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Asymmetric Responses of the Underground Economy to Tax Changes: Evidence From New Zealand Data

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Abstract: We consider the relationship between taxes and the size of the underground economy in New Zealand. Previous studies indicate that a positive relationship exists in this and certain other countries. We address the following question: "Is the response of the underground economy to an *increase* in taxes the same as its response to a *decrease* in taxes?" To answer this question we modify an existing methodology for testing for both "timing symmetry" and "pattern symmetry". Paying careful attention to the non-stationarity and cointegration of our annual data, we test for the presence of such symmetry in the tax-evasion relationship for New Zealand. We find that although the effect on the underground economy of an upward movement in the effective tax rate is numerically greater than that of a downward tax movement, this difference is not statistically significant in either the short-run or the long-run. Elasticity and multiplier calculations allow us to quantify some of the effects of changes in taxation policy on hidden output.

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1. Introduction

It is widely agreed that there is a relationship between taxes and the amount of tax evasion, or the size of the "underground economy". Several theoretical models have been suggested to support this hypothesis: *e.g.*, Allingham and Sandmo (1972), Yitzhaki (1974), Perncavel (1979), Koskela (1983), Watson (1985), Kesselman (1989) and Trandel and Snow (1999). The associated empirical literature is more limited, partly due to the difficulty of obtaining reliable time-series data on the size of the underground economy. However, support for this point in various economies comes from Clotfelter (1983), Crane and Norzad (1987), Schneider (1994), Hill and Kabir (1996), Caragata and Giles (1998), Giles and Caragata (1999), Giles and Johnson (1999), and Giles and Tedds (2000).

In this paper we use data generated recently for the New Zealand underground economy by Giles (1999), to test the hypothesis that the response of this illegal activity to *increases* in taxes is symmetric with the corresponding response to *decreases* in taxes. Intuitively, there is no reason for this hypothesis to be supported, necessarily. For example, agents may move relatively quickly into the underground sector to avoid an increased tax burden, but if their illegal activities go undetected, they may exhibit inertia in the face of a subsequent tax decrease. Tax changes also have implications for measured output, and the causal relationship between the latter and underground output has been explored empirically by Giles (1997), so there are additional flow-on effects of this sort to be considered.

We have annual time-series data over the period 1968 to 1994 for the relative size of the underground economy, *i.e.*, (UE/GDP), where UE is underground output. This series was obtained through the construction of a "Multiple Indicator, Multiple Causes" (MIMIC) model, which in turn was calibrated using a nonlinear currency-demand model. The results of Caragata and Giles (1998), Giles and Caragata (1999), Giles and Johnson (1999), and Giles and Tedds (2000), provide extensive evidence of a positive relationship between (UE/GDP) and the tax burden, as measured by the aggregate "effective tax rate", (TR/GDP), where "TR" denotes total tax revenue.

The possibility of an asymmetric response of the underground economy to changes in the effective tax rate is obviously of considerable interest from a policy-making perspective, but there appears to be no empirical evidence to date relating to this particular issue. Accordingly, taking account of the non-stationarity of our time-series data, we have adapted the general econometric methodology adopted in the context of gasoline mark-up pricing by Bacon (1991), Karrenbrock (1991), Manning (1991), Borenstein *et al.* (1992) and Quinn (1997), to test for symmetry in the underground economy-effective tax rate relationship.

Section 2 discusses the characteristics of our data; and in Section 3 we outline the formulation and estimation of the models that provide the framework for our analysis. Section 4 describes our symmetry tests and discusses some of the economic implications of our results. Our concluding comments appear in Section 5.

2. Data Properties and Implications

As noted above, we use two aggregate ratios, (UE/GDP) and (TR/GDP), in our analysis. The annual New Zealand data for the period 1968 to 1994 for the former variable are generated by Giles (1999), and the latter data are compiled from official data released by Statistics New Zealand and Revenue New Zealand. Both series are available on the web at **http://www.uvic.ca/econ/uedata.html**, and they are depicted in Figure 1.

