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# Factor substitution, factor-augmenting technical progress, and trending factor shares: the Canadian evidence

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# Factor substitution, factor-augmenting technical progress, and trending factor shares: The Canadian evidence


## Abstract

Revised productivity accounts recently released by Statistics Canada are used to estimate a Klump-McAdam-Willman normalized CES supply-side system for the half-century 1961–2010. The model permits distinct rates of factor-augmenting technical change for capital and labour that distinguish between short-term versus long-term effects, as well as a non-unitary elasticity of substitution and time-varying factor shares. The advantage of the Canadian data for this purpose is that they provide a unified treatment of measurement issues that have had to be improvised in the US and European data used by previous researchers. In contrast to the previous US results, we find an elasticity of substitution not significantly less than unity, and an absence of capital-augmenting technical change in both the short and long run. Technical change is thus solely labour augmenting, consistent with Uzawa's steady state growth theorem. The model also yields plausible TFP estimates, and successfully captures trends in factor shares that have been the subject of recent study in international data.



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Debates over the sources and direction of technical change have long been at the heart of the theory and empirics of economic growth. Uzawa's (1961) celebrated steady state growth theorem says that balanced growth requires that technical change be labour augmenting. Given that it concerns balanced growth, this does not preclude *any* capital bias to technological progress, only that it disappear in the long run. For a textbook treatment see Acemoglu (2009, Sec. 7.2.3) who summarizes (p. 64) the implications of Uzawa's theorem as, "... balanced growth can only be generated by an aggregate production function that features Harrod-neutral technological change. ... Suppose the production function takes the special form $F(A_K(t)K(t), A_L(t)L(t))$. ... balanced growth is only possible if $A_K(t)$ is constant after date $T$."

However distinguishing biases in the direction of technical change empirically in aggregate data is not easy, especially in such a way as to allow for possible differences in short term versus long term biases. This paper applies an empirical framework designed to accomplish this to a recently-released Canadian data set. Perhaps surprisingly in view of conflicting results obtained by researchers using other data sets, we find that the Canadian data appear to be compatible with the steady state growth theorem. Indeed, we find an absence of capital-augmenting technical change even in the short term.

# 1 Background

Estimating a production function jointly with its implied marginal productivity conditions (factor demands) is a well established empirical methodology with a long history, going back at least to Bodkin and Klein (1967). The approach is especially valuable for a constant elasticity of substitution (CES) production function, where the marginal productivity conditions are more sophisticated than the constant factor shares implied by a Cobb-Douglas function. However CES functional forms require nonlinear estimation. As well, there is now a sizable literature establishing that the proper empirical implementation of CES models requires normalization around "baseline" values of the factor shares.

Articles by Klump, McAdam, and Willman (2007a, 2007b) (henceforth KMW) were the first to show how to estimate such a normalized CES system. The first article used annual US data 1953–1998, while the second compared results from annual US data 1953–2002 and quarterly euro-area data 1970–2003. Although KMW and coauthors have further explored and developed their methodology in subsequent work (see in particular their 2012 survey paper), the 2007 articles continue to be the principal examples of the estimation of normalized CES systems.[1]

For each of the US and euro area, KMW had no choice but to assemble the necessary data from disparate sources, as they document in some detail. For both jurisdictions they sought to model the aggregate private sector economy. For the US this was best defined as the private nonresidential sector. Given the objective of modeling the aggregate economy over several decades, both their construction of appropriate time series and the interpretation of their estimation results were disciplined by a desire for compatibility with long run balanced growth.

Our contribution in the present paper is to estimate a normalized CES system using a Canadian data set that, as recently revised, seems to be ideal for this purpose. Whereas the US and euro-area data sets used by KMW were, of necessity, somewhat improvised in their construction, the Canadian data are the unified and coherent culmination of many years of work by the professional staff of Statistics Canada (StatCan). One purpose of these productivity accounts is to enable StatCan to estimate total factor productivity, estimates that provide a useful comparison with the TFP implications of our estimated CES system.

