi_1 - 1$	-0.0969	-0.0926
	(0.0216)	(0.0247)
interest elasticity[a], $-\eta$	-0.0602	-0.0615
	(0.0241)	(0.0257)
intercept parameter[b], A	0.4712	0.4815
	(0.0455)	(0.0571)
time trend, μ		-0.0004
		(0.0013)
Welfare cost $w(r)$ (percent of income)		
at $r = 0.03$	0.112	0.117
at $r = 0.05$	0.181	0.190
at $r = 0.06$	0.215	0.225
at $r = 0.13$	0.444	0.465
at $r = 0.14$	0.476	0.498

Notes: Standard errors are in parentheses.

[a] For comparison, the Baumol-Tobin square-root rule for money holding predicts an interest elasticity of -0.5. Lucas (2000, Fig. 2) found that this value provided a good fit to U.S. M1, 1900–1994, for this money demand specification.

[b] Obtained by nonlinear least squares estimation of the ECM. For comparison, Lucas (2000) used the calibrated value $A = 0.0488$ and (in his Sections 4 and 5) a theoretical benchmark of $A = 0.05$.

Table 14: The Welfare Cost of Inflation: M1 under Semi-log Money Demand

	ADL/ECM(1,1,2,2)	
Restricted equilibrium error imposes $-\lambda_m = \lambda_y = 1$: $p_t - m_t + y_t + \log B - \xi r + \mu t$	Case III $\mu = 0$	Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient, $\psi = \phi_1 - 1$	-0.0087	-0.0409
	(0.0096)	(0.0208)
interest semielasticity,[a] $-\xi$	-51.9204	-14.6257
	(50.8506)	(5.7523)
intercept parameter,[b] B	0.5490	0.5293
	(0.5928)	(0.1179)
time trend, μ		-0.0114
		(0.0030)
Welfare cost $w(r)$ (percent of income)		
at $r = 0.03$	0.488	0.261
at $r = 0.05$	0.774	0.603
at $r = 0.06$	0.864	0.794
at $r = 0.13$	1.048	2.051
at $r = 0.14$	1.051	2.196

Notes: Standard errors are in parentheses.

[a] For comparison, Lucas (2000, Fig. 3) found that the value $\xi = 7$ provided a good fit to U.S. money demand data, 1900–1994.

[b] Obtained by nonlinear least squares estimation of the ECM. For comparison, Lucas (2000) calibrates B to a value of 0.3548.

Table 15: The Welfare Cost of Inflation: NewM1 under Semi-log Money Demand

	ADL/ECM(2,1,0,1)	
Restricted equilibrium error imposes $-\lambda_m = \lambda_y = 1$:	Case III	Case IV
$p_t - m_t + y_t + \log B - \xi r + \mu t$	$\mu = 0$	$\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient, $\psi = \phi_1 + \phi_2 - 1$	-0.0328	-0.0334
	(0.0249)	(0.0255)
interest semielasticity[a], $-\xi$	-13.5577	-13.3747
	(5.3480)	(5.3870)
intercept parameter[b], B	0.3965	0.4034
	(0.0813)	(0.1007)
time trend, μ		-0.0005
		(0.0037)
Welfare cost $w(r)$[c](percent of income)		
at $r = 0.03$	0.185	0.187
at $r = 0.05$	0.433	0.437
at $r = 0.06$	0.573	0.579
at $r = 0.13$	1.538	1.565
at $r = 0.14$	1.654	1.684

Notes: Standard errors are in parentheses.

[a] In comparison, Lucas (2000, Fig. 3) found that the value $\xi = 7$ provided a good fit to U.S. money demand data, 1900–1994.

[b] Obtained by nonlinear least squares estimation of the ECM. For comparison, Lucas (2000) calibrates B to a value of 0.3548.

[c] In comparison BLNW (2017, p. 26) find, for US welfare costs based on NewM1 and semi-log money demand, a median estimate of 0.18 percent.