In Table 1 we show the results of testing the data for non-stationarity, allowing for the possibilities of I(2), I(1) or I(0) processes. We have used both the "augmented" Dickey-Fuller (ADF) tests, in which the null hypothesis is non-stationarity, and the tests of Kwiatowski *et al.* (KPSS) (1993) in which the null hypothesis is stationarity. We have used a 10% significance level to compensate for the low powers of these tests, although the results are not sensitive to this choice.

In applying the ADF tests, the augmentation level (p) has been chosen by the default method in the SHAZAM (1997) package, as Dods and Giles (1995) show that this approach leads to low size-distortion when "pre-testing" in samples of our size. We have used the sequential strategy of Dolado *et al.* (1990) to determine the inclusion/exclusion of drift and trend terms in the Dickey-Fuller regressions. In Table 1, t_{dt} denotes the ADF unit root "t-test" with drift and trend terms included in the fitted regression; F_{ut} is the corresponding ADF "F-test" for a unit root and zero trend; t_d is the unit root "t-test" with a drift but no trend in the fitted regression; F_{ud} is the corresponding "F-test" for a unit root and a zero drift; and t is the ADF unit root test when the fitted regression has no drift or trend term included. Finite-sample critical values for our "t-tests" and "F-tests" come from MacKinnon (1991), and from Dickey and Fuller (1979, 1981), respectively.

In the case of the KPSS tests, where the null is stationarity, and the alternative hypothesis is non-stationarity, we have used both a zero value for the Bartlett window parameter, *l*, as well as $l = 5$. The latter value is implied by the KPSS "*l8* rule" for our sample sizes[1]. KPSS give asymptotic critical values for the test with null hypotheses of both level-stationarity and trend-stationarity. Cheung *et al.* (1995) provide response-surface results that allow us to calculate finite-sample critical values in the trend-stationary case.

The results in Table 1 indicate clearly that both (UE/GDP) and (TR/GDP) are I(1), and hence are non-stationary. Accordingly, it is meaningful to test for possible cointegration between the two series in each case, and in Table 2 we see the results of applying both the cointegrating regression ADF (CRADF) test and the Leybourne-McCabe (1993) test. In the former case, the null is "no cointegration", and finite-sample critical values are available from MacKinnon (1991). We see that there is good evidence of cointegration at the 10% significance level. In the latter case, only asymptotic critical values are available from Leybourne and McCabe, so the finite-sample p-value for our test statistic has been bootstrapped. The bootstrap simulation was undertaken with the SHAZAM (1997) package, and used 10,000 replications. Again, we see that there is clear evidence of cointegration between the tax burden and the size of the underground economy in New Zealand, so there must be Granger-causality between these two variables, in one direction or both[2]. In addition, this cointegration has implications for the way in which we should model the relationship between the underground economy and the effective tax rate, as is discussed in the next section.

3. Model Formulation and Estimation

As (UE/GDP) and (TR/GDP) are cointegrated, we can model the long-run equilibrium relationship between them by fitting an Ordinary Least Squares (OLS) regression to these data, *without* differencing them prior to estimation - indeed, OLS will be super-consistent. However, the short-run dynamics of the relationship, which is what we are interested in primarily, can be captured by estimating an error-correction model (ECM) of the following basic type:

$$\Delta y_t = [\alpha + \lambda t] + \sum\beta_i \Delta y_{t-i} + \sum\gamma_j \Delta x_{t-j} + \phi z_{t-1} + \varepsilon_t , \qquad \textbf{(1)}$$

$$(i = 1, ..., q \; ; \; j = 0, 1, 2, ..., r \; ; \; t = 1, 2, ..., T)$$

where the "error-correction term" (ECT) series, z_{t-1}, is the lagged value of the OLS residual series from the "cointegrating regression":

$$y_t = \mu + [\rho t] + \theta x_t + u_t \,. \qquad \textbf{(2)}$$

In both equations (1) and (2), terms in square brackets here and below are optional, "t" denotes a linear time-trend, and in our case y = (UE/GDP) and x = (TR/GDP).

