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Taxes, Risk-Aversion, and the Size of the Underground Economy: A Nonparametric Analysis With New Zealand Data

David E. A. Giles & Betty J. Johnson

Department of Economics, University of Victoria

Victoria, B. C., V8W 2Y2, Canada

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Abstract: We use nonparametric regression analysis to investigate the relationship between the effective tax rate and the relative size of the underground economy, using New Zealand data. The theoretical underpinnings of such a relationship are established by extending some of the predictions of the recent model of Trandel and Snow (1999) to allow for the form of the available aggregate data. Time-series evidence indicates that these data are non-stationary and cointegrated, and this is taken into account in our estimation. The theoretical framework produces an ambiguous prediction regarding the sign of the relationship we are studying. However, our nonparametric empirical analysis produces a positive and "S-shaped" relationship, and this supports earlier empirical studies that imposed such functional forms. The estimated model is used to simulate the effects of hypothetical tax changes on the size of the New Zealand underground economy, and to draw policy conclusions.

Keywords: Tax evasion, underground economy, risk aversion, tax rates, nonparametric regression.

JEL Classifications: C14; C22; H26

Contact Author: Professor David Giles, Department of Economics, University of Victoria, PO Box 1700, STN CSC, Victoria, BC, Canada, V8W 2Y2
FAX (250) 721-6214; Voice (250) 721-8540; e-mail dgiles@uvic.ca

1. Introduction

There is a long-standing hypothesis that there is a relationship between taxes and the degree of tax evasion, or the size of the "underground economy". Various theoretical models have been proposed in support of this hypothesis, but the associated empirical literature is relatively sparse. In part this is due to the difficulty of obtaining meaningful time-series data for the size of the underground workforce or underground output. In this paper, based on recent developments in the theoretical literature on tax evasion, we examine in some detail the empirical relationship between the relative size of the New Zealand underground economy and the effective tax rate in that country, using annual time-series data for the period 1968 to 1994. The relative size of the underground economy (UE) is measured as (UE/GDP), and the data are those generated by Giles (1999) using a structural Multiple Indicator Multiple Causes (MIMIC) model. The (aggregate) effective tax rate is defined as (TR/GDP), where TR is total tax revenue.

As is outlined below, there is theoretical justification for the hypothesis of a positive relationship between the size of the underground economy and the tax rate, but this prediction is not readily testable directly due to the form of the data that are generally available. In this paper we examine the nature of this relationship empirically, making proper allowance for the non-stationarity of our time-series data, and using nonparametric estimation in order to avoid distorting the conclusions by "imposing" an assumed functional form on the analysis. Accordingly, this study extends and corroborates that of Caragata and Giles (1998) and Giles and Caragata (1999) for New Zealand - those authors adopted an explicit parametric analysis in their investigation of the tax burden-underground economy relationship. Specifically, they considered various "S-shaped functions, and favoured a logistic functional form to "explain" (UE/GDP) as a positive function of (TR/GDP).

Our objective here is to abstract from any such functional constraints, and to investigate some of the practical implications of the recent theoretical model of Trandel and Snow (1999) by estimating this relationship using nonparametric methods. Section 2 provides a general theoretical background; and in Section 3 we re-formulate certain implications of the Trandel-Snow model in terms of an hypothesis that can be examined empirically using macroeconomic data. Section 4 discusses the available data, including issues of non-stationarity and possible cointegration; and Section 5 deals with the estimation issues and results. Some of the economic implications of the estimated model, including some simple simulation results, are described in Section 6; and our concluding comments appear in Section 7.

2. Theoretical considerations

Schneider and Enste (1998) and Giles and Caragata (1999) discuss some of the previous empirical evidence pertaining to the effect of taxes on the underground economy, stemming from early contributions by Clotfelter (1983) and Crane and Nourzad (1987), to the more recent results of Schneider (1994), Johnson *et al.*(1998), Cebula (1997), and Hill and Kabir (1996). This evidence overwhelmingly supports the hypothesis of a positive relationship between taxes and the size of the underground economy, though the data used and the details of the analysis vary enormously from study to study.

