The Timing and Scale of Investment Under Uncertainty

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Abstract

This paper studies optimal investment in sunk assets for a firm facing uncertain future demand and costs. The investment problem is decomposed into decisions over scale and timing which are influenced by convex adjustment costs and the feasibility of delaying investment. The value of the resulting real option to delay behaves as expected in response to increases in demand and cost uncertainty but has new timing and scale interpretations. The larger is the option value, the more capital would be installed if delay were not profitable. The timing of investment, however, is controlled by the expected trajectory of capital prices relative to the firm's discount rate. The analysis also suggests an empirical model for aggregate investment which explains 96% of the variation in real annual investment for Canada over the period (1981-94) using regressors formed from only three variables: business confidence, capital prices and real bond rates.

1 Introduction

The microeconomic analysis of investment remains an active research area, despite the very considerable progress that has been made over the last few decades. Much of the recent work is conducted within the "real options" paradigm, where the joint impact of uncertainty and the sunk nature of capital investment is explicitly modeled. Uncertainty is thought to have two distinct effects which work in opposition to each other. Consequently, the net effect of uncertainty on investment is regarded as ambiguous.

The first effect of uncertainty was established by Hartman (1972) and Abel (1983) and arises from the fact that profit functions are convex in prices and operating costs. By a direct application of Jensen's inequality (Jensen (1906)), the introduction of uncertainty into prices or operating costs will therefore increase expected profits, raising the rate of return and increasing the value of investment. This is a pure uncertainty effect arising from the ability of the firm to optimise over output levels in response to short-run fluctuations in the strength of demand.

The second effect focuses on the irreversibility of investment, and has a lineage that dates back to Arrow (1968). The link between irreversibility and uncertainty was made by McDonald and Siegel (1986) and is extensively covered by Dixit and Pindyck (1994). This line of work shows that the ability to delay investment confers on the firm a call option over the future income stream from the proposed real asset. A rational firm will therefore delay investment until the expected return compensates for the cost of capital plus the opportunity cost of "killing" the delay option.

The relationship between these two effects has been studied in an option value framework by Abel et al (1996). Their model highlights the influence of two different real options: the option to delay further investment, and the option to reverse previous investment. Since the value of both of these options increases with uncertainty, but they have opposing effects on investment, the impact of uncertainty is again found to be ambiguous.

Our analysis also treats both types of uncertainty in a single model, but does so with a different purpose than Abel et al (1996). The aim here is to separately identify and analyse the timing and scale decisions that jointly determine the optimal rate of investment. This distinction is important for at least two reasons. First, it closely approximates reality in the sense that decisions about fixed investment are usually made periodically rather than continuously. Secondly, it admits the possibility that quite different investment profiles can give rise to the same rate of investment. At one extreme a firm may add large amounts of capacity very infrequently, so that excess capacity or congestion occur frequently. An alternative strategy of adding capacity in line with expected demand each period could produce the same rate of investment but look very different to analysts, consumers and portfolio investors. By decomposing the optimal rate of investment into its scale and timing dimensions, this paper shows how the uncertainty over future parameters affects these distinct decisions.

The model is a very simple blend of the key ideas from the adjustments costs and real options branches of the investment literature. The adjustment costs approach of Hartman (1972) and Abel (1983) is well suited to studying the scale aspects of investment, but it needs to be modified to admit the timing issues highlighted in the real options literature. Conversely, the real options approach is very useful in analysing timing but tends to abstract from issues of scale. Within this latter tradition, the models of incremental investment by Pindyck (1988) and Dixit and Pindyck (1994, chapter 11) are most similar to this paper but are not readily adapted to looking at the question of optimal scale.

Indeed, the whole class of continuous time models, such as those that follow naturally from the geometric Brownian motion demand processes used in most of the real options literature, are ill-suited to our purpose. An explicit and non-trivial periodicity is necessary to separate the timing and scale aspects of the investment decision. Consequently, we follow Abel *et al* (1996) in using a simple discrete time model, but one which is sufficiently flexible to represent the important option value effects.

Two innovative features are embedded in the model, both of which relate to the firm's choice between investing in the current period and delaying. The focus on investment scale leads us to depart from the implicit assumption in the real options literature that next period's project is the same as this period's. Instead, the firm forms an expectation about the size of the project that would be installed in the next period, in the event that it delays investment. Secondly, in comparing the value to the firm of current versus future investment, we apply an expected rate of economic depreciation rather than an exogenously specified rate.