Table 16: The Welfare Cost of Inflation: M2 under Semi-log Money Demand

	ADL/ECM(1,1,0,1)	
Restricted equilibrium error imposes $-\lambda_m = \lambda_y = 1$:	Case III	Case IV
$p_t - m_t + y_t + \log B - \xi r + \mu t$	$\mu = 0$	$\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient, $\psi = \phi_1 - 1$	-0.0515	-0.0660
	(0.0248)	(0.0290)
interest semielasticity,[a] $-\xi$	-6.4891	-5.4388
	(3.9083)	(2.8746)
intercept parameter,[b] B	0.7679	0.6513
	(0.1627)	(0.1183)
time trend, μ		0.0020
		(0.0018)
Welfare cost $w(r)$ (percent of income)		
at $r = 0.03$	0.197	0.143
at $r = 0.05$	0.503	0.370
at $r = 0.06$	0.695	0.514
at $r = 0.13$	2.449	1.895
at $r = 0.14$	2.729	2.125

Notes: Standard errors are in parentheses.

[a] For comparison, Lucas (2000, Fig. 3) found that the value $\xi = 7$ provided a good fit to U.S. money demand data, 1900–1994.

[b] Obtained by nonlinear least squares estimation of the ECM. For comparison, Lucas (2000) calibrates B to a value of 0.3548.

Table 17: The Welfare Cost of Inflation: M1 under Selden-Latané Money Demand

	ADL/ECM(1,1,2,2)	
Restricted equilibrium error imposes $-\lambda_m = \lambda_y = 1$: $p_t - m_t + y_t - \log(C + \delta r_t + \mu t)$	Case III $\mu = 0$	Case IV $\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient, $\psi = \phi_1 - 1$	-0.0115	-0.0112
	(0.0064)	(0.0066)
interest parameter, δ	345.5170	355.8270
	(241.8990)	(268.3860)
intercept parameter,[a] C	0.2677	0.3650
	(0.8875)	(1.2539)
time trend, μ		-0.0018
		(0.0122)
Welfare cost $w(r)$ (percent of income)		
at $r = 0.03$	0.784	0.686
at $r = 0.05$	0.926	0.823
at $r = 0.06$	0.977	0.872
at $r = 0.13$	1.197	1.084
at $r = 0.14$	1.218	1.105

Notes: Standard errors are in parentheses.

[a] Obtained by nonlinear least squares estimation of the ECM.

Table 18: The Welfare Cost of Inflation: NewM1 under Selden-Latané Money Demand

	ADL/ECM(2,1,2,2)	
Restricted equilibrium error imposes $-\lambda_m = \lambda_y = 1$:	Case III	Case IV
$p_t - m_t + y_t - \log(C + \delta r_t + \mu t)$	$\mu = 0$	$\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient, $\psi = \phi_1 + \phi_2 - 1$	-0.0342	-0.0293
	(0.0295)	(0.0293)
interest parameter, δ	58.6959	63.9576
	(26.9121)	(35.4307)
intercept parameter[a], C	2.2659	2.4874
	(0.5893)	(0.9836)
time trend, μ		-0.0058
		(0.0153)
Welfare cost $w(r)$[b] (percent of income)		
at $r = 0.03$	0.235	0.213
at $r = 0.05$	0.454	0.413
at $r = 0.06$	0.561	0.511
at $r = 0.13$	1.198	1.093
at $r = 0.14$	1.274	1.162

Notes: Standard errors are in parentheses.

[a] Obtained by nonlinear least squares estimation of the ECM.

[b] In comparison BLNW (2017, p. 26) find, for US welfare costs based on NewM1, "...median estimates equal to 0.21 per cent for the Selden-Latané specification ..."

Table 19: The Welfare Cost of Inflation: M2 under Selden-Latané Money Demand

	ADL/ECM(2,1,1,1)	
Restricted equilibrium error imposes $-\lambda_m = \lambda_y = 1$:	Case III	Case IV
$p_t - m_t + y_t - \log(C + \delta r_t + \mu t)$	$\mu = 0$	$\mu \neq 0$
Parameters of the error correction term:		
speed-of-adjustment coefficient, $\psi = \phi_1 + \phi_2 - 1$	-0.0537	-0.0641
	(0.0245)	(0.0284)
interest parameter, δ	7.1668	6.6609
	(4.9685)	(4.1593)
intercept parameter,[a] C	1.4195	1.6001
	(0.2524)	(0.2797)
time trend, μ		-0.0026
		(0.0029)
Welfare cost $w(r)$ (percent of income)		
at $r = 0.03$	0.132	0.100
at $r = 0.05$	0.328	0.252
at $r = 0.06$	0.448	0.347
at $r = 0.13$	1.512	1.222
at $r = 0.14$	1.682	1.366

Notes: Standard errors are in parentheses.

[a] Obtained by nonlinear least squares estimation of the ECM.