Of primary interest here is the extensive theoretical literature that considers the role of taxes in determining the size of the underground economy. In fact, much of this literature deals with theoretical models that are quite narrow in their perspective. In particular, in the spirit of the seminal contribution of Allingham and Sandmo (1972), much of this literature relates to models of "pure tax evasion". In such models, income is earned from only one source, and some of this income is not declared to the taxation authority. Many of these models also assume that the penalties for evasion are imposed as a fraction of undeclared income, rather than as a fraction of the evaded tax, and/or that the tax system is linear[1] (*e.g.*, Yitzhaki, 1974). Allowing for tax progressivity and risk-averse tax-paying agents, the theoretical models of Pencavel (1979) and Koskela (1983) predict that increasing the tax rate reduces the amount of tax evasion. However, this result becomes ambiguous if flexibility is allowed with respect to labour's hours worked, and much of this literature is of questionable interest in terms of empirical verifiability.

On the other hand, there is also a more appealing theoretical literature relating to two-sector "underground economy" models. In these models there are two sources of potential income, and the probability that any evasion will be detected by the authorities differs between the two sectors. In one sector, all earned income is "visible" with respect to taxation liability, while in the other sector the possibility of tax evasion results in lower before-tax wages. Examples of such contributions are those of Watson (1985), Kesselman (1989) and Trandel and Snow (1999). Models of this type are more appealing from an empirical viewpoint as their underlying assumptions more closely match reality, and so they provide interesting testable hypotheses.

The form of these hypotheses is, however, somewhat complicated. For example, the sign of the relationship between the tax rate and the degree of tax evasion depends, among other things, upon what is assumed about agents' risk aversion. The recent results of Trandel and Snow (1999) illustrate this point. If taxes are progressive and if the agents' preferences exhibit decreasing absolute risk aversion, and non-increasing relative risk aversion, then their model predicts a positive relationship between the tax rate and the share of the total labour force that is active in the evasive sector of the economy. A positive relationship is also predicted between the degree of tax-progressivity and the relative size of the underground labour force. We will examine aspects of the Trandel-Snow model in our empirical analysis below.

3. Formulating a testable hypothesis

In practice, aggregate data on the size of the underground economy are generally estimated in terms of the value of "hidden" output, rather than the size of the associated labour force. Indeed, to facilitate international comparisons, these output figures (however they are derived) are often reported as a percentage of measured GDP. For example, see Schneider and Enste (1998) for some extensive cross-country comparisons, and Giles (1999) for a continuous recent time-series for the New Zealand underground economy. This form of the data necessitates some manipulations of the predictions of the Trandel-Snow model before testing them empirically.

Of course, the institutional characteristics of the New Zealand taxation system[2] render this theoretical model somewhat stylized. However the apparent quality[3] of the underground economy data, and the fact that they were generated without the use of effective tax rate information (so that the modelling of a relationship between this rate and the underground economy is not a spurious exercise), mean that the New Zealand situation provides a useful empirical basis for testing some of the implications of the Trandel-Snow model[4].

In this theoretical two-sector model, workers are identical and each one supplies a unit of labour to sector 'e' (or sector 'n'), in which evasion is (or is not) undertaken. The model does not allow for workers who participate in both sectors of the economy simultaneously. Income earned per worker in sector 'i' is denoted 'y_i'; 'a' is the share of the workforce in sector 'e'; and 't' is the constant marginal tax rate (above a positive threshold level of income, 'b'). Faced with a probability 'p' that evaded income will be detected, and penalised on the basis of additional tax owed, workers in the underground sector choose a level of undeclared income, 'x', to maximise expected utility. As labour is mobile, this means finding optimal a* and x* values to equate the expected utility of working in each sector. Trandel and Snow prove, *inter alia*, that if agents' preferences exhibit decreasing absolute, and non-decreasing relative, risk aversion, then $(\partial a^*/\partial t) > 0$. That is, the size of the underground economy, measured in *employment* terms, grows with the value of the marginal tax rate that is faced in both sectors. Of course, as these preferences are not observable, but are merely revealed through the agents' actions, the sign of the relationship between the marginal tax rate and the employment-size of the underground economy is itself effectively an empirical issue.

We now extend their analysis by reinterpreting this prediction in terms of *average* tax rates (which differ between the evading and non-evading sectors), as well as in an aggregate income (or output) context, rather than an employment context. These extensions are important from an empirical viewpoint, given the form in which such data are generally available. The (actual) average tax rate faced by workers in the non-evading sector is $\tau_n = t[y_n(a^*) - b] / y_n(a^*)$. Similarly the (expected) average tax rate in the evading sector is $\tau_e = t\{ y_e(a^*) - b - [1 - p(1 + m)]x^*\} / y_e(a^*)$. As Trandel and Snow (1999; p 221) note, $\tau_n > \tau_e$. This follows immediately if tax evasion is to be a better-than-fair gamble, for this requires that $(1 - p) > pm$.