These assumptions, combined with the simplicity of the basic model, enable us to derive a new expression for the value of the delay option (henceforth referred to as either the real option or the option value). Consistent with the previous literature, the real option is greater, the more uncertain is future demand. Unlike existing work, however, we do not interpret larger option values as reducing the "incentive to invest". If anything, the reverse is true. Larger option values arise because the anticipated scale of investment is growing, so the incentive to invest is getting stronger. Investment remains withheld, however, until the expected growth rate in capital prices exceeds the firms discount rate. Thus, large option values indicate the potential for a lot of investment, which can occur suddenly if firms change their view about the expected time path of capital prices. The new interpretation our model gives to the real option value is one of the main contributions of the paper.

An additional advantage of this analysis is that the model also suggests a new empirical approach to explaining the level of real investment at the industry and national levels of aggregation. Specifically, the model predicts that investment is determined by three variables: business confidence, the price of capital goods, and a dummy variable that is zero when the option value is positive and one otherwise. The real option dummy is formed using current and expected future capital prices, and a measure of the discount rate. Thus, the three underlying explanatory variables are business confidence, capital prices and real interest rates. A linear regression based solely on the specification suggested by this model explains 96% of the variation in real investment in Canada over the period 1981-1994.

The outline of the remainder of the paper is as follows. The next section explains the basic set-up, in which the firm determines its optimal period t investment level by trading off convex adjustment costs against expected returns. The main theoretical results of the paper are in Section 3. This is followed in Section 4 by a brief analysis of the way that economic regulation affects the firm's investment decisions. The model is empirically implemented in Section 5 using aggregate investment for Canada, and some concluding comments are offered in Section 6.

2 The Model

We specify a production function for period t which relates output Q_t to inputs of capital K_t and labor L_t as follows:

$$Q_t = F(K_t, L_t)$$

where $F(.)$ is homogeneous[1] of degree one in K_t, and concave and non-decreasing in each input. At the beginning of each period the firm selects its preferred level of labor and installs additional capital equal to I_t with the result that the capital stock evolves according to

$$K_{t+1} = (1 - \delta_t)K_t + I_t$$

where δ_t is the rate of economic depreciation over period t. By installing capital the firm incurs an investment cost equal to $C(I_t, q_t) = q_t C(I_t)$ where q_t is the price of capital goods at time t. We assume that $C(I_t)$ is increasing, that the first and second partial derivatives $C'(I)$ and $C''(I)$ exists for all possible I, that $C''(I) = k > 0$, a constant known to the firm, and that $C(0) = 0$. Apart from the restriction that $C''(I) = k$, which assists us to derive clean results below, these assumption are standard in the adjustment cost literature.

At the beginning of each period the firm selects I_t and L_t with the objective of maximising the following expected sum of discounted cash flows

$$E_t \sum_{t=1}^{\infty} r^{t-1}(p_t Q_t - w_t L_t - q_t C(I_t))$$

where the expectation E_t is formed using the period t information set, which includes all t dated magnitudes. In this equation, r is the firm's real discount rate, and $p_t = p(Q_t)$ and w_t are the price and wage rates at time t respectively. We assume that future values of Q_t, w_t and q_t have random components, possibly in addition to some deterministic trends. Our use of Q_t as the measure of demand strength is not standard in the investment literature. This choice is convenient for our discussion of regulated firms below and switching to a random p_t would not affect any of our results.

Using the homogeneity of the production function, the maximum profit attainable in period t can be written as

$$h(K_t, Q_t, w_t) = K_t g(Q_t, w_t)$$

[1]This assumption was used by Hartman (1972) in deriving the first uncertainty result within the adjustment costs paradigm. Although slightly restrictive, its impact is modified by two other features of the model. First, adjustment costs are convex so there is some decline in the marginal productivity of new capital. Second, the real option derivation only requires comparison of two levels of investment with the result that homogeneity behaves like a linear approximation to the production function between these levels.

where $g(.)$ is convex in both arguments as a consequence of the firm adjusting the variable input (labour) to maximise period t profits given current demand and cost information. The current value of the firm at any time t depends on the installed capital base and expectations about the future evolution of the random variables as follows:

$$E_t(V_t) = K_t E_t \sum_{t=1}^{\infty} r^{t-1}(1-\delta)^{t-1} g(Q_t, w_t). \quad (1)$$