The estimation of (1) requires the prior determination of the lag lengths, "q" and "r". Further, if no current value of Δx_t appears in (1), then OLS estimation can be used. Otherwise an Instrumental Variables (IV) estimator should be considered to ensure consistent parameter estimates. Given that we are interested in possible asymmetric effects, on y, relating to the impact of *upward movements* in x, as opposed *to downward movements* in x. Accordingly, we have modified (1) into a two-equation system, in the following way[3]:

$$\Delta y^{+}_{t} = [\alpha^{+} + \lambda^{+} t] + \sum \beta^{+}_{i} \Delta y^{+}_{t-i} + \sum \gamma^{+}_{j} \Delta x^{+}_{t-j} + \phi^{+} z_{t-1} + \varepsilon^{+}_{t} \qquad \textbf{(3)}$$

$$\Delta y^{-}_{t} = [\alpha^{-} + \lambda^{-} t] + \sum \beta^{-}_{k} \Delta y^{-}_{t-k} + \sum \gamma^{-}_{l} \Delta x^{-}_{t-l} + \phi^{-} z_{t-1} + \varepsilon^{-}_{t}$$

$$(i = 1, \ldots, q^{+} \,;\ j = 0, 1, 2, \ldots, r^{+} \,;\ k = 1, 2, \ldots, q^{-};\ l = 0, 1, 2, \ldots, r^{-};\ t = 1, 2, \ldots, T)$$

where:

$\Delta x^{+}_{t} = \Delta x_t$, if $\Delta x_t > 0$; zero otherwise

$\Delta x^{-}_{t} = \Delta x_t$, if $\Delta x_t < 0$; zero otherwise

$\Delta y^{+}_{t} = \Delta y_t$, if $\Delta x_t > 0$; zero otherwise

$\Delta y^{-}_{t} = \Delta y_t$, if $\Delta x_t < 0$; zero otherwise

and $z_t = (y_t - \mu - [\rho t] - \theta x_t)$.

This form of our model allows us to test, in a full maximum likelihood (ML) framework, for the type of asymmetric influences in which we are interested. We have obtained a parsimonious specification of the two-equation system, (3), by allowing for up to four lags on the various regressors, and then simplifying the model on the basis of both Akaike's Information Criterion, and the apparent significance of the individual coefficients. The resulting ML estimation results[4] (again obtained with the SHAZAM econometrics package) appear in Table 3. The final model does *not* include an intercept or trend in either equation, and the ECT is based on a cointegrating regression that excludes a trend term. Less satisfactory results were obtained when a trend was included in the cointegrating regression and/or in the ECM itself.

4. Asymmetry Tests & Economic Implications

4.1 Testing results

Also shown in Table 3 are the results of various tests of the specification of our estimated model. These include the Jarque-Bera test for the Normality of the errors; the FRESET test of DeBenedictis and Giles (1998) for omitted regressors and/or wrong functional form; and the Lagrange Multiplier for the serial independence of the errors[5]. Overall, these tests indicate that our estimated model is quite well specified, although some of the FRESET test statistic values are less than satisfactory in the case of the second equation.