Proposition 1. *Suppose that a fixed, non-zero, range of income is untaxed, that preferences exhibit decreasing absolute and non-decreasing relative risk aversion, and that tax evasion is a better-than-fair gamble. Then the size of the underground economy rises, if either the average tax rate in the non-evading sector increases, or if the (expected) average tax rate in the evading sector falls.*

Proof. First, consider the non-evading sector. Let $f_n(a^*) = \tau_n - t[y_n(a^*) - b] / y_n(a^*) = 0$. By the implicit function theorem, $(\partial a^*/\partial \tau_n) = - [(\partial f_n/\partial \tau_n) / (\partial f_n/\partial a^*)]$. Now, $(\partial f_n/\partial \tau_n) = 1$, and $(\partial f_n/\partial a^*) = - [tby'_n(a^*) / y_n(a^*)^2]$. So, $(\partial a^*/\partial \tau_n) = [y_n(a^*)]^2 / [tby'_n(a^*)] > 0$, because $y'_n(a) > 0$.

In the evading sector, let $f_e(a^*) = \tau_e - t\{y_e(a^*) - b - [1 - p(1 + m)]x^*\} / y_e(a^*) = 0$. Again, by the implicit function theorem, $(\partial a^*/\partial \tau_e) = - [(\partial f_e/\partial \tau_e) / (\partial f_e/\partial a^*)]$. In this case, $(\partial f_e/\partial \tau_e) = 1$, and $(\partial f_e/\partial a^*) = - [tby'_e(a^*)] / [y_e(a^*)]^2 - [t\{1 - p(1 + m)\}x^*y'_e(a^*)] / [y_e(a^*)]^2$. As $y'_e(a) < 0$, it follows that $(\partial a^*/\partial \tau_e) <0$ provided that $[1 - p(1 + m)] > 0$. This last condition is simply $(1 - p) > pm$, which holds if tax evasion is a better-than-fair gamble. ●

If 'N' is the size of the total labour force, then total *declared* (and hence measured) equilibrium gross income generated in the non-evading sector is $[N(1 - a^*)y_n]$. In the evading sector, declared equilibrium income will be $[Na^*(y_e - x^*)]$, and this will correspond to measured gross income generated by that sector if evasion is not detected. However, in the event of detection, measured income from this sector[5] will be $[Na^*y_e]$. Similarly, equilibrium evaded income will be $[Na^*x^*]$ in the absence of detection, and otherwise it will be zero. So, the expected *relative* size[6] of the underground economy in aggregate income (output) terms is:

$$u = [(1 - p)a^*x^*] / [(1 - a^*)y_n + a^*(y_e - x^*)]. \qquad \textbf{(1)}$$

Proposition 2. *When a fixed, non-zero, range of income is untaxed and preferences exhibit decreasing absolute and non-decreasing relative risk aversion, an increase in the marginal tax rate may either increase or decrease the expected relative size of the underground economy, measured in income terms.*

Proof. Denoting $(\partial a^*/\partial t)$ by $a^{*\prime}$, and $(\partial x^*/\partial t)$ by $x^{*\prime}$, it follows from (1) that:

$$(\partial u/\partial t) = (1 - p)\{A(a^*x^{*\prime} + x^*a^{*\prime}) - Ba^*x^*\}/ A^2, \qquad \textbf{(2)}$$

where $A = [(1 - a^*) y_n + a^*(y_e - x^*)]$,

and $B = [(1 - a^*)(\partial y_n/\partial t) + a^{*\prime} (y_e - x^* - y_n) + a^*\{(\partial y_e/\partial t) - x^{*\prime}\}]$.

Now, $A > 0$. Also, under the stated conditions, $a^{*\prime} > 0$, so $(\partial y_n/\partial t) > 0$, and $(\partial y_e/\partial t) < 0$, by the chain rule[7]. Clearly, from (2), regardless of the sign of $x^{*\prime}$, the sign of $(\partial u/\partial t)$ is ambiguous. ●

So, the nature of the effect of a change in the *marginal* tax rate on the expected *relative* size of the underground economy is an empirical issue. Not surprisingly, it also follows directly from this result, and from the definitions of the average tax rates, τ_n and τ_e, that a similar ambiguity arises with respect to the signs of $(\partial u/\partial \tau_n)$ and $(\partial u/\partial \tau_e)$.