As a byproduct, our analysis yields estimates of the elasticity of substitution for Canada. Although perhaps of only incidental interest, this nevertheless seems to fill something of a dearth in the literature. Canada is a notable omission from, for example, the empirical studies summarized in KMW (2012, Table 2).

Given that exploiting this data set is at the centre of the analysis, we begin by describing it. We then turn to our implementation of the KMW methodology.

## 2 Data

Our data source is Table 383-0021 of the Canadian socioeconomic database (CANSIM), which provides data for the aggregate business sector defined as the whole economy less public administration, non-profit institutions, and the rental value of owner-occupied dwellings. This definition of the private sector economy appears to be broadly comparable to that used by KMW (2007a, 2007b, 2008) in constructing their US and euro-area data.

The update to Table 383-0021 released in Spring 2014, which provides complete data for the years 1961–2010, differs from previous versions in its treatment of capital cost. Previously this was obtained simply as the residual of GDP less labour compensation, and so by definition factor payments exhausted output, consistent with an implicit assumption of perfect competition. In the revised Table 383-0021 an external rate of return is used to calculate capital cost for service industries, with the result that capital income and cost differ. (See Baldwin, Gu, Macdonald, Wang, and Yan (2014) for details.) In turn there is a discrepancy between output and factor payments that can be interpreted as a markup arising from imperfect competition, consistent with this feature of the US data constructed by KMW.[2]

Insert Figure 1 around here

Figure 1 portrays some time series features of these Canadian aggregates. Panel (a) shows the levels of business sector real GDP, its component factor payments labour compensation and capital cost, and the implied residual markup, all in 2007 dollars. In these data the markup is always positive, in contrast to the US markup inferred by KMW (2007a), which is only positive on average. Panels (c)–(f) of Figure 1 plot the log-differences of these aggregates. These do not trend markedly, suggesting that—at least at a descriptive level—these business sector aggregates are reasonably approximated by long run constant growth processes, consistent with the neoclassical growth model. This is confirmed by the Dickey-Fuller tests of Table 1 which strongly reject the unit root hypothesis for these log-differences, suggesting that they can be treated as stationary.

Insert Table 1 around here

Consider next the factor shares: Figure 1(b) plots the ratios to GDP of each of labour compensation, capital cost, and the markup. These average 61.0%, 34.0%, and 4.9% respectively, consistent with what growth economists commonly regard as plausible factor shares and a rate of profit in developed countries. The unit root evidence in the lower portion of Table 1 suggests that the labour and capital shares trend, either stochastically or deterministically. This is perhaps surprising given their "great ratio" status, and is inconsistent with long run balanced growth. It is, however, consistent with what KMW (2007a, 2007b, 2008) found of their US and euro-area factor shares. The tendency in Figure 1(b) for capital's share to increase and labour's share to decrease is also consistent with recent international evidence. For a focus on labour's factor share see Karabarbounis and Neiman (2014); for capital's share see Piketty and Zucman (2014) who find (p. 1302) that "...capital shares have increased in all rich countries" between 1970 and 2010. Both articles conjecture an elasticity of substitution greater than one as part of the explanation for these trends, something our estimation results do not support, as we shall see.

The apparent nonstationarity of factor shares over the sample period is one motivation for modelling the production sector so as to permit the short run to depart from the long run, and to allow factor shares to vary systematically with other influences in a way that is not permitted by a Cobb-Douglas specification.

Turning to a detailed consideration of the factor payments that are the numerators of these factor shares, each of capital cost and labour compensation is the product of price and quantity. In measuring the quantity of labour, Jorgenson has long argued the importance of accounting for changing labour force composition. We therefore use StatCan's quality-adjusted *Labour Input* series, consistent with KMW (2007a). Figure 2(a) plots this *Labour Input* series and panel (b) plots the implied real wage calculated as the ratio of real labour compensation to *Labour Input*. Both trend upward, as should be the case in data for a growing economy. (For this purpose labour compensation is deflated using the GDP deflator. Because *Labour Input* is an index the units of the implied real wage on the vertical axis of Figure 2(b) have no economic interpretation, and so it is also expressed as an index.)

Insert Figure 2 around here

In the case of capital, we use the *Capital Stock* series of CANSIM Table 383-0021 instead of StatCan's alternative quality-adjusted *Capital Input* series. *Capital Stock* is constructed from investment and investment price indexes that are benchmarked to 2007, following a geometric depreciation pattern. Capital cost is then defined to be the product of capital stock and the user cost of capital. This measure excludes profits from capital income (Baldwin et al., 2014) and is thus consistent with the "capital income" of KMW (2007a).

As a check on this choice, Figure 2(c) plots the ratios of each of *Capital Stock* and *Capital Input* to real GDP. (Because all these variables are indexes with 2007 = 100, the ratios are unity in 2007.) Figure 2(d) shows the implied real prices of capital services, calculated as the ratios of real capital cost (obtained by deflating with the GDP deflator) to each of the *Capital Input/Stock* series. Balanced growth and the commonly-accepted stylized facts of growth require that the capital-output ratio and the real factor price of capital be stable in the long run. Incompatible with this, the quality-adjusted *Capital Input* measure yields an increasing capital-output ratio and decreasing real price of capital services. Instead, our favoured non-quality-adjusted *Capital Stock* measure yields comparatively stable series that are more compatible with the balanced growth conditions.[3]

# 3 The KMW Framework

The many issues surrounding the specification and estimation of CES supply-side systems, including normalization, have been thoroughly exposited in a series of articles: for a comprehensive survey see KMW (2012) and the references therein. Here we merely summarize the essentials needed to understand our analysis.

## 3.1 Growth Specifications of the Factor Efficiencies

One expression for a constant returns to scale CES production function is

$$Y_t = \left[(E_t^N N_t)^{-\rho} + (E_t^K K_t)^{-\rho}\right]^{1/\rho}. \tag{1}$$

Notation is conventional—$N$ and $K$ are labour and capital, $\rho$ is the substitution parameter—except for the labour- and capital-augmenting efficiency levels $E_t^N$ and $E_t^K$, each of which is specified generically as

$$E_t^i = E_0^i e^{g_i(t)} \qquad (i = N, K).$$

The associated growth rates are

$$\frac{\mathrm{d}\log E_t^i}{\mathrm{d}t} = \frac{\mathrm{d}g_i(t)}{\mathrm{d}t} \qquad (i = N, K).$$

The textbook case of constant growth at instantaneous rate $\gamma_i$ is $g_i(t) = \gamma_i t$. But even if this provides a good approximation to growth in the long run—and this is an open question—in the shorter run it may be unduly restrictive. Rather than impose constant growth as a maintained hypothesis, KMW permit more general growth trajectories by using the Box-Cox transformation to specify $g_i(t)$ as

$$g_i(t) = \gamma_i \left( \frac{t^{\lambda_i} - 1}{\lambda_i} \right) \qquad (i = N, K) \tag{2}$$

so that the rates of technological progress depend on the curvature parameters $\lambda_i$:

$$\frac{\mathrm{d}g_i(t)}{\mathrm{d}t} = \gamma_i t^{\lambda_i - 1} = \begin{cases} \to \infty \text{ as } t \to \infty & \text{if } \lambda_i > 1 \quad \text{(accelerating growth);} \\ \gamma_i & \text{if } \lambda_i = 1 \quad \text{(constant growth);} \\ \to 0 \text{ as } t \to \infty & \text{if } \lambda_i < 1 \quad \text{(decelerating growth).} \end{cases}$$

Of course, although this Box-Cox specification permits accelerating growth in either or both of the efficiency levels $E_t^i$, this is implausible empirically in the long run, as we (and KMW) find.

Within the range $\lambda_i < 1$ lies the special case $\lambda_i = 0$. As $\lambda_i \to 0$ the Box-Cox function yields the logarithmic transformation

$$g_i(t) = \gamma_i \left( \frac{t^{\lambda_i} - 1}{\lambda_i} \right) \to \gamma_i \log t \text{ as } \lambda_i \to 0$$

so that

$$\frac{\mathrm{d}g_i(t)}{\mathrm{d}t} = \gamma_i \frac{\log t}{\mathrm{d}t} = \gamma_i \frac{1}{t} \to 0 \text{ as } t \to \infty.$$

Thus the rate of factor-$i$-augmenting technological progress decelerates to zero if $\lambda_i < 1$. If $0 < \lambda_i < 1$ this deceleration is slower than when $g_i(t) = \gamma_i \log t$, while if $\lambda_i < 0$ it is faster. Hence, although $\lambda_i = 0$ might be regarded as a benchmark rate of deceleration, in terms of the qualitative properties of the growth trajectory it is of no special interest. If appropriate it does, however, simplify the numerics of nonlinear estimation by replacing the Box-Cox function with the log function.

Within the general patterns of technological progress permitted by this specification, several special cases have long been of interest to growth economists.

**Hicks neutrality** The efficiencies of all factors improve at a common rate: $\gamma_N = \gamma_K > 0$, $\lambda_N = \lambda_K = 1$.

**Harrod neutrality** Technological progress is solely labour-augmenting,

- in both the short and long run: $\gamma_K = 0$, $\gamma_N > 0$, $\lambda_N \geq 1$;
- only in the long run: $\lambda_K < 1$, $\gamma_N > 0$, $\lambda_N \geq 1$.
- If Harrod neutrality is defined to mean constant labour-augmenting technological progress, then the restriction $\lambda_N \geq 1$ would specialize to $\lambda_N = 1$.[4]

**Solow neutrality** Technological progress is solely capital-augmenting,

- in both the short and long run: $\gamma_N = 0$, $\gamma_K > 0$, $\lambda_K \geq 1$;
- only in the long run: $\lambda_N < 1$, $\gamma_K > 0$, $\lambda_K \geq 1$.
- If Solow neutrality is defined to mean constant capital-augmenting technological progress, then the restriction $\lambda_K \geq 1$ would specialize to $\lambda_K = 1$.

The ability to distinguish empirically between short-term versus long-term biases in technical change is important. As the opening passage of this paper noted, short-run capital-augmenting technical change is not inconsistent with Uzawa's (1961) steady state growth theorem, as long as it disappears in the long run.

Of course, the CES parameterization that is the maintained hypothesis of the KMW methodology reduces to Cobb-Douglas under a unitary elasticity of substitution, in which case distinct factor efficiencies are not separately identifiable and all technological progress can be formulated as labour augmenting. In the words of Jones (2005), "...it is well-known that for a neoclassical growth model to exhibit steady-state growth, either the production function must be Cobb-Douglas or technical change must be labor-augmenting in the long run." Thus in the KMW framework where the elasticity of substitution is $\sigma = 1/(1+\rho)$, the restriction $\rho = 0$ or $\sigma = 1$ is also a sufficient condition for the steady state growth theorem to hold.

## 3.2 A Preliminary Look at the Empirical Evidence on Factor Biases

The estimation results obtained by KMW (2007a) using US data are not entirely consistent with the steady state growth theorem, but their euro-area results are (KMW 2007b). Their key parameter estimates are reproduced in the first two columns of Table 2, and are contrasted with a preliminary look, in the third column, at our Canadian results that are reported in greater detail in Section 4.2. The estimates for the elasticity of substitution $\sigma$ of 0.556 (US) and 0.669 (euro area) are both significantly below unity, establishing that a Cobb-Douglas specification would be inadequate for these countries. In contrast our Canadian $\hat{\sigma} = 0.9030$ is within two standard errors of unity, and so is not grossly at odds with Cobb-Douglas. That all three point estimates are below unity is consistent with the larger empirical literature on aggregate substitution elasticities, which only occasionally finds values greater than unity for individual countries.[5]

Insert Table 2 around here

Turning to factor biases in technological advance, KMW's $\hat{\gamma}_K = 0.004$ for the US, although small, is statistically significant, but this capital bias dissipates rapidly ($\hat{\lambda}_K = -0.018$), leaving labour augmentation at the higher rate of $\hat{\gamma}_N = 0.015$ as the dominant source of technological advance. The fly in this ointment is that $\hat{\lambda}_N = 0.439$, well below unity, so that even labour-augmenting technological progress dissipates. It is in this respect that their US results are inconsistent with the steady state growth theorem. Furthermore this inconsistency is remarkably robust to the alternative measures of labour input and income explored in the working paper version of their article (KMW 2004), as well as to the longer 1953–2002 sample of KMW (2007b, Table 3).

In the euro area the capital bias of $\hat{\gamma}_K = 0.002$ is not statistically significant, and so may not even be present in the short run, and in any case dissipates in the long run ($\hat{\lambda}_K = 0.376$), although not as rapidly as in the US. More importantly, although labour-augmenting technology advances at a low rate ($\hat{\gamma}_N = 0.003$), it persists in the long run in a manner consistent with constant growth: $\hat{\lambda}_N = 1.184$ is well within one standard error of unity.

Our Canadian results are also consistent with the steady state growth theorem. Capital-augmenting technological progress is non-existent even in the short run: $\hat{\gamma}_K = -0.0307$ is not significantly different from zero. Labour-augmenting progress is substantial ($\hat{\gamma}_N = 0.0296$) and consistent with sustained growth: $\hat{\lambda}_N = 0.7965$ is well within two standard errors of unity.

These Canadian results are of special interest because of their unique congruence with recent developments in the microtheoretic foundations of aggregate production. Following Kortum (1997), Jones (2005) has argued that a compelling foundation for an aggregate production function is to view it as a reduced form that reflects a range of underlying production techniques. Firms discover new techniques by searching for new ideas, which are drawn from a distribution. Kortum (1997) showed that, in such search-theoretic models of growth, exponential growth only arises if ideas are drawn from a Pareto distribution, at least in the upper tail.

This motivated Jones (2005) to investigate the implications of beginning with the assumption that ideas are drawn from a Pareto distribution as a modeling primitive. The global production function is an aggregation over ideas of idea-specific "local" production techniques. For a given idea the ability to substitute between factors is limited (the elasticity of substitution is below unity). Greater substitution is afforded by drawing new ideas from the distribution. Jones shows that long run balanced growth requires that technological progress in local production techniques be purely labour-augmenting, while the global production function must be Cobb-Douglas. "In other words, an assumption Kortum (1997) suggests we make if we want a model to exhibit steady-state growth leads to important predictions about the shape of the production function and the direction of technical change." (Jones, 2005, p. 518). Our Canadian results are remarkable in their consistency with the Jones predictions: the elasticity of substitution of $\hat{\sigma} = 0.9030$ is not significantly different from one but, in the absence of imposing $\sigma = 1$ a priori, we find technological progress to be solely labour augmenting. Neither of KMW's US or euro-area results are so fully in accordance with Jones.

With this preliminary look at the economic implications of our results, we now turn to a more detailed discussion of the methodology.

# 4 The Normalized CES System

Although the CES production function (1) is in a form similar to its typical textbook presentation, it is not suitable for empirical implementation because the substitution parameter $\rho$ (or $\sigma$) and the parameters governing technical change are not separately identified. For this, two things are necessary: first, the production function must be estimated jointly with the implied factor demands and, second, the resulting three-equation system must be estimated in normalized form. For Monte Carlo evidence supporting the ability of such a normalized system to identify these distinct elements see León-Ledesma, McAdam, and Willman (2010).

## 4.1 System Specification

So expressed, the KMW (2007a, equs. (6), (7), (8)) normalized system is as follows.[6]

$$\log\left(\frac{w_t N_t}{p_t Y_t}\right) = \log\left(\frac{1-\pi}{1+\mu}\right) + \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\bar{Y}}{N_t/\bar{N}}\right) - \log\zeta - g_N(t,\bar{t})\right] \tag{3a}$$

$$\log\left(\frac{q_t K_t}{p_t Y_t}\right) = \log\left(\frac{\pi}{1+\mu}\right) + \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\bar{Y}}{K_t/\bar{K}}\right) - \log\zeta - g_K(t,\bar{t})\right] \tag{3b}$$

$$\log\left(\frac{Y_t}{N_t}\right) = \log\left(\frac{\zeta\bar{Y}}{\bar{N}}\right) + g_N(t,\bar{t}) - \frac{\sigma}{1-\sigma}\log\left\{\pi\exp\left[\frac{1-\sigma}{\sigma}\left(g_N(t,\bar{t}) - g_K(t,\bar{t})\right)\right]\left(\frac{K_t/\bar{K}}{N_t/\bar{N}}\right)^{(\sigma-1)/\sigma} + (1-\pi)\right\} \tag{3c}$$

The first two equations are the marginal productivity conditions in factor share form while, in the third equation, the maintained hypothesis of constant returns to scale permits the production function to be expressed in labour intensive form. The expressions $g_i(t, \bar{t})$ are the normalized versions of the Box-Cox growth terms (2), defined as

$$g_i(t, \bar{t}) = \frac{\bar{t}\gamma_i}{\lambda_i}\left[\left(\frac{t}{\bar{t}}\right)^{\lambda_i} - 1\right] \qquad (i = N, K),$$

where $\bar{t}$ is the arithmetic mean of the time trend series. This normalization does not alter the economic interpretations we have given for the growth parameters $\gamma_i$, $\lambda_i$. Notice, for example, that $\lambda_i = 1$ still yields constant growth, $g_i(t, \bar{t}) = \gamma_i(t - \bar{t})$, just with a redefined time index.

In addition to being parameterized in terms of the elasticity of substitution $\sigma$ instead of $\rho$, several parameters appear in the system (3) that do not appear in the original production function (1). The distribution parameter $\pi$ is the share of capital in total factor payments and so should roughly correspond to the sample mean of $q_tK_t/(w_tN_t + q_tK_t)$, which is 0.358 in our sample. Indeed, for data sets for which nonlinear estimation proves problematic, convergence can be aided by setting $\pi$ to this sample mean. Like KMW we did not find this necessary, and allow $\pi$ to be freely estimated.[7]

The parameter $\mu$ is a markup that provides for a wedge between GDP and factor payments, allowing for imperfect competition, and should roughly correspond to the mean profit share of $(p_tY_t - w_tN_t - q_tK_t)/p_tY_t$ which, as mentioned in connection with Figure 1(b), is 0.049 in our sample.

Finally, in principle the point of normalization should be a "fixed point" at which baseline values of the factors $N$ and $K$ yield a baseline value of production $Y$. In practice the geometric means $\bar{N}$, $\bar{K}$, $\bar{Y}$ are used as these baseline values, but the nonlinearity of the system means that $\bar{N}$, $\bar{K}$ will yield $\bar{Y}$ only approximately, not exactly. The "normalization constant" $\zeta$ treats this discrepancy. Although it has no particular economic interpretation, $\zeta$ will be closer to unity the better the approximation that the sample means provide to a true fixed point of the estimated model.

## 4.2 Estimation Results

Like KMW, we estimated the CES system (3) as a nonlinear system of seemingly unrelated regressions.[8] The results are presented in Table 3, which is constructed for ready comparability with the US results in Table 1 of KMW (2007a). The first four columns correspond to the US models indicated in the table notes, whereas the final two columns—Models 6 and 7—are for restricted versions of the system that are not considered in KMW.[9]

Insert Table 3 around here

In all models $\zeta$, $\pi$, and $1 + \mu$ are estimated unrestricted, and the estimates are consistent with the values for these parameters that we have just discussed. In all models capital's share of factor payments, $\pi$, is estimated to be close to the sample mean of 0.358. The markup factor $1 + \mu$ implies an estimate of $\mu$ that is close to the mean profit share of 0.049. And the normalization constant $\zeta$ is close to unity, indicating that the sample means used for normalization are close to a true fixed point.

As well, for models in which the elasticity of substitution $\sigma$ is estimated unrestricted, most estimates are in the range of 0.9–1. The lone exception is Model 6, discussed further below, which yields an estimate $\hat{\sigma} = 0.5727$ that is more in line with the US and euro-area values reviewed in Table 2.