Accepting the model's specification, however, the results in Table 3 imply that the model exhibits "simple timing symmetry", because our specification search suggests that "r^+" = "r^-". Of course, the estimated values of γ^+_1 and γ^-_1 are *numerically* different from each other, and this has economic implications that we pursue in the next sub-section. The different dynamic specifications of the two equations in the model, together with the inclusion of error-correction terms, further complicate the question of "timing". This is also explored more fully below. We can check for "pattern symmetry" in the model, by testing H_0: $\gamma^+_1 = \gamma^-_1$. This cross-equation restriction is readily examined by using a Wald test in the context of the joint ML estimation of the model. The resulting test statistic, whose null distribution is asymptotically chi-square with one degree of freedom, takes the value of 0.3963. Its "p-value" is 0.529 against a two-sided alternative[6], so there is no basis for rejecting "pattern symmetry". However, given our small sample size, we have also bootstrapped this test, using 10,000 replications. This yielded a finite-sample "p-value" for the Wald test of 0.584, and the same conclusion.

4.2 Estimated elasticities

The nature of the relationship between the relative size of the underground economy and the effective tax rate can be explored and interpreted in terms of the estimated short-run and long-run elasticities. These measures, and some basic tests relating to them appear in Table 4. As the estimation results in Table 3 indicate, the *current* value of Δ(TR/GDP) does not appear as a regressor in the model, so the short-run elasticities relate to a one-year lag. Separate elasticities emerge from the two equations of model (3) – one for periods when Δ(TR/GDP) > 0, and one for periods when Δ(TR/GDP) < 0 in each case. All of the elasticities shown in Table 4 are calculated at the sample means of the data. These means relate to the *sub-sample* for which the effective tax rate is positive in the case of η^+, φ^+ and ξ^+, and the *sub-sample* for which it is negative in the case of η^-, φ^- and ξ^-.

As our model contains an ECT, both the one-period and long-run elasticities must take this into account. In Table 4, η denotes a "naïve" one-period short-run elasticity, ignoring the effect of the ECT in the model; φ is a "full" one-period elasticity, taking account of the ECT; and ξ is a full long-run equilibrium elasticity, taking account of the ECT and the full dynamic specification of the two equations in the model. The values of η^+ and η^- are included only for completeness, to show the effect on the elasticity calculations of (wrongly) ignoring the ECT in each equation. As we see, this results in an under-statement of the one-period elasticity values in each case.

The test results in Table 4 indicate that both in the short-run or the long-run, the "tax increase elasticities" exceed the "tax decrease elasticities" numerically, but these differences are not statistically significant. As expected, all of the estimated elasticities are positive, and $\xi > \varphi$ for both positive and negative tax rate changes. Generally the elasticity estimates are significantly different from zero, at least at the 10% level, and the long-run elasticities (and φ^+) are *not* significantly different from unity.

Full details of the short-run and long-run elasticities, estimated at each actual sample point, appear in Table 5. Some interesting economic implications emerge. For example, in 1994 (UE/GDP) = 11.3% and (TR/GDP) = 34.5%, so if the latter had been *reduced* to 30% (say) then the underground economy would have *fallen* to 10.5% of GDP within one year, and to just over 10% of GDP in the long-run, *ceteris paribus*. Similarly, if the effective tax rate had been raised to 39% (say) then the underground economy would have *risen* to 12.46% of GDP within one year, and to 12.53% of GDP in the long-run, *ceteris paribus*. In each case these calculations assume that the change in the tax burden is achieved without any change in the "tax-mix", which is a rather simplistic assumption[7]. As can be seen from Table 5, there are quite marked variations in all of the elasticities over the sample period, and these generally relate to movements in the underlying business cycle. In particular, the *largest* elasticity values are associated with years in which measured and/or underground output experienced *downturns*.

This last result is consistent with the evidence from Giles and Caragata (1999) that during a recession, the "hard-core" component of the underground economy is much greater than during the peak of a boom. The elasticities implied by their aggregate logistic model[8] also appear in Table 5 by way of comparison, and we see that these values are close to the short-run "tax decrease" elasticities developed in this paper, and generally lie between our two short-run elasticities[9]. This is encouraging, especially as the model estimated by Giles and Caragata has a rather restrictive functional form and has no dynamic components[10]. It seems from Table 5 that the elasticities reported earlier by Giles and Caragata as a by-product of their analysis may be somewhat conservative, and somewhat more variable than ours. The

New Zealand underground economy may be somewhat more responsive to changes in taxation policy than was previously thought.

4.3 Multiplier analysis

Table 5 gives the dynamic multipliers, d_k, k = 1, 2, ……….., showing the impacts on (UE/GDP) after k periods associated with a one-unit change in (TR/GDP). The cumulative effects of these multipliers up to and including k periods, D_k, are also shown in that table, as are the eigenvalues (λ_j) of the matrix "F" constructed from the implicit coefficients in the two "undifferenced" equations that make up our model[11]. These eigenvalues indicate that in the case of an *increase* in the effective tax rate, the dynamic multipliers for the relative size of the underground economy follow a path that is *oscillatory*, but *damped*, with a frequency of 0.681, and a period of 9.227 years. In contrast to this, in the case of a *decrease* in the effective tax rate, the underground economy dynamic multipliers follow a *non-oscillatory*, *stable*, time-path.

So, although the hypotheses of pattern and timing symmetry could not be rejected above, we see that there are actually important differences between the upward and downward responses in the underground economy to changes in the tax rate. All of the variables in our model are expressed as percentages, so the one-period effect of a percentage-point increase in the effective tax rate is an increase in (UE/GDP) of approximately a quarter of a percentage point. The corresponding effect of a one percentage-point decrease in the tax rate is a decrease of just over one sixth of a percentage point in (UE/GDP). Further, after two years, the cumulative effect of a tax increase is approximately double that of a tax decrease, though after three or four years these effects are roughly equal (but opposite in direction, of course). In the latter case we see that a one percentage-point change in the tax rate has a maximum cumulative effect of a little less than a third of a percentage-point on the relative size of the underground economy.

It is especially interesting that an *increase* in the effective tax rate apparently results in an oscillatory response in the relative size of the underground economy, whereas a *decrease* in the tax rate does not. In the former case, the explanation may be that although we know[12] that there will be an immediate *increase* in (UE/GDP) when (TR/GDP) rises, we also know from Scully (1996) that in the New Zealand context there is also a significant subsequent *reduction* in real measured GDP growth. The empirical causality evidence from Giles (1997) then implies that this will be followed by a downturn in the size of UE, in real terms. Thus, depending on the lag structures associated with these various interrelationships, an oscillatory response in the *relative size* of the underground economy, (UE/GDP), is not surprising. The apparent absence of this phenomenon in the case of a tax *decrease* may be the result of asymmetries in the perceived probability of detection on the parts of those engaging in underground activity, as opposed to those who are not, and/or differences in the associated habit-formation processes. In particular, the incentive effects may be more readily identifiable for tax-payers when the tax rate is cut, as opposed to increased. Agents may take longer to determine how to avoid tax increases (and the transaction costs may be higher), than to determine how to spend tax savings when taxes are cut.

5. Conclusions

In this paper we have considered the possibility of different response patterns in the underground economy to upward, as opposed to downward, movements in the effective tax rate, and explored this possibility empirically with New Zealand data. The results of our econometric testing suggest that, although there is no evidence of statistically significant departure from either "timing symmetry" or "pattern symmetry" in this context, there are marked differences in the point elasticities associated with upward and downward changes in the tax rate. Moreover, when the dynamic multipliers are considered in detail, important differences also arise in the associated response paths.

If representative of the situation in other economies, these results have important implications for the effectiveness of fiscal policy as a tool for dealing with the growing problem[13] of the underground economy in many countries. In particular, they suggest that the erosion of the tax-base that apparently follows an increase in the tax rate may not as easily be recovered in the face of a subsequent tax reduction. On the other hand, our empirical results also suggest that in the latter case any such recovery is likely to proceed steadily, and cumulatively, as the dynamic effects of this change in fiscal policy work through the system over a four to five year period.