Accordingly, in modelling the relationship between the macroeconomic aggregates (UE/GDP) and (TR/GDP), the Trandel-Snow model predicts an *ambiguous* partial derivative, and this issue is an empirical one. Their model and its predecessors are, of course, silent on the matter of the functional form of any such relationship between the tax rate and tax evasion. This underscores the relevance of adopting a nonparametric approach in our empirical analysis below.

4. Data Issues

As noted already, we use two aggregate ratios, (UE/GDP) and (TR/GDP). Annual data for the period 1968 to 1994 for the former variable are taken from Giles (1999), while the latter data are compiled from official data released by Statistics New Zealand and Revenue New Zealand. Both series are available on the web at **http://www.uvic.ca/econ/uedata.html**, and are displayed in Figure 1. As can be seen from the results in Table 1, we have tested each series for non-stationarity, allowing for the possibilities of I(2), I(1) or I(0) data. We have used both the "augmented" Dickey-Fuller (ADF) tests, in which the null hypothesis is non-stationarity, as well as the tests of Kwiatowski *et al.* (KPSS) (1993) in which the null hypothesis is stationarity of the data. A 10% significance level has been adopted to deal with the well-known low powers of these tests, although the results are not sensitive to this choice.

In the case of the ADF tests, the augmentation level (p) has been chosen by the default method in the SHAZAM (1997) package, as Dods and Giles (1995) show that this approach leads to low size-distortion in the presence of moving-average errors with samples of the size being used here. We have followed the sequential strategy of Dolado *et al.* (1990) to deal with the issue of the inclusion/exclusion of drift and trend terms in the Dickey-Fuller regressions. So, in Table 1, t_{dt} denotes the ADF unit root "t-test" with drift and trend terms included in the fitted regression; F_{ut} is the corresponding ADF "F-test" for a unit root and zero trend; t_d is the unit root "t-test" with a drift but no trend in the fitted regression; F_{ud} is the corresponding "F-test" for a unit root and a zero drift; and t is the ADF unit root test when the fitted regression has no drift or trend term included. Finite-sample critical values for our "t-tests" come from MacKinnon (1991), and those for the "F-tests" are given by Dickey and Fuller (1979, 1981).

In the case of the KPSS tests, where the null is stationarity, and the alternative hypothesis is non-stationarity, we have used both a zero value for the Bartlett window parameter, l, as well as $l = 5$. The latter value is implied by the KPSS "*l8* rule" for our sample size[8]. KPSS provide asymptotic critical values for the test with null hypotheses of both level-stationarity and trend-stationarity. Cheung *et al.* (1995) provide response-surface information that facilitates finite-sample critical values in the trend-stationary case[9].

The results in Table 1 indicate clearly that both (UE/GDP) and (TR/GDP) are I(1), and hence are non-stationary[10]. Accordingly, it is meaningful to test for possible cointegration between the two series, and in Table 2 we see the results of applying both the cointegrating regression ADF (CRADF) test and the Leybourne-McCabe (1993) test. In the former case, the null is "no cointegration", and finite-sample critical values are available from MacKinnon (1991). We see that there is good evidence of cointegration at the 10% significance level. In the latter case, only asymptotic critical values are available from Leybourne and McCabe, so the finite-sample p-value for out test statistic has been bootstrapped. Again, we see that there is clear evidence of cointegration[11] between the tax burden and the size of the underground economy. Such cointegration implies the existence of Granger causality between these two variables. We have tested for causality between (UE/GDP) and (TR/GDP) in both linear and logarithmic terms, using the Toda-Yamamoto (1995) approach. Somewhat surprisingly we found no evidence of Granger causality[12], but the theoretical considerations discussed in section 2 make it clear that (UE/GDP) should be the dependent variable in our empirical analysis.

5. Nonparametric estimation

In view of the above cointegration results, we may legitimately model the long-run relationship between (UE/GDP) and (TR/GDP) by using the "levels" of these two ratio variables. No differencing of the data is needed. As another option, we could construct an error-correction model, which would be appropriate if we were interested in the short-run dynamics of the relationship. Consistent with the theoretical literature, our nonparametric estimation is focussed only on the first of these two possibilities, and our model takes the simple form:

$$(UE/GDP)_t = m\{(TR/GDP)_t\} + \varepsilon_t \ , \qquad (3)$$

where m is the conditional mean of the dependent variable and the ε_t's are Normal, independent and homoskedastic.

Estimation was undertaken with the NONPAR routine in the SHAZAM (1997) econometrics package, using the Nadaraya-Watson estimator with a Normal kernel, and the bandwidth parameter was chosen by Silverman's (1986, p.45) "optimal" method. Some experimentation verified that the results were not particularly sensitive to either of these choices. We also considered versions of (3) that included the growth of real GDP, and lagged values of (UE/GDP) or (TR/GDP) as additional explanatory variables. However, none of these other regressors were significant, and our preferred simple estimates are given in Table 3.

The model's within-sample "predictions", with the (TR/GDP) variable sorted into ascending order, appear in Figure 2. The general shape of this relationship supports the earlier use of a logistic function by Caragata and Giles (1998) and by Giles and Caragata (1999). Moreover, the estimated derivative of (UE/GDP) with respect to (TR/GDP), which varies with each data-point, is positive at all but two points in the sample. Negative slopes (with values of -0.054 and -0.044 respectively) are estimated only in 1990 and 1991, but these values are not significantly different from zero. In other years the estimated nonparametric slopes range in value from 0.026 (in 1994) to 0.372 (in 1972).

Table 4 shows the corresponding predicted values for (UE/GDP), together with lower and upper limits for the corresponding 95% prediction interval, and the estimated elasticities. The predictions range in value from 7.52% to 9.51% of GDP. These values should be compared with the "actual" sample values for (UE/GDP) given by Giles (1999), which range from 6.84% to 11.31%.

6. Further economic implications

The corresponding estimated elasticities between the effective tax rate and the (UE/GDP) ratio also appear in Table 4. As these two variables are already expressed in percentage terms, these elasticities must be interpreted with care. For example, 1986 was an interesting year in the history of New Zealand taxation policy, with the introduction of the Goods and Services Tax (GST) in October, and the simultaneous major changes to sales taxes and to the personal income tax and corporate income tax schedules[13]. In 1986 the effective tax rate was 29.85% and the estimated underground economy elasticity was 0.303. This means that a 10% cut in the effective tax rate would lead to a 3.03% drop in the %(UE/GDP). A 10% cut in the effective tax rate means reducing it from 29.85% to 26.87%. The %(UE/GDP) in New Zealand in 1986 was 9.23%. So, it is predicted that the size of the underground economy would have dropped by 3.03% (that is, it would have dropped from 9.23% of GDP to 8.95% of GDP) in that year[14] had the tax burden been reduced by 10%, without any change in the "tax-mix".

In Table 5 we see the results of "simulating" the estimated relationship in (3) to get predicted values for (UE/GDP) as the effective tax rate ranges *hypothetically* in value from 17% to 38%. It is especially interesting to note that as the tax rate is decreased, the effect on the underground economy "flattens out" at a tax rate of about 20%. This accords remarkably well with the conclusions of Caragata and Giles (1998) and of Scully (1996). The former authors concluded that the "optimal" effective tax rate is approximately 21% in terms of the "acceleration rate" of the impact of tax changes on the underground economy being maximized. Scully's results suggested a growth-maximizing effective tax rate of 20% in the case of New Zealand. Interestingly, therefore, there is a close consistency between the tax rate that needs to be targeted from an economic growth viewpoint, and the one that needs to be targeted from a compliance viewpoint.

7. Conclusions

In this paper we have used nonparametric time-series regression to examine some of the predictions of a class of theoretical models of the underground economy. In particular, we have interpreted the recent model of Trandel and Snow (1999) in aggregate terms, and have provided empirical evidence concerning the partial relationship between the relative (output) size of the underground economy and the effective tax rate, using New Zealand data. This theoretical model predicts an ambiguous sign for the above relationship, but empirically we find a positive, "S-shaped" relationship that fits the data well over our sample period, 1968 to 1994.

When we simulate the model over a plausible range of tax rate values we obtain results that accord closely with other related results by Caragata and Giles (1998) and Scully (1996), each of whom address the issue of an "optimal effective tax rate" for New Zealand from different perspectives. In particular, our nonparametric model suggests that the responsiveness of the underground economy to simple changes in the tax burden falls markedly when the effective tax rate drops below about 20%.