Table 20: The Welfare Cost of Inflation (percent of income)

| Illustrative interest rates (percent) | M2 1915–2016 | | | Comparator studies[a] | | |
| | Loglinear | Semi-log | Selden-Latané | Lucas (2000) M1 1900–1994[b] | | Ireland (2009) M1 1980:I–2006:IV |
				Loglinear	Semi-log	Semi-log
3	0.112	0.197	0.132	0.845[c]	0.097[c]	0.013[e]
5	0.181	0.503	0.328	1.091[c]	0.247[c]	0.036[e]
6	0.215	0.695	0.448	1.195[d]	0.340	0.051
13	0.444	2.449	1.512	1.760	1.172	0.219[e]
14	0.476	2.729	1.682	1.826	1.302	0.251
interest parameter:	$-\eta = -0.0602$	$-\xi = -6.4891$	$\delta = 7.1668$	$-\eta = -0.5$	$-\xi = -7$	$-\xi = -1.7944$
intercept parameter:	$A = 0.4712$	$B = 0.7679$	$C = 1.4195$	$A = 0.0488$	$B = 0.3548$	$B = 0.1686$

[a] Another comparator study is BLNW (2017), who use NewM1 1915–2016 and find (p. 26) "…welfare losses at the average short rate that has prevailed over the sample period …equal to 0.21 per cent for the Selden-Latané specification, and 0.18 for the other two."

[b] The intercept values $A = 0.0488$ and $B = 0.3548$ do not actually appear in Lucas (2000) but are given in Ireland (2009, p. 1040).

[c] Using Lucas's parameter values, Ireland (2009, p. 1041) remarks that if "…the steady-state real interest rate equals 3 percent, so that $r = 0.03$ prevails under a policy of zero inflation or price stability, then …this policy costs the economy the equivalent of 0.85 percent of income when money demand is log-log, but only 0.10 percent of income when money demand has the semi-log form. Likewise, an ongoing 2 percent inflation [so $r = 0.05$] costs the economy 1.09 percent of income under [a loglinear form], but only 0.25 percent of income under [a semi-log form]."

[d] "At a six percent interest rate, for example, the log-log curve implies a welfare cost of about one percent of income …" (Lucas 2000, p. 251).

[e] From Table 6 of Ireland (2009, p. 1050) based on static OLS estimation of semi-log money demand. Ireland also presents welfare costs based on dynamic OLS, but the values are similar.

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Table 21: Structural Change: M2 under Loglinear Money Demand

Equilibrium error: $p_t + \lambda_m m_t + \lambda_y y_t + \log A - \eta \log r_t$	Homogeneity restrictions $-\lambda_m = \lambda_y = 1$	
	under test	imposed
Parameters of the error correction term:		
speed-of-adjustment coefficient, $\psi = \phi_1 - 1$	-0.0702	-0.0541
	(0.0261)	(0.0226)
interest elasticity, $-\eta$	-0.1295	-0.1572
	(0.0467)	(0.0650)
intercept parameter, A		0.3728
		(0.0577)
transactions elasticity, λ_y	0.6957	1
	(0.3753)	
$-\lambda_m$	0.8822	-1
	(0.1837)	
Wald tests of homogeneity restrictions (p-values):		
(1) Unitary money elasticity: $\lambda_m = -1$	0.5212	
(2) Unitary income elasticity: $\lambda_y = 1$	0.4175	
(3) y and $-m$ share common coefficient: $-\lambda_m = \lambda_y$	0.3380	
(4) Given (3), common coefficient is 1	0.3066	
(5) Joint test of (2) and (3): $-\lambda_m = \lambda_y = 1$	0.3259	
Structural change coefficients:[a]		
Impulse indicators		
1920	0.1086	0.1032
1921	-0.1075	-0.1141
Step indicators		
1931	-0.0696	-0.0666
1933	0.0993	0.1011
1946	0.0638	0.0676
1948	-0.0894	-0.0824
Welfare cost $w(r)$ (percent of income)		
at $r = 0.03$		0.3620
at $r = 0.05$		0.5568
at $r = 0.06$		0.6250
at $r = 0.13$		1.2446
at $r = 0.14$		1.3260

Notes: Standard errors are in parentheses.

[a] Impulse and step indicators found to be statistically significant using the default EViews settings for the indicator saturation algorithm: optimally-determined chronological blocking and the Schwarz information criterion.