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Footnotes

1. This rule sets l = INT [8(T/100)$^{1/4}$].

2. This causality is discussed in more detail by Giles and Caragata (1998) and Caragata and Giles 1999), and Giles and Johnson (1999).

3. Bacon (1991), Karrenbrock (1991), Manning (1991), Borenstein *et al.* (1992) and Quinn (1997) also modify (1) in various ways to allow for asymmetric response-effects. Our approach is slightly different from any of theirs.

4. The estimation of a *joint* system is supported (at the 10% level, say) by the likelihood ratio test for a diagonal contemporaneous error covariance matrix. The test statistic is 3.510 and is asymptotically chi-square with one degree of freedom, so its "p-value" is 0.0610.

5. The FRESET test is an alternative to the familiar RESET test, but a globally-valid Fourier approximation (rather than a locally-valid polynomial approximation) to the conditional regression mean is used. This results in superior power. The test statistic FRE_i involves "i" sine and cosine terms, so its asymptotic distribution is $\chi^2_{(2i)}$. The LM_i test statistic relates to an alternative hypothesis of a simple autoregressive or moving average process of order "i", so its asymptotic distribution is $\chi^2_{(i)}$.

6. If our prior feeling is that $\gamma^+_1 > \gamma^-_1$, then the one-sided z-test (whose statistic is the square root of the Wald statistic) reported in the third line of Table 4 is appropriate.

7. The effects on the New Zealand underground economy when the tax-mix is changed (with or without a change in the tax burden) are simulated in detail by Caragata and Giles (1998).

8. The results reported by Giles and Caragata (1999) are based on a simple *static* logistic functional relationship between (UE/GDP) and (TR/GDP), so their elasticities relate to the *current-period* impact. By way of further comparison, Giles and Johnson (1999) estimate a static nonparametric regression model of this relationship for New Zealand, and obtain current-period elasticities which range from 0.467 to 1.186 over this same period.

9. The logistic elasticities in Table 5 are calculated on a comparable basis to the others in that table. That is, the actual sample values of (TR/GDP) and (UE/GDP) are used. Giles and Caragata (1999) plot the logistic elasticity against the effective tax rate in their Figure 2(d), and report a value of 0.76 when the effective tax rate is at its 1994 value of 34.5%. In that case the elasticities were computed using *hypothetical* values of (TR/GDP), and the corresponding *predicted* values of (UE/GDP).

10. On the other hand, it should also be noted that their results were quite insensitive to the choice of functional form – logistic, Gompertz, cumulative Normal, and extreme-value functions were all considered.

11. See Section 1.2, and especially equation [1.2.3] of Hamilton (1994) for full details.

12. See Caragata and Giles (1998), Giles and Caragata (1999), and Giles and Johnson (1999).

13. For example, see Schneider and Enste (1998) and Giles and Tedds (2000).

Table 1. ***Unit root test results***

a. Augmented Dickey-Fuller tests [a]

		n	p	t_{dt}	F_{ut}	t_d	F_{ud}	t	Outcome
					U = (UE/GDP)				
H_0 :	I(2)	23	2	-3.44	n.a.	n.a.	n.a.	n.a.	Reject I(2)
H_0 :	I(1)	24	2	-2.70	3.66	-1.63	2.14	1.06	**I(1)**
					T = (TR/GDP)				
H_0 :	I(2)	23	2	-2.62	3.43	-2.65	n.a.	n.a.	Reject I(2)
H_0 :	I(1)	24	2	-2.42	3.21	-1.40	2.73	1.63	**I(1)**

b. KPSS tests [b]

		n	Level-Stationary		Trend-Stationary		Outcome
			l=0	***l=5***	***l=0***	***l=5***	
				U = (UE/GDP)			
H_0 :	I(0)	27	1.519	0.520	0.131	0.110	I(1)
				T = (TR/GDP)			
H_0 :	I(0)	27	0.099	0.073	0.129	0.081	I(1)

Notes: **a.** The outcomes are based on exact 10% critical values from MacKinnon (1991).

b. The outcomes are based on exact 10% critical values from Cheung *et al.* (1995), and the KPSS 10% asymptotic critical values.

Table 2. ***Cointegration test results***

a. Cointegrating Regression Augmented Dickey-Fuller "t-tests"

n	**p**	**No Trend**		**Trend**		**Outcome**[a]
		R^2	**t**	**R^2**	**t**	
27	0	0.40	-3.434	0.58	-3.856	**Cointegration**
			(-3.57)		(-4.15)	
			[-3.20]		[-3.77]	

b. Leybourne-McCabe tests

n	**h_1**	**Bootstrapped p-value**	**Outcome**[b]
27	0.122	0.07	**Cointegration**
	(0.31)		
	[0.23]		

Notes: **a.** MacKinnon's (1991) exact 5% (10%) critical values appear in parentheses (brackets). "p" is the augmentation level.

b. Leybourne and McCabe's (1993) asymptotic 5% (10%) critical values appear in parentheses (brackets). The bootstrapped p-value is based on 10,000 replications.

Table 3. ***Model estimation results***[a,b]

Dependent Variable	Regressors					
	Dep$_{-1}$	**Dep$_{-2}$**	**Dep$_{-3}$**	**ΔT^{+}_{-1}**	**ΔT^{-}_{-1}**	**ECT**
ΔU^{+}	0.462 (1.82)	n.a.	n.a.	0.181 (1.61)	n.a.	-0.406 (-1.85)
JB: 2.143 [0.342]	**FRE$_1$**: 6.894 [0.032]	**FRE$_2$**: 6.706 [0.152]	**FRE$_3$**: 2.430 [0.876]			
LM$_1$: 0.673 [0.250]	**LM$_2$**: 0.027 [0.489]	**LM$_3$**: 0.525 [0.300]	**LM$_4$**: 0.561 [0.287]	**LM$_5$**: 0.566 [0.286]	**LM$_6$**: 1.114 [0.133]	
ΔU^{-}	n.a.	0.429 (4.08)	0.248 (2.21)	n.a.	0.098 (1.26)	-0.395 (-5.89)
JB: 0.570 [0.752]	**FRE$_1$**: 4.017 [0.134]	**FRE$_2$**: 14.603 [0.006]	**FRE$_3$**: 13.306 [0.038]			
LM$_1$: 0.175 [0.430]	**LM$_2$**: 0.593 [0.277]	**LM$_3$**: 0.202 [0.420]	**LM$_4$**: 0.072 [0.471]	**LM$_5$**: 1.154 [0.124]	**LM$_6$**: 1.257 [0.104]	

Notes: **a.** "Dep" denotes "dependent variable", "U" denotes "(U/GDP)", "T" denotes "(TR/GDP)", and "ECT" denotes "error correction term". Asymptotic "t-ratios" appear in parentheses.

b. "JB" denotes "Jarque-Bera Normality test", "FRE" denotes "FRESET test", "LM" denoted "Lagrange Multiplier test". "p-values" appear in square brackets.

Table 4. ***Tests on coefficients and elasticities***

Null Hypothesis[a]	**z-Statistic**	**Estimated Value**	**s.e.**	**p-value**[b]
$\gamma^{+}_{1} = 0$	1.606	0.181	0.113	0.054
$\gamma^{-}_{1} = 0$	1.262	0.098	0.078	0.104
$\gamma^{+}_{1} - \gamma^{-}_{1} = 0$	0.630	0.084	0.133	0.265
$\eta^{+} = 0$	1.606	0.319	0.198	0.054
$\eta^{-} = 0$	1.262	0.131	0.104	0.104
$\eta^{+} - \eta^{-} = 0$	0.861	0.188	0.219	0.195
$\varphi^{+} = 0$	2.449	0.458	0.187	0.007
$\varphi^{-} = 0$	2.302	0.235	0.102	0.011
$\varphi^{+} - \varphi^{-} = 0$	1.09	0.224	0.206	0.139
$\xi^{+} = 0$	2.354	0.486	0.206	0.010
$\xi^{-} = 0$	2.521	0.327	0.130	0.005
$\xi^{+} - \xi^{-} = 0$	0.646	0.159	0.246	0.254

Notes: **a.** η denotes a "naïve" one-period elasticity, ignoring the effect of the ECT in the model; φ is a "full" one-period elasticity, taking account of the ECT; and ξ is a full long-run equilibrium elasticity.

b. The p-values are for a one-sided (positive) alternative hypothesis in each case.

Table 5. ***Estimated elasticities at each sample point***

Year	φ^+	φ^-	ξ^+	ξ^-	**Logistic**[a]
1969	0.806	0.542	0.854	0.753	0.518
1970	0.888	0.597	0.941	0.830	0.587
1971	0.896	0.602	0.949	0.837	0.656
1972	0.882	0.592	0.934	0.824	0.601
1973	0.809	0.544	0.857	0.756	0.536
1974	0.735	0.494	0.778	0.687	0.547
1975	0.847	0.569	0.898	0.792	0.674
1976	0.788	0.529	0.835	0.736	0.539
1977	0.877	0.589	0.929	0.820	0.625
1978	0.865	0.581	0.917	0.808	0.719
1979	1.003	0.674	1.063	0.938	0.700
1980	0.830	0.558	0.880	0.776	0.642
1981	0.901	0.606	0.955	0.843	0.680
1982	0.936	0.629	0.992	0.875	0.734
1983	0.817	0.549	0.865	0.763	0.612
1984	0.809	0.544	0.857	0.756	0.549
1985	0.807	0.542	0.856	0.755	0.577
1986	0.811	0.545	0.859	0.758	0.624
1987	0.691	0.464	0.732	0.646	0.530
1988	0.812	0.546	0.861	0.759	0.722
1989	0.934	0.627	0.989	0.873	0.732
1990	0.959	0.644	1.016	0.896	0.866
1991	1.059	0.711	1.122	0.990	0.876
1992	1.071	0.720	1.135	1.001	0.803
1993	0.919	0.618	0.974	0.859	0.753
1994	0.790	0.531	0.837	0.739	0.647
Mean **(c.v.)**	**0.867** **(0.105)**	**0.583** **(0.105)**	**0.919** **(0.105)**	**0.810** **(0.105)**	**0.656** **(0.153)**

Note: **a.** The elasticities in this column are those implied by the simple logistic model fitted to these same data by Giles and Caragata (1999). Note that these are calculated over a longer sample than in Table 4.

Table 6. ***Dynamic multipliers and stability properties***[a]

j, k	Increase in Tax Rate			Decrease in Tax Rate		
	d_k	D_k	λ_j	d_k	D_k	λ_j
1	0.2609	0.2609	(0.5281 + 0.4279i)	0.1753	0.1753	0.8589
	(0.1065)	(0.1065)		(0.0761)	(0.0761)	
2	0.0941	0.3550	(0.5281 - 0.4279i)	0.0094	0.1846	0.3485
	(0.0580)	(0.1186)		(0.0384)	(0.0535)	
3	-0.0211	0.3339		0.1075	0.2922	-0.6028
	(0.0559)	(0.0920)		(0.0445)	(0.0921)	
4	-0.0658	0.2681		0.0202	0.3124	
	(0.0685)	(0.0627)		(0.0311)	(0.0768)	
5	-0.2507	0.0174		0.0045	0.3170	
	(0.1495)	(0.1207)		(0.0172)	(0.0640)	

Note: **a.** Standard errors appear in parentheses.

Figure 1. Effective tax rate and underground economy, New Zealand 1968-1994