The possibility that the adjustment of the underground economy to changes in the tax burden is asymmetric (in the upward and downward directions) is an important issue that is not discussed here. This is currently being explored by the authors in the context of both the New Zealand and Canadian economies.

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Footnotes

1. A "linear" tax system is one in which the tax rate is a fixed proportion of income - it is a "flat tax" system with a zero exemption-threshold. If the tax system is linear then any fraction of undeclared income can also be represented as a fraction of evaded tax.

2. The tax schedule in New Zealand was simplified considerably during our sample period. With respect to corporate taxes, the tax structure assumed in the Trandel-Snow model is closely approximated. In the case of personal income taxes the statutory rate is certainly progressive, with a very simple scale (in recent years), but with a zero "threshold" level.

3. The accuracy of Giles' (1999) aggregate time-series measure of the underground economy in New Zealand is supported by the independent micro-evidence, based on Revenue New Zealand business audit records, discussed by Giles (1998).

4. In addition, as can be seen in Figure 1, both the effective tax rate and the relative size of the underground economy in New Zealand exhibit considerable cyclical variation over our sample period. This implies that our analysis is based on "informative" empirical evidence.

5. The penalty for evasion is not counted as part of generated "income", but this does not affect the conclusions below.

6. Here we are considering the expected value of the ratio of underground to measured income. Alternatively, one could consider the ratio of expected underground income to expected measured income. A corresponding analysis of this different concept yields the same implications as below.

7. Recall that $y'_n(a) > 0$ and $y'_e(a) < 0$.

8. This rule sets l = INT $[8(T/100)^{1/4}]$.

9. For our sample size the asymptotic and finite-sample KPSS critical values are very similar, so the unavailability of the latter in the case of a level-stationary null should not be of concern.

10. We also tested the logarithms of the two variables for unit roots, and found *log*(UE/GDP) to be I(0) and *log*(TR/GDP) to be I(1). Accordingly, there can be no cointegration between these two transformed variables, and it would be inappropriate to model the relationship between them without differencing the later variable. We have not pursued this possibility here.

11. The tests for cointegration are actually tests for a *linear* cointegrating relationship between (UE/GDP) and (TR/GDP). In our case, the OLS residuals from this cointegrating relationship suggest that a *non-linear* relationship may be more appropriate. (The Durbin-Watson statistic is 1.280, with an exact p-value of 0.015; the LM tests for fifth-order and sixth-order autocorrelation are significant at the 1% level; and the $RESET_2$ test statistic is 4.976, with a p-value of 0.035.) This further supports our use of a nonparametric specification below.

12. This negative result no doubt reflects our (necessarily) small sample size and/or the need for a multivariate testing framework. For example, in the context of our underground economy data, the causality results of Giles (1997) are found by Johnson (1998) to be sensitive to the dimensionality of the underlying VAR model.

13. Initially the GST was levied at a rate of 10%, wholesale taxes were abolished and the top marginal personal income tax rate was reduced from 66% to 48%. Other major changes to these rates have taken place subsequently.

14. Note that this is a drop of 3.03%, and **not** a drop of 3.03 percentage points.

Table 1. ***Unit root test results***

a. Augmented Dickey-Fuller tests [a]

		T	p	t_{dt}	F_{ut}	t_d	F_{ud}	t	Outcome
					UE/GDP				
H_0 :	I(2)	23	2	-3.44	n.a.	n.a.	n.a.	n.a.	Reject I(2)
[H_A :	I(1)]								
H_0 :	I(1)	24	2	-2.70	3.66	-1.63	2.14	1.06	I(1)
[H_A :	I(0)]								
					TR/GDP				
H_0 :	I(2)	23	2	-2.62	3.43	-2.65	n.a.	n.a.	Reject I(2)
[H_A :	I(1)]								
H_0 :	I(1)	24	2	-2.42	3.21	-1.40	2.73	1.63	I(1)
[H_A :	I(0)]								

b. KPSS tests [b]

		T	Level-Stationary		Trend-Stationary		Outcome
			l=0	***l=5***	***l=0***	***l=5***	
				UE/GDP			
H_0 :	I(0)	27	1.519	0.520	0.131	0.110	I(1)
[H_A :	I(1)]						
				TR/GDP			
H_0 :	I(0)	27	0.099	0.073	0.129	0.081	I(1)
[H_A :	I(1)]						

Notes: **a.** The outcomes are based on finite-sample 10% critical values from MacKinnon (1991).

b. The outcomes are based on finite-sample 10% critical values from Cheung *et al.* (1995), and the KPSS 10% asymptotic critical values.