We are primarily interested in how the firm's investment decisions depend on uncertainty in its future environment. Several preliminary observations will assist the exposition below. First, we will consider the effect of increasing the variance of the underlying random variables without changing their expected values; *i.e.* by introducing a mean preserving spread into the variable. This will increase the expected value of a convex function of the random variable, by Jensen's inequality. We can use (1) to illustrate this idea: uncertainty about future values of Q_t or w_t increases the expected value of the firm since $g(Q_t, w_t)$ is convex. Secondly, since the only way that Q_t and w_t enter the firm's decision making in the remainder of this paper, and since uncertainty in each of them has the same effect, we can conveniently restrict attention to uncertainty in Q_t and interpret this as "operational uncertainty" which could equally arise through variation in w_t. Finally, for notational convenience we define the following variables

$$R_1 = g(Q_t, w_t) \quad (2)$$

$$R_2 = E_t \sum_{t=2}^{\infty} r^{t-1}(1-\delta)^{t-1} g(Q_t, w_t) \quad (3)$$

and observe that R_1 is not random, while R_2 is a convex function of the random operational variable Q_t.

The final preliminary needed before we address the investment decision is the rate of depreciation of capital over the current period. Depreciation in periods beyond the current one are not relevant when the firm decides whether to invest now or wait one period. Consequently, the manner in which the firm establishes the δ in (3) does not concern us. Period t depreciation is important however, and we consider this now.

2.1 Economic Depreciation

It is well known that the economic depreciation of capital arises from the erosion of two stocks: total willingness to pay for the services of the asset, and total service life of the asset. Reductions in either or both of these stocks reduce the value of the firm holding the asset, and we can therefore define the rate of depreciation of capital over period t reference to the expected value of the net cashflows.

$$\delta_t = E_t \frac{V_t - V_{t+1}}{V_t}$$

which can be rewritten by substituting (2) and (3) into (1) as follows

$$\begin{aligned} \delta_t &= \frac{K_t(R_1 + R_2) - K_t R_2}{K_t(R_1 + R_2)} \\ &= \frac{R_1}{R_1 + R_2} \end{aligned} \tag{4}$$

This shows the relationship between economic depreciation over the period and the share of total lifetime discounted net cash-flows that the asset delivers over the period. The use of economic depreciation is important here because it has direct links to the value of the firm. Furthermore, it is apparent from (4) that δ_t is itself a random variable, since it depends on R_2. For future reference we note that δ_t is a decreasing convex function of R_2 which is itself an increasing convex function of Q_t. Hence operational uncertainty, which has the effect of increasing the expected value of R_2 will reduce the expected rate of economic depreciation δ_t. Furthermore, since lower depreciation rates increase R_2, as can be seen in (3), the effect of operational uncertainty on the value of the firm is unambiguously positive.

3 The Scale and Timing of Investment

Having observed the random variables q_t and Q_t at the beginning of period t, the firm selects the level of investment to maximise the expected value of the firm. The result of this periodic capacity optimisation problem can be written as

$$E_t(V_t^*) = \max_{I_t}(K_t + I_t)(R_1 + R_2) - q_t C(I_t) \tag{5}$$

and the solution is obtained by selecting the optimal level of investment I_t^* for which

$$C'(I_t^*) = \frac{R_1 + R_2}{q_t}$$

where it should be noted that q_t has been observed and hence is not random. The optimal scale for investment at time t is therefore given by

$$I_t^* = C'^{-1}\left(\frac{R_1 + R_2}{q_t}\right) \tag{6}$$

Now, using the curvature of $C(.)$, and the convexity of R_2 we can derive the impact of uncertainty on I_t^*. Using Jensen's inequality, it is apparent that uncertainty in the form of a mean preserving spread in future values of Q_t will increase R_2. This will increase the scale of investment because $C(I)$ is convex, a result first observed by Hartman (1972). Uncertainty in future capital prices has no effect on I_t^* because only (known) current prices enter (6).

Our assumptions on $C(I)$ allow some convenient refinement of (6). Since $C'' = k > 0$, we have $C'(x) = kx$ and $C'^{-1}(x) = \frac{x}{k}$ so that:

$$I_t^* = \frac{R_1 + R_2}{kq_t} \tag{7}$$

Having characterised the level of investment that would be chosen if the firm were to invest at time t, we now need to consider whether this action is optimal, given the irreversibility of investment and the feasibility of delaying it. To address this we need to compare $E_t(V_t^*)$ with the maximum value of the firm when it delays investment by one period, denoted $E_t(V_{t+1}^*)$ to indicate that the expectation is formed one period ahead of the date at which investment would occur.

In evaluating $E_t(V_{t+1}^*)$ it is important to recognise that the size of investment which would be optimal at time t may differ from that which the firm would expect to install in the event that it delays investment to period $(t+1)$. If demand is growing quickly, for example, the optimal strategy may involve a choice between adding more capacity to an existing facility, or waiting until next period and building an entirely new facility with some spare capacity[2]. Consequently, we express the expected value from delaying investment by one period as

$$E_t(V_{t+1}^*) = K_t(R_1 + R_2) + \max_{I_{t+1}} I_{t+1} \frac{R_2}{(1-\delta_t)} - rq_{t+1}C(I_{t+1}) \tag{8}$$

where the expectation operator has been suppressed for q_{t+1}. Note that R_2 is adjusted upwards by the factor $(1-\delta_t)^{-1}$ to account for the fact that capital installed in period $(t+1)$ will start depreciating one year later than capital installed in period t. Using (4), however, we can deduce that when economic depreciation is used $\frac{R_2}{(1-\delta_t)} = (R_1 + R_2)$ which will henceforth be used in evaluating (8). The first term on the RHS of (8) is the known profit that will accrue over the current period and has no impact on the investment decision. Using the remaining terms, we can see that the level of investment that is expected to be optimal at time $(t+1)$ is defined as

$$\begin{aligned} I_{t+1}^* &= C'^{-1}\left(\frac{R_1 + R_2}{rq_{t+1}}\right) \\ &= \frac{R_1 + R_2}{krq_{t+1}} \end{aligned} \tag{9}$$

By the same arguments used above in respect of (6), we can deduce that operational uncertainty increases (decreases) the scale of I_{t+1}^* because of the convexity of $C(I)$. In the case of anticipated investment in the next period, however, q_{t+1} is an additional source of uncertainty. Using the fact that (9) is convex in q_{t+1}, we can see that uncertainty over future capital prices will act in the same direction as the operational uncertainty, increasing I_{t+1}^*.

Before turning to the question of timing, we define the "optimal scale gap" by $\Delta I_t^* = (I_{t+1}^* - I_t^*)$ and note that, from (7) and (9), this is given by:

$$\Delta I_t^* = \frac{R_1 + R_2}{k}\left(\frac{q_t}{rq_{t+1}} - 1\right). \tag{10}$$

[2]In addition, the decision between current and future investment may well involve a choice between different vintages of technology. While serious examination of this issue is beyond the scope of this paper, the model used here would appear to be well suited to studying technological choice in competitive industries.

It is important for what follows to determine the curvature properties of (10) with respect to the random operational variable Q and capital price variable q_{t+1}. It is clear that ΔI_t^* is convex is q_{t+1} since this term enters through the demoninator of (9). This means that the expected size of the optimal scale gap increases with uncertainty in future capital prices. The situation is more complicated for operational uncertainty, however. To see this, note that the impact of changes in future values of Q is felt through R_2, so we can write

$$\frac{\partial^2 \Delta I_t^*}{\partial Q} = \frac{\partial^2 R_2}{\partial Q^2}\left(\frac{q_t}{r q_{t+1}} - 1\right) k^{-1}$$

We know that R_2 is convex in Q but this property only carries through to ΔI_t^* if the bracketted term is positive. We summarise the situation as follows.

Lemma 1 *The optimal scale gap ΔI_t^* is increased by operational uncertainty if $q_t > rE_t q_{t+1}$ and conversely.*

Having determined the optimal investment strategies for periods t and $(t+1)$, and the way that uncertainty affects these strategies, we are now ready to examine the timing issue. As emphasised by the real options literature, the choice between investing and delaying involves a forward looking evaluation of the profitability of each choice. Assuming that $E_t(V_t^*) > 0$, the firm will nevertheless only invest at time t if $E_t(V_t^*) \geq E_t(V_{t+1}^*)$, in which case the expected value of the firm is not increased by deferring investment for one period. To analyse the influences on this timing decision, we subtract (5) from (8), evaluating each at the optimal scale, and rearrange as follows

$$\begin{aligned}
\Delta V_t^* &= E_t(V_{t+1}^*) - E_t(V_{t+1}^*) \\
&= I_{t+1}^*(R_1 + R_2) - rq_{t+1}C(I_{t+1}^*) - I_t^*(R_1 + R_2) + q_t C(I_t^*) \\
&= \Delta I_t^*(R_1 + R_2) - rq_{t+1}C(I_{t+1}^*) + q_t C(I_t^*) \qquad (11)
\end{aligned}$$

We want to study the conditions under which the real option value ΔV_t^* is negative (positive) implying that immediate investment of I_t^* will (will not) occur. To analyse this, we use the derivation presented in the Appendix to rewrite (11) as follows.

$$\Delta V_t^* = \frac{(R_1 + R_2)^2}{2k} \left(\frac{q_t}{q_{t+1}} - r \right) \tag{12}$$

Consider first the effect of uncertainty in future demand and capital costs on the size of the option value ΔV_t^*. For a given structure of capital prices such that $\frac{q_t}{q_{t+1}} > r$, operational uncertainty increases the expected value of R_2, increasing ΔV_t^* and reinforcing the incentive to delay. Apart from the qualifying capital price condition, this result is consistent with previous literature in the real options tradition: greater operational uncertainty leads to higher option values through the convexity of R_2. Uncertainty over future capital prices increases the expected value of $\frac{q_t}{q_{t+1}}$ which also leads to higher option values. This finding is consistent with the results of Dixit and Pindyck (1994) who modeled uncertainty over capital prices and demand jointly, but without any explicit scale decision.

The most unusual aspect of (12) however, is the interpretation it gives to the roles of expectations about capital costs and demand. *Ceteris paribus*, the larger is expected revenue $(R_1 + R_2)$, the larger is the option value and the optimal level of investment I_t^*. So, growth in current and expected future demand increases both I_t^* and ΔV_t^* with the result that excess demand increases and existing capacity gets more congested. Equation (12), however, says that while strong demand is a necessary condition for investment; it is not sufficient. Until the expected future capital prices get high enough relative to the current price of capital, the option value will not drop to zero, and investment will be withheld.

What information is therefore conveyed by the size of the option value? From (12) and (7) it is clear that the size of the option value is postively correlated with the amount of capital that would be invested if waiting was not a better strategy. It does not provide any information about how soon investment will occur however. The timing of investment is completely controlled by the expected trajectory of capital prices.

This is relevant for a standard claim in the existing real options literature in which the size of the option value is interpreted as being related to the incentive to invest. A typical statement of this is: "an increase in the variance of future returns...increases the value of the call option, which decreases the incentive to invest" (Abel *et al* (1996)). Our results suggest that this is not an accurate characterisation, because the desired scale of investment increases with the real option value, albeit conditional on the timing decision. So, investment becomes more and more profitable as the real option increases, and (unless demand expectations turn sour) this greater investment will eventually occur as soon as the outlook for capital prices improves sufficiently.

Consideration of the other side of the investment goods market reinforces the view that higher option values herald more investment. Increases in the real option value signal greater potential demand for capital goods and stronger incentives for suppliers of capital to cut prices. Thus, under plausible conditions we would expect sustained increases in real option values to be followed by almost instantaneous falls to zero as the outlook for capital prices changes and investment occurs.

An important advantage of characterising optimal investment through the pair of simple equations (7) and (12) is that it facilitates several further lines of inquiry. We now offer a brief discussion of two extensions.

4 The Impact of Regulation

The above analysis has assumed that the firm is able to operate without reference to any official constraints. In this section we briefly consider the implications of economic regulation designed to prevent the firm from exercising any market power that it may otherwise have. Regulators typically fix the price (or equivalently the rate of return) and require that the firm serve all resulting demand, often imposing financial penalties for failing to do so. The maximum profit function for a regulated firm can therefore be written as follows:

$$h(K_t, Q_t, w_t) = \max_{L_t}[p_t F(K_t, L_t) - w_t L_t - f(Q_t - F(K_t, L_t))]$$

where $f(x)$ is the financial penalty imposed by the regulator for failing to provide x units of service when $x > 0$, and $f(x) = 0$ otherwise. When $f(Q_t - F(K_t, L_t)$ is large enough, the firm will always choose to set $F(K_t, L_t) = Q_t$ and the short run profit function reduces to

$$h(K_t, Q_t, w_t) = p_t F(K_t, L_t^*) - w_t L_t^* = K_t g(Q_t, w_t)$$

where L_t^* is chosen to ensure that $F(K_t, L_t^*) = Q_t$ and that $w_t = p_t F_L$. The firm's objective in period t is to maximise the expected discounted stream of such future contributions, and to invest in the way that supports this objective. The critical difference between the environment faced by the regulated firm and our previous analysis of an unregulated firm is that the former cannot optimise its output in each period. This has important implications for investment.

Given the regulated price p_t, the revenue component of $h(.)$ is simply a linear function of Q_t for regulated firms, while the cost component is increasing and convex in Q_t given our assumptions on $F(.)$. Thus, since $g(.)$ is the difference between these functions, it is concave in Q_t, and also in w_t by identical reasoning. Since the maximum profit a regulated firm can achieve in any period is concave in Q_t and w_t, uncertainty in these variables reduces the expected value of $g(.)$.

This is in stark contrast to the profit function of an unregulated firm which is convex in input prices and demand. Thus, by changing the shape of $g(.)$, economic regulation reverses the impact of operational uncertainty on the profitability of the firm. Assuming that investment by the regulated firm remains voluntary, and that economic depreciation is used, all of the optimal scale results derived above carry over to the regulated case, as does the option value definition. The interpretation of these expressions is altered, however, by the fact that R_2 is *concave* in future demand (and labour prices) for regulated firms.

Inspecting (7) and (12) with this in mind, we can see that operational uncertainty reduces both I_t^* and ΔV_t^* rather than increasing them as it does for unregulated firms. Regulation therefore unambiguously reduces both the scale of investment and the size of the option value. This does not necessarily affect the frequency of investment, however, because that is controlled by the expected time path of capital prices.

5 Macroeconomic Application

One of the most useful aspects of the approach taken here is the fact that it yields a testable prediction about the determinants of real investment. From (7) we see that changes in desired investment (in the absence of timing constraints) are related to changes in expected future revenues $(R_1 + R_2)$, current capital prices q_t and the cost of adjustment parameter k. The best available measures of changes in expected future revenues are indices of business confidence (which could also be thought of as indexing Keynes' famous "animal spirits"), and these are available for most developed economies. Capital prices are similarly readily available, so an estimable form of (7) can be written as

$$\ln(I_t) = \alpha + \beta_1 \ln(C_t) + \beta_2 \ln(q_t) + \varepsilon_t \tag{13}$$

where $e^\alpha = k$ and $\varepsilon_t \tilde{} N(0, \sigma^2)$. This specification will only hold when the real option (12) is non-positive, however, suggesting the need to construct a dummy variable defined by:

$$D_t = \begin{cases} 1 & \text{when } \frac{q_t}{q_{t+1}} < r_t \\ 0 & \text{when } \frac{q_t}{q_{t+1}} \geq r_t \end{cases} \tag{14}$$

For firm level investment, D_t will be a strict control on investment with the implication that each of the variables in regressions explaining firm level investment would be premultiplied by D_t. Since none of the variables in (13) and (14) are necessarily firm specific, however, this model could be readily estimated at an industry level. Indeed, notwithstanding the effects of aggregation, it may well have explanatory power for aggregate national investment. In these more general settings, however, a strict application of (14) would be inappropriate. This is because firms will vary in their expectations about whether D_t takes the value one or zero. Consequently, while interaction of D_t with the other variables in (13) would be a sensible addition to the model for aggregated data, the existing variables should be retained to account for heterogeneous expectations about capital prices.

Based on these ideas, we will estimate the following model:

$$\ln(I_t) = \alpha + \beta_1 \ln(C_t) + \beta_2 \ln(q_t) + \beta_4 D_t \ln(C_t) + \beta_5 D_t \ln(q_t) + \varepsilon_t \tag{15}$$

It remains to consider the definition of D_t where the main issue is how to model firms' expectations about future capital prices. For the purpose of illustrating the model, we have used the actual values of q_{t+1} which is equivalent to assuming that firms correctly predict this variable[3]. We allow

[3]Very similar results were obtained by using a backward looking measure q_{t-1}/q_t. This is equivalent to assuming that firms use the most recently observed capital price ratio as their estimate of the next ratio.

for slight errors in this prediction by weakening the condition under which $D_t = 1$ to include cases in which the discount rate is up to 1% below the expected capital price ratio.

The data for this application were all drawn from the CANSIM database. We use real annual GDP expenditure fixed investment (series D14456), the business conditions survey of Canadian manufacturers (series D262728) averaged over the quarters of each year, a chained price index of gross fixed capital formation (series D15621) averaged over the quarters of each year. The real discount rate r_t was formed by subtracting annual Canadian CPI inflation (formed using series D15614) from the USA long term bond rate (series B54403) averaged over the months of each year.

The following results were obtained using OLS estimation in the SHAZAM (1997) software:

$$\ln(\widehat{I}_t) = \underset{(8.00)}{4.47} \; \underset{(-3.14)}{-0.32\ln(C_t)} \; \underset{(15.07)}{+1.78\ln(q_t)} \; \underset{(5.65)}{+0.94 D_t \ln(C_t)} \; \underset{(-5.56)}{-0.67 D_t \ln(q_t)}$$

where the figures in parentheses are t ratios with 9 degrees of freedom. The R^2 from this model is 0.96, the DW statistic is 1.49 which allieviates any concerns about a possibly spurious regression. There is no evidence of heteroscedasticity based on several standard LM tests. An F test on the joint significance of the terms involving D_t emphatically rejects the hypothesis that these have no effect. Figure 1 plots the actual and fitted values from this model and displays a remarkably good fit, especially considering the absence of any lagged variables in the estimated equation.

Based on these regression results, the investment equation derived from this model has significant explanatory power even well beyond the level of the firm. Given the persistent difficulties that economists have had over the specification of investment equations, further investigation of this option based approach to explaining aggregate investment would appear to be warranted. Apart from applications to other economies, our implementation could be readily improved on with more attention to the precise timing of the variables, and more sophisticated econometrics particularly in respect of the treatment of the dummy variable. In addition it may also be useful is to consider a full model of the capital goods sector with (15) being the demand side of this market.

6 Conclusion

This paper has developed a simple new model of investment which combines adjustment costs and delay options in a discrete time framework. The model focusses on analysing the distinct decisions over the scale and timing of investment which jointly determine the investment rate. The real option value derived from this model highlights the importance of expected changes in the price of capital for investment, and leads to a new interpretation of large option values. These have previously be thought of as signalling a reduced incentive to invest, but the analysis here suggests that if anything the reverse is true. If option values are large and growing, investment is getting more profitable and more funds will eventually be committed as long as demand remains strong. The trigger which causes firms to commit arises from a change in the outlook for capital prices.

This approach also yields an estimable investment equation in which the option value is represented by a dummy variable formed by relating capital prices to discount rates. Using this approach, a linear regression on data formed from only three variables (business confidence, capital prices and bond rates) is able to explain 96% of the variation in real investment in Canada over the period 1981-1994.

Further work along both theoretical and empirical lines seems warranted. The model could be extended to include strategic interactions between competitors in which both capital investment period specific pricing contribute to the value of the firm. An additional theoretical task, which was only briefly considered here, is to derive the implications of real options for regulated industries. This is rather urgent given the fact that regulators routinely set the operating environments in which firms either invest, or do not. There would also appear to be considerable scope for further useful empirical work based on this model, including applications to other data sets, refinement of the econometric methodology and the inclusion of option-based investment equations in macroeconometric models.

7 References

Arrow, K. J. (1968) "Optimal Capital Policy with Irreversible Investment" in J.N Wolfe (ed.) Value Capital and Growth, Essays in Honour of Sir John Hicks, Edinburgh University Press: Edinburgh.

Abel, A.B. (1983) Optimal Investment Under Uncertainty, American Economic Review, 73, pp. 228-233.

Abel, A.A., A.K. Dixit, J.C. Eberly and R.S. Pindyck (1996) "Options, The Value of Capital and Investment", Quarterly Journal of Economics, 111, pp. 753-777.

Dixit, A.K. and R.S. Pindyck (1994) "Investment Under Uncertainty", Princeton University Press: Princeton.

Hartman, R. (1972) "The Effects of Price and Cost Uncertainty on Investment", Journal of Economic Theory, 5, 258-266.

Jensen, J.L.W.V. (1906) Sur les Fonctios Convexes et les inegaites entre les valeurs moyennes, Acta Math., 30, pp.175-193.

Majd, S. and R.S. Pindyck (1987) "Time to Build, Option Value and Investment Decisions", Journal of Financial Economics, 18, pp. 7-27.

McDonald, R. and D. Siegel (1986) "The Value of Waiting to Invest" Quarterly Journal of Economics, 101, pp. 707-728.

SHAZAM, (1997) Users Reference Manual, McGraw Hill: New York.

8 Appendix

From (11), the definition of the option value is given by

$$\Delta V_t^* = \Delta I_t^*(R_1 + R_2) - rq_{t+1}C(I_{t+1}^*) + q_t C(I_t^*) \tag{16}$$

and we know from (10) that

$$\begin{aligned} \Delta I_t^* &= \frac{R_1 + R_2}{k} \frac{(q_t - rq_{t+1})}{rq_t q_{t+1}} \\ &= \frac{R_1 + R_2}{k} \left(\frac{q_t}{rq_{t+1}} - 1 \right). \end{aligned} \tag{17}$$

Observe the following consequences of the restrictions imposed on the adjustment cost function, namely that $C''(I) = k \neq 0$.

$$C'(I) = kI \tag{18}$$

$$C(I) = \frac{k}{2} I^2 \tag{19}$$

where the second result uses $C(0) = 0$ to eliminate the constant of integration. Now we can use (7) and (9) in combination with (19) to obtain the following results

$$C(I_t^*) = \frac{k}{2} \left(\frac{R_1 + R_2}{kq_t} \right)^2 \tag{20}$$

$$C(I_{t+1}^*) = \frac{k}{2} \left(\frac{R_1 + R_2}{rkq_{t+1}} \right)^2 \tag{21}$$

Now substitute (17), (20), and (21) into (16) to get

$$\begin{aligned} \Delta V_t^* &= \frac{(R_1 + R_2)^2}{k} \left(\frac{q_t}{rq_{t+1}} - 1 \right) - rq_{t+1} \frac{k}{2} \left(\frac{R_1 + R_2}{rkq_{t+1}} \right)^2 + q_t \frac{k}{2} \left(\frac{R_1 + R_2}{kq_t} \right)^2 \\ &= \frac{(R_1 + R_2)^2}{k} \left(\frac{q_t}{rq_{t+1}} - 1 \right) - \frac{(R_1 + R_2)^2}{2rkq_{t+1}} + \frac{(R_1 + R_2)^2}{2kq_t} \end{aligned}$$

$$
\begin{aligned}
&= \frac{(R_1+R_2)^2}{k}\left(\frac{q_t}{rq_{t+1}}-1\right)-\frac{(R_1+R_2)^2}{2k}\left(\frac{q_t}{rq_{t+1}}-1\right) \\
&= \frac{(R_1+R_2)^2}{2k}\left(\frac{q_t}{rq_{t+1}}-1\right) \\
&= \frac{(R_1+R_2)^2}{2k}\left(\frac{q_t}{q_{t+1}}-r\right)
\end{aligned}
$$

This establishes the result cited in (12).

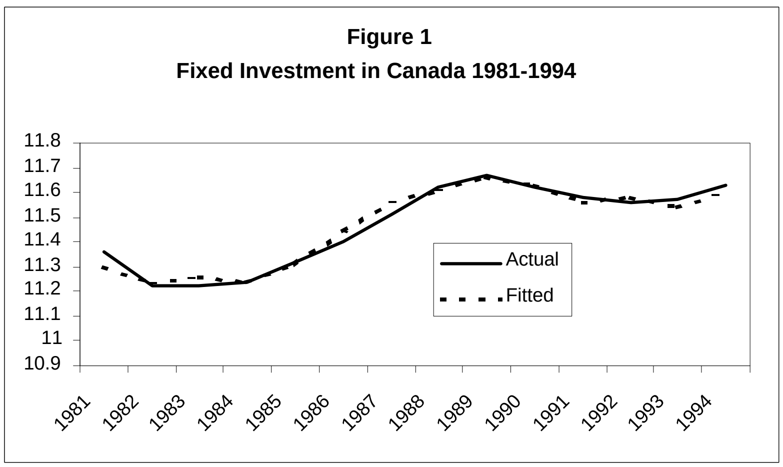
Figure 1
Fixed Investment in Canada 1981-1994
11.8
11.7
11.6
11.5
11.4
11.3
11.2
11.1
11
10.9
Actual
Fitted
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994

Figure 1: