Robust specification testing in regression: the FRESET test and autocorrelated disturbances

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Revised, May 1998

Abstract

We consider the robustness of traditional and new versions of the RESET test to AR(1) or MA(1) regression errors. The new FRESET tests of DeBenedictis and Giles (1988) out-perform the usual RESET test.

***Keywords* :** RESET test; Regression specification; Non-independent errors

***JEL classification* :** C12; C52

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1. Introduction

Testing a regression model's specification is an important task in econometrics. Among the many tests that are used, Ramsey's (1969) RESET test of the model's conditional mean (and hence of the chosen regressors and functional form) remains popular. Generally, this "variable addition" test is applied via a locally valid, low-order polynomial approximation to the conditional mean. It is well known (*e.g.*, Ramsey and Schmidt, 1976; Thursby and Schmidt, 1977) that the power of the RESET test depends on the form of this approximation.

Recently, DeBenedictis and Giles (1988) show that this traditional RESET test can have very low power – even less than its size, so it is a "biased" test. They propose a modified RESET test using a globally valid Fourier approximation to the model's conditional mean, rather than a Taylor series (polynomial) approximation. This new "FRESET" test is apparently unbiased, and generally much more powerful than the RESET test. Contrary to earlier suggestions (*e.g.*, Thursby, 1979, 1982), we know (*e.g.*, Pagan, 1984; Porter and Kashyap, 1984; Godfrey, 1988) that the RESET test is sensitive to non-independent regression errors. This is also true for the FRESET test. This paper studies the effects of autoregressive and moving-average regression errors on the properties of the FRESET and RESET tests. We also consider the Newey-West (1987) robust covariance estimator in the construction of these tests.

The next section discusses the RESET and FRESET; a Monte Carlo experiment, and its results, are discussed in section 3; and the last section provides conclusions and recommendations in favour of the FRESET test over the RESET test when the regression errors are autocorrelated.

2. The RESET and FRESET tests

The usual RESET test involves augmenting the regression with powers of the OLS predictions of the original specification, and testing the joint significance of these terms. The model of interest is

$$y = X\beta + \varepsilon \quad , \qquad (1)$$

where X is (T x k), of rank k, and ε is a Normal, zero-mean disturbance. If (1) is mis-specified so that E [ε | X] = ξ $\neq 0$, the RESET test approximates ξ by Zθ, and tests H_0: $\theta = 0$ in the model

$$y = X\beta + Z\theta + u \ . \tag{2}$$

If the ε_t's are independent, X is non-stochastic, and Z is random only by being a function of the OLS estimator, b, of β, then by Milliken-Graybill (1970), the RESET statistic is exactly F-distributed under H_0. Generally, Z has t'th row vector given by $Z_t = [(Xb)_t^2, (Xb)_t^3, \ldots, (Xb)_t^{p+1}]$, so the F-statistic has p and (T-k-p) degrees of freedom. Following Ramsey and Gilbert (1972) and Thursby (1989), p = 1, 2, 3 is common, and p = 3 is supported by DeBenedictis and Giles (1998). As Z is random, the distribution of the RESET test is non-standard under the alternative. If X is random (*e.g.*, if (1) is dynamic) and/or the errors are autocorrelated, the RESET statistic will *not* be F under the null, though scaling it by its numerator degrees of freedom will generally make it asymptotically chi-square.

Gallant (1981) notes that Taylor approximations have only *local* validity, but Fourier approximations have *global* validity, so DeBenedictis and Giles (1998) approximate ξ with a Fourier expansion, defining $Z_t = [\sin(w_t), \cos(w_t), \sin(2w_t), \cos(2w_t), \ldots, \sin(p'w_t), \cos(p'w_t)]$, for some p', where $(Xb)_t$ is transformed to w_t in [$-\pi$, $+\pi$]. A sinusoidal transformation (Box, 1966), $w_t = 2\pi \sin^2[(Xb)_t] - \pi$, defines the FRESETS test. A linear transformation (Mitchell and Onvural, 1996), $w_t = \pi\{2(Xb)_t - [(Xb)_{max} - (Xb)_{min}]\}/\{(Xb)_{max} - (Xb)_{min}\}$, defines the FRESETL test. DeBenedictis and Giles (1998, 398-400) detail the Fourier expansion, and some refinements. If the Milliken-Graybill conditions hold, the FRESET statistics are F with 2p' and (T-k-2p') degrees of freedom under the null.