**The Maintained System: Model 4**

Estimation results for the fully unrestricted system (3) are reported as Model 4; the estimates for the elasticity of substitution $\sigma$ and the growth parameters $\gamma_N$, $\lambda_N$, $\gamma_K$, $\lambda_K$ are those that appeared in Table 2. To reiterate, $\sigma = 1$ is not rejected and technical change is labour-augmenting in a manner consistent with long run constant growth.

How well does this system explain the data? Consider first the $R^2$s, reported for the successive equations as $R^2_N$, $R^2_K$, $R^2_Y$. In the case of the production function (3c), $R^2_Y = 0.977$ is of limited interpretive value because the dependent variable $\log(Y_t/N_t)$ is nonstationary and the high $R^2_Y$ is to some extent an artifact of this nonstationarity. The marginal productivity conditions (3a) and (3b), where $R^2_N = 0.552$, $R^2_K = 0.502$, are of more interest in this respect, since the dependent variables are the (log) factor shares and trend only modestly. These $R^2$s are notable given that the specifications are so heavily disciplined by theory. Each is interpretable as a demand function for its respective factor. As purely static constructs, neither makes provision for the intertemporal considerations, such as adjustment costs in the case of investment, that empirical researchers often find necessary to introduce into factor demand specifications.

Goodness-of-fit is, of course, not the only criterion by which these equations can be judged: as well, the residuals should be stationary. Table 3 reports ADF tests for each equation of all our models. (The substantive conclusions of these tests are not particularly sensitive to the number of augmenting lags.) Focusing on Model 4, the unit root null is generally rejected, at least at a 10% significance level, indicating that all three equations can reasonably be regarded as balanced specifications yielding stationary residuals. Evidently the influences appearing on the right hand side of the marginal productivity conditions (3a) and (3b) successfully explain the nonstationarity in the raw capital and labour shares indicated by the ADF tests of Table 1.

To consider the observed and fitted factor shares explicitly, Figure 3 superimposes the fitted values yielded by Model 4 on the observed factor shares of the earlier Figure 1(b).

Figure 3 makes clear that, despite having the parameter $\pi$ corresponding to capital's factor share, the model nevertheless allows factor shares to vary through time. Furthermore the model successfully captures the modest upward and downward trends, respectively, in the capital and labour shares over the sample period. This is perhaps surprising given the estimated elasticity of substitution of $\hat{\sigma} = 0.9030$. The conventional intuition is that, as real wages increase while the real user cost of capital remains comparatively stable, the relative price of capital falls. Production substitutes away from expensive labour in favour of capital. Under a unitary elasticity of substitution (as in a Cobb-Douglas production function) the balance of these forces is such that factor shares remain constant. This conventional intuition[10] suggests that $\sigma > 1$ is needed for the shift toward capital to be substantial enough to overcome its declining relative price and increase its factor share. In our estimated model labour-augmenting technical change is operating to negate this intuition, yielding the trending factor shares that have been the subject of much recent study of international data (Karabarbounis and Neiman 2014; Piketty 2014, Chap. 6; Piketty and Zucman 2014) at the same time that, consistent with much within-country evidence, $\sigma < 1$.

Insert Figure 3 around here

**The Kmenta Approximation and Total Factor Productivity**

Model 4 Kmenta follows KMW (2007a) in using the Kmenta (1967) Taylor series approximation to the nonlinear system as a basis for estimation. It helps establish that the estimation results are largely robust to this alternative estimation strategy, including the goodness-of-fit and ADF statistics. However, although we report Model 4 Kmenta for completness, we will place less emphasis on the Kmenta versions of our models than did KMW because of the more recent evidence in León-Ledesma, McAdam, and Willman (2010, p. 1355) that "...identification of the substitution elasticity there remains poor and that of technical change bleak." Indeed, the "bleak" identification of the parameters $\gamma_N$, $\gamma_K$ is revealed in the much larger standard errors of Model 4 Kmenta than we obtained from our direct nonlinear estimation of Model 4.

Instead the primary utility of the Kmenta approximation is that, unlike the original nonlinear CES production function, it permits a growth accounting-type decomposition of output growth into one component attributable to observed factor inputs and another attributable to unobserved technical change. The latter therefore serves as a measure of total factor productivity, which has the expression (KMW 2007a, equ. (9))

$$\log(\text{TFP}) = \pi g_K(t,\bar{t}) + (1-\pi) g_N(t,\bar{t}) - \frac{1-\sigma}{\sigma}\frac{\pi(1-\pi)}{2}[g_N(t,\bar{t}) - g_K(t,\bar{t})]^2.$$

This can be evaluated even when estimation itself does not use the Kmenta approximation.

So obtained, the average TFP growth rate is reported for each of our models, and is generally in the neighbourhood of 1%. This value is, of course, entirely plausible, although it is higher than the 0.51% growth rate over this period of StatCan's TFP series in Table 383-0021.

Our estimate of around 1% TFP growth makes an interesting comparison with previous estimates that arose from earlier versions of the data set and alternative methodologies for constructing the series related to capital services. Specifically, StatCan conforms with the internationally-accepted "bottom up" methodology for treating capital services which, based on the previous version of Table 383-0021, yielded an estimate of 0.28% for TFP growth 1961–2011. Diewert and Yu (2012) contrasted this with an estimate of 1.03% yielded by their "top down" approach. (For a comparison of the two methodologies, see the exchange between Gu (2012) and Diewert (2012).) Whereas our analysis, by taking Table 383-0021 as published, accepts the StatCan construction of the capital services series, most of our models nevertheless yield TFP growth rates closer to that of Diewert and Yu.

Turning to comparisons with other countries, KMW (2007a, Table 1; 2007b, Tables 3, 4) find rates of TFP growth of 1.2-1.4% for the US and 0.28–0.31% for the euro area. In relation to these US estimates our Canadian estimates are consistent with the broader empirical TFP literature, which typically finds lower rates for Canada than for the US.

These estimation results for the maintained model suggest a number of possible restrictions on the system, to which we now turn.

**Constant Factor-Augmenting Growth**

The estimates $\hat{\lambda}_N = 0.7965$, $\hat{\lambda}_K = 0.9870$ yielded by Model 4 are both within two standard deviations of unity, suggesting that the constant growth restrictions $\lambda_N = \lambda_K = 1$ may be a reasonable special case to impose. This is Model 2. However these joint restrictions are not particularly supported by the data: the likelihood ratio statistic is $2(314.216 - 310.780) = 6.872$ ($p$-value=0.032).

Model 7 imposes $\lambda_K = 1$ alone and shows that, although this restriction is supported, the substantive findings are unaffected by it—although it does yield a very low implied rate of TFP growth. Most importantly, the growth rate $\gamma_K$ continues to be statistically insignificant, confirming that the Canadian data do not exhibit capital-augmenting technical change in either the short or long run.

Model 3 is the pure Cobb-Douglas case in which a unitary elasticity of substitution is superimposed on the constant growth restrictions $\lambda_N = \lambda_K = 1$ of Model 2. Notice that setting $\sigma = 1$ in the marginal productivity conditions (3a) and (3b) causes the square-parenthesis terms to disappear, leaving factor shares varying around constant means, as should be true of Cobb-Douglas factor shares. The result is that the $R^2$s must be zero, as is confirmed in the estimation results. The growth rates $\gamma_N$, $\gamma_K$ appear only in the production function (3c), and are not separately identified. Consequently in this special case capital and labour must share a common growth rate, and $\gamma_N = \gamma_K$ in the estimation results. However this Cobb-Douglas special case is clearly not supported by the data, as indicated by the dramatic reduction in its loglikelihood value relative to both Models 2 and 4. Note as well the marked deterioration in the ADF statistics for the marginal productivity conditions.

**Eliminating Capital-Augmenting Technical Change**

Finally, given that $\gamma_K$ is statistically insignificant in all the other models, Model 6 sets $\gamma_K = 0$, eliminating capital-augmenting technical change in both the short and long run. However this eliminates not only $\gamma_K$ but also $\lambda_K$