Table 2. ***Cointegration test results***

a. Cointegrating Regression Augmented Dickey-Fuller "t-tests"

T	**p**	**No Trend**		**Trend**		**Outcome**[a]
		R^2	**t**	**R^2**	**t**	
27	0	0.40	-3.434 (-3.57) [-3.20]	0.58	-3.856 (-4.15) [-3.77]	Cointegration

b. Leybourne-McCabe tests

T	**h_1**	**Bootstrapped p-value**	**Outcome**[b]
27	0.122 (0.31) [0.23]	0.07	Cointegration

Notes: **a.** MacKinnon's (1991) finite-sample 5% (10%) critical values appear in parentheses (brackets).
b. Leybourne and McCabe's (1993) asymptotic 5% (10%) critical values appear in parentheses (brackets). The bootstrapped p-value is based on 10,000 replications.

Table 3. ***Nonparametric estimation of the relationship between (UE/GDP) & (TR/GDP)***
(Simple nonparametric regression)

Bandwidth Parameter	0.548		
R^2 (Adjusted R^2)	0.492	(0.428)	
Cross-Validation Mean Square Error	0.794		
AIC (SC) [FPE]	0.842	(1.016)	[0.844]
Residuals Analysis			
Durbin-Watson Statistic	1.433		
Runs Test, Normal Statistic (p-value)	-0.975	(0.330)	
Coefficient of Skewness (Standard Deviation)	0.956	(0.448)	
Coefficient of Excess Kurtosis (Standard Deviation)	0.665	(0.872)	
Jarque-Bera, Chi-Square, asy. $\chi^2(2)$ (p-value)	3.681	(0.159)	
Chi-Square Goodness of Fit, asy. $\chi^2(3)$ (p-value)	6.094	(0.107)	
LM(1), asy. Standard Normal (p-value)	1.006	(0.157)	
LM(2), asy. Standard Normal (p-value)	0.208	(0.418)	
LM(3), asy. Standard Normal (p-value)	0.205	(0.419)	
LM(4), asy. Standard Normal (p-value)	0.152	(0.440)	
RESET(2), AIC (SC) [FPE]	0.843	(1.129)	[0.849]
RESET (2,3), AIC (SC) [FPE]	0.965	(1.462)	[0.988]
RESET(2,3,4), AIC (SC) [FPE]	1.013	(1.594)	[1.045]

Table 4. ***Ranked nonparametric within-sample predictions and 95% confidence limits***

Lower	Predicted (UE/GDP)%	Upper	(TR/GDP)%	Elasticity
7.080	7.525	7.970	23.643	0.467
7.124	7.559	7.994	23.859	0.531
7.155	7.584	8.013	24.003	0.577
7.510	7.911	8.312	25.287	1.053
7.655	8.053	8.452	25.693	1.178
7.667	8.065	8.464	25.726	1.186
8.163	8.557	8.952	26.996	1.157
8.495	8.888	9.281	28.075	0.754
8.506	8.899	9.292	28.123	0.736
8.620	9.013	9.407	28.696	0.536
8.665	9.059	9.454	28.992	0.453
8.706	9.103	9.499	29.328	0.379
8.716	9.113	9.511	29.423	0.362
8.755	9.157	9.559	29.852	0.303
8.794	9.205	9.616	30.412	0.267
8.804	9.217	9.631	30.563	0.264
8.804	9.217	9.631	30.564	0.264
8.824	9.244	9.665	30.904	0.267
8.832	9.256	9.680	31.047	0.272
8.854	9.286	9.717	31.403	0.290
9.003	9.478	9.952	33.432	0.295
9.017	9.497	9.977	33.681	0.259
9.026	9.511	9.996	33.884	0.224
9.034	9.529	10.024	34.220	0.158
9.033	9.540	10.046	34.527	0.094
8.941	9.525	10.109	35.843	-0.167
8.908	9.513	10.119	36.080	-0204