3. Monte Carlo Experiment

Monte Carlo simulation is used to examine the robustness of the RESET and FRESET tests to non-independent errors. We consider various mis-specifications through variable omission, and allow ε in (1) to be either AR(1) or MA(1). The data-generating process (DGP) involves either $\varepsilon_t = \rho\varepsilon_{t-1} + \nu_t$ or $\varepsilon_t = \nu_t + \lambda\nu_{t-1}$, where ν_t is white noise. Given the results of DeBenedictis and Giles (1998), we set p = p' = 3 and we consider T = 20 and 50. Table 1 shows the DGP's and corresponding fitted models, these being mis-specified if $\gamma \neq 0$. These models correspond to Model 1 of DeBenedictis and Giles (1998), and Models 6 to 8 of Thursby and Schmidt (1997, 638). The variables x_2 , x_3 and x_4 are from Ramsey and Gilbert (1972) and Thursby and Schmidt (1977). Values of 0, ±0.3, ±0.6, ±0.9 are considered for ρ and λ, and values of γ in [-8.0, +8.0] generate different degrees of model mis-specification. Nominal significance levels of 5% and 10% are considered, but unless the errors are independent, the tests will exhibit size-distortion. Also considered are the probabilities that each test rejects the null of interest. We term these probabilities "powers", though strictly the power is the rejection probability when $\theta \neq 0$. Unless $\xi = Z\theta$ (*e.g.*, if just x_2 were used instead of prediction powers in Model 1), these concepts differ, and our "power curves" have non-standard shapes. The simulations involve 5,000 replications with SHAZAM (1993) code on a DEC Alpha3000/400.

Naturally, the "powers" increase with nominal significance level, and slightly with T. The result patterns are insensitive to these choices. Illustrative rejection proportions for the null hypotheses appear in Tables 2 to 6. The Newey-West correction increases the true significance levels and "powers" of the tests, often generating substantial size-distortions. Generally, RESET exhibits the least, and FRESETL the most, size-distortion. For the FRESET tests the direction of this distortion opposes the sign of the autocorrelation.

Generally, "powers" decrease with increasing autocorrelation, and the "gains" from the Newey-West correction typically reflect the associated positive size-distortions. The FRESET tests perform well, without such a correction, especially in the important case of positive AR(1) errors. This can be seen in Tables 3 and 4, especially for FRESETL, and particularly when the relative size-distortions are noted. There is strong support for the FRESETS test under MA(1) contamination, as we see in Tables 5 and 6.

4. Conclusions

Strictly, our results relate to situations of *joint* model mis-specifications, so care must be taken in interpreting notions such as "size-distortion" here. Other forms of joint mis-specification are under study with the FRESET tests, including non-normality, heteroskedasticity, and non-stationarity of the regressors. Giles and DeBenedictis (1998) show that the global approximation underlying the FRESET tests leads to substantial improvements over the traditional RESET test for regression mis-specification. We have shown that the merits of the FRESET tests are quite robust to common forms of non-independent errors. The FRESETS and FRESETL tests, *without* a Newey-West correction, are recommended if MA(1) or positive AR(1) errors are suspected, respectively. The results to date, and the ease of its application, favour the FRESET test.

Acknowledgments

We are grateful to Judith Giles, Nilanjana Roy, Ken Stewart and Ken White for their comments.

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Table 1

Models

Model	Specification	Error Term
1	DGP: $y_t = 1.0 - 0.4x_{3t} + x_{4t} + \gamma x_{2t} + \varepsilon_t$	$\varepsilon_t = \rho\varepsilon_{t-1} + v_t$
	Null: $y_t = \beta_0 + \beta_3 x_{3t} + \beta_4 x_{4t} + \varepsilon_t$	$E[v_t] = 0$, $E[v_t^2] = \sigma_v^2$, $Cov[v_t, v_s] = 0$, and $\lvert\rho\rvert < 1$
2	DGP: $y_t = 1.0 - 0.4x_{3t} + x_{4t} + \gamma x_{2t} + \varepsilon_t$	$\varepsilon_t = v_t + \lambda v_{t-1}$.
	Null: $y_t = \beta_0 + \beta_3 x_{3t} + \beta_4 x_{4t} + \varepsilon_t$	$E[v_t] = 0$, $E[v_t^2] = \sigma_v^2$, $Cov[v_t, v_s] = 0$, and $\lvert\rho\rvert < 1$

Table 2

Rejection Rates: ρ = 0, T=50, 10%

	No Correction			Newey-West		
γ	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL
-6.0	1.000	1.000	0.999	1.000	1.000	1.000
-5.0	1.000	1.000	0.999	1.000	1.000	1.000
-4.0	1.000	1.000	0.998	1.000	1.000	1.000
-3.0	1.000	1.000	0.998	1.000	1.000	1.000
-2.0	0.992	1.000	0.995	0.999	0.999	0.998
-1.0	0.617	0.882	0.806	0.810	0.945	0.949
0.0	0.104	0.102	0.101	0.237	0.371	0.374
1.0	0.556	0.766	0.817	0.702	0.938	0.928
2.0	0.593	0.999	0.992	0.851	0.999	1.000
3.0	0.881	1.000	0.999	0.995	1.000	1.000
4.0	0.987	1.000	0.997	1.000	1.000	1.000
5.0	0.999	1.000	0.996	1.000	1.000	1.000
6.0	1.000	1.000	1.000	1.000	1.000	1.000

Table 3

Rejection Rates: AR(1), ρ = -0.3, ρ = -0.3, T=50, 10%

	ρ = -0.3						ρ = 0.3					
	No Correction			Newey-West			No Correction			Newey-West		
γ	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL
-8.0	1.000	0.999	0.997	1.000	1.000	1.000	1.000	0.998	0.998	1.000	1.000	1.000
-7.0	1.000	1.000	0.998	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	1.000
-6.0	1.000	0.998	0.999	1.000	1.000	1.000	1.000	0.998	0.998	1.000	1.000	1.000
-5.0	1.000	0.998	0.996	1.000	1.000	0.999	1.000	0.999	0.995	1.000	1.000	0.999
-4.0	1.000	0.998	0.996	1.000	1.000	0.999	1.000	0.998	0.994	1.000	1.000	1.000
-3.0	1.000	0.999	0.997	1.000	1.000	1.000	1.000	0.999	0.994	1.000	1.000	0.998
-2.0	0.987	0.991	0.992	0.999	0.998	0.999	0.983	0.992	0.992	0.996	0.998	0.999
-1.0	0.598	0.810	0.808	0.778	0.944	0.937	0.576	0.757	0.785	0.783	0.929	0.940
0.0	0.100	0.121	0.134	0.221	0.364	0.376	0.094	0.077	0.070	0.220	0.310	0.310
1.0	0.501	0.764	0.715	0.624	0.921	0.902	0.495	0.771	0.720	0.710	0.934	0.927
2.0	0.558	0.988	1.000	0.845	0.998	1.000	0.546	0.994	1.000	0.814	0.999	1.000
3.0	0.861	1.000	1.000	0.993	1.000	1.000	0.854	0.997	1.000	0.993	1.000	1.000
4.0	0.984	0.996	1.000	1.000	1.000	1.000	0.980	0.998	1.000	1.000	1.000	1.000
5.0	0.999	0.998	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	1.000	1.000
6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
8.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000

Table 4

Rejection Rates: AR(1), ρ = -0.9, ρ = -0.9, T=50, 10%

	ρ = -0.9						ρ = 0.9					
	No Correction			Newey-West			No Correction			Newey-West		
γ	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL
-8.0	1.000	1.000	0.994	1.000	1.000	0.999	1.000	0.998	0.997	1.000	1.000	1.000
-7.0	1.000	0.998	0.995	1.000	1.000	0.999	1.000	0.999	0.996	1.000	1.000	1.000
-6.0	0.999	0.997	0.994	1.000	1.000	0.999	1.000	0.997	0.993	1.000	1.000	0.999
-5.0	0.991	0.991	0.990	1.000	0.997	0.998	0.998	0.994	0.989	1.000	0.999	0.998
-4.0	0.962	0.991	0.987	0.999	0.997	0.997	0.981	0.990	0.979	0.997	0.999	0.996
-3.0	0.875	0.957	0.956	0.985	0.985	0.987	0.892	0.960	0.944	0.971	0.995	0.988
-2.0	0.677	0.860	0.849	0.888	0.963	0.947	0.585	0.701	0.696	0.798	0.947	0.948
-1.0	0.296	0.483	0.486	0.382	0.668	0.670	0.098	0.125	0.128	0.281	0.452	0.446
0.0	0.098	0.308	0.326	0.133	0.389	0.399	0.012	0.006	0.005	0.023	0.029	0.026
1.0	0.377	0.437	0.427	0.418	0.629	0.587	0.054	0.137	0.121	0.242	0.461	0.447
2.0	0.412	0.784	0.816	0.538	0.931	0.939	0.148	0.730	0.773	0.328	0.954	0.975
3.0	0.586	0.957	0.981	0.804	0.991	0.999	0.465	0.951	0.990	0.754	0.996	1.000
4.0	0.774	0.984	0.999	0.952	0.997	1.000	0.794	0.993	1.000	0.960	0.999	1.000
5.0	0.909	0.993	1.000	0.993	0.999	1.000	0.954	0.992	1.000	0.998	0.999	1.000
6.0	0.972	1.000	1.000	1.000	1.000	1.000	0.994	1.000	1.000	1.000	1.000	1.000
7.0	0.990	0.996	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000
8.0	0.999	0.998	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000

Table 5

Rejection Rates: MA(1), ρ = -0.3, ρ = -0.3, T=50, 10%

	ρ = -0.3						ρ = 0.3					
	No Correction			Newey-West			No Correction			Newey-West		
γ	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL
-8.0	1.000	0.998	0.998	1.000	1.000	1.000	1.000	0.998	0.996	1.000	1.000	1.000
-7.0	1.000	0.999	0.998	1.000	1.000	1.000	1.000	0.999	0.998	1.000	1.000	0.999
-6.0	1.000	0.996	0.997	1.000	1.000	1.000	1.000	0.997	0.998	1.000	1.000	1.000
-5.0	1.000	0.998	0.995	1.000	1.000	0.999	1.000	0.999	0.995	1.000	1.000	1.000
-4.0	1.000	0.998	0.994	1.000	1.000	1.000	1.000	0.998	0.996	1.000	1.000	1.000
-3.0	1.000	0.999	0.995	1.000	1.000	0.999	1.000	0.999	0.997	1.000	1.000	1.000
-2.0	0.985	0.990	0.993	0.997	0.999	1.000	0.989	0.992	0.995	0.999	0.999	0.999
-1.0	0.576	0.758	0.792	0.789	0.930	0.940	0.615	0.808	0.806	0.784	0.934	0.929
0.0	0.105	0.083	0.085	0.227	0.339	0.333	0.106	0.128	0.127	0.233	0.377	0.389
1.0	0.523	0.769	0.723	0.721	0.925	0.927	0.517	0.789	0.730	0.654	0.929	0.911
2.0	0.561	0.991	1.000	0.827	0.998	1.000	0.558	0.988	1.000	0.840	0.997	1.000
3.0	0.858	0.998	1.000	0.992	1.000	1.000	0.863	0.999	1.000	0.995	1.000	1.000
4.0	0.982	0.996	1.000	1.000	1.000	1.000	0.986	0.997	1.000	1.000	1.000	1.000
5.0	0.998	0.996	1.000	1.000	1.000	1.000	0.999	0.996	1.000	1.000	0.999	1.000
6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
8.0	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 6

Rejection Rates: MA(1), ρ = -0.9, ρ = -0.9, T=50, 10%

	ρ = -0.9						ρ = 0.9					
	No Correction			Newey-West			No Correction			Newey-West		
γ	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL	RESET	FRESETS	FRESETL
-8.0	1.000	0.998	0.997	1.000	1.000	0.999	1.000	0.997	0.996	1.000	1.000	1.000
-7.0	1.000	1.000	0.996	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	1.000
-6.0	1.000	0.999	0.998	1.000	1.000	0.999	1.000	0.998	0.997	1.000	1.000	0.999
-5.0	1.000	0.998	0.996	1.000	1.000	0.999	1.000	0.996	0.995	1.000	0.999	0.999
-4.0	0.999	0.998	0.992	1.000	1.000	1.000	1.000	0.997	0.996	1.000	0.999	0.999
-3.0	0.992	0.996	0.993	1.000	1.000	0.999	0.997	0.996	0.991	0.789	0.999	0.998
-2.0	0.901	0.968	0.974	0.968	0.997	0.998	0.931	0.978	0.978	0.986	0.995	0.996
-1.0	0.362	0.526	0.545	0.599	0.823	0.844	0.464	0.627	0.614	0.634	0.857	0.831
0.0	0.108	0.089	0.086	0.202	0.278	0.277	0.111	0.150	0.161	0.218	0.370	0.371
1.0	0.348	0.526	0.482	0.592	0.832	0.828	0.390	0.608	0.554	0.490	0.838	0.802
2.0	0.433	0.965	0.994	0.667	0.997	1.000	0.445	0.962	0.989	0.706	0.993	0.999
3.0	0.718	0.996	1.000	0.941	1.000	1.000	0.737	0.995	1.000	0.961	0.999	1.000
4.0	0.919	0.996	1.000	0.999	0.999	1.000	0.936	0.997	1.000	0.999	1.000	1.000
5.0	0.986	0.995	1.000	0.999	0.999	1.000	0.994	0.998	1.000	1.000	1.000	1.000
6.0	0.998	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000
7.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.982	0.982
8.0	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000