Table 5. ***Simulated values of (UE/GDP)% for various effective tax rates***

(TR/GDP) (%)	(UE/GDP) (%)	
	Nonparametric	Logistic
17	7.314	6.519
18	7.307	6.668
19	7.306	6.821
20	7.312	6.977
21	7.331	7.136
22	7.370	7.299
23	7.445	7.465
24	7.584	7.634
25	7.821	7.808
26	8.170	7.984
27	8.559	8.164
28	8.870	8.348
29	9.060	8.536
30	9.171	8.728
31	9.252	8.923
32	9.340	9.122
33	9.438	9.326
34	9.518	9.533
35	9.545	9.745
36	9.517	9.960
37	9.451	10.180
38	9.367	10.405

Figure 1. Effective tax rate and underground economy, New Zealand 1968-1994

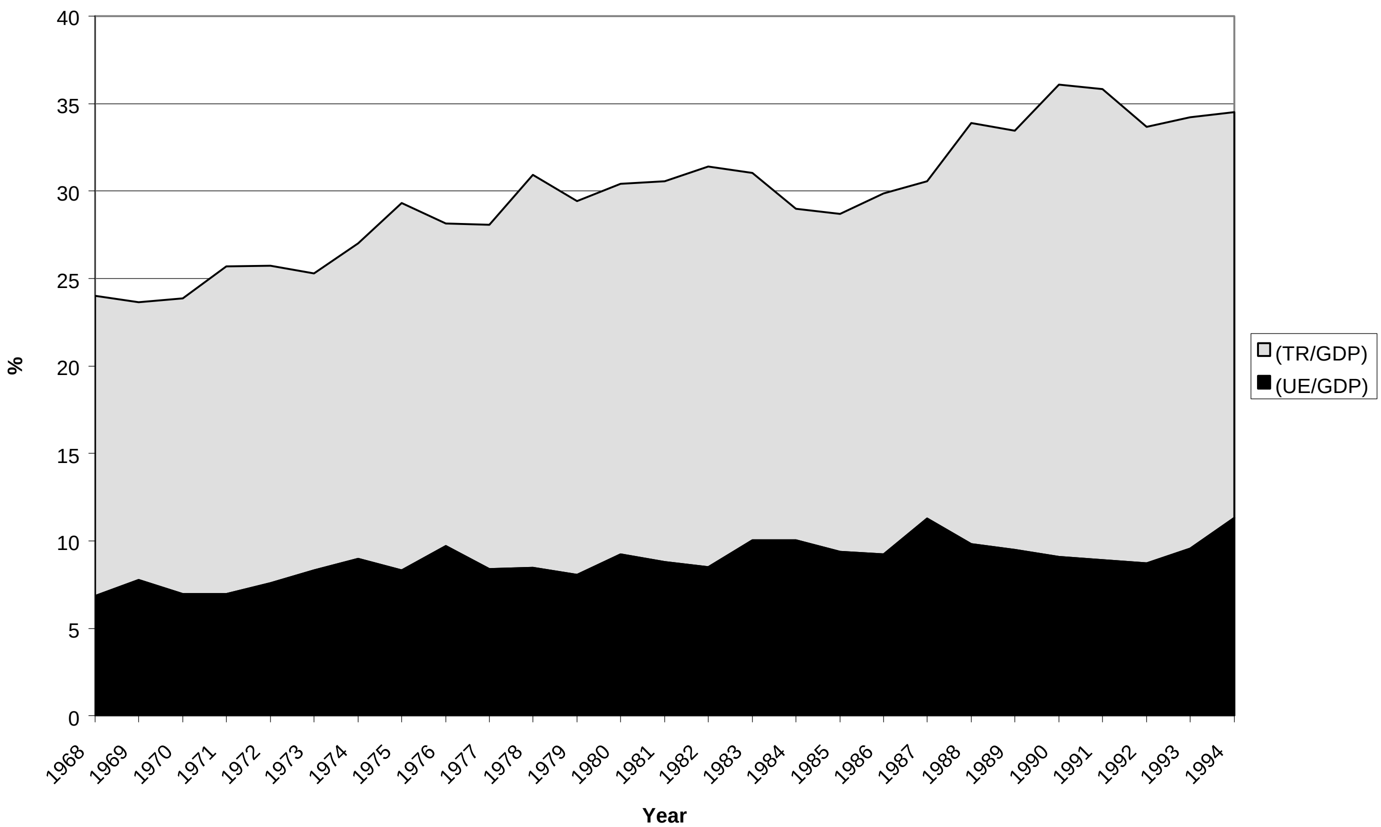

Figure 2. Non-parametric relationship between underground economy and effective tax rate: New Zealand, 1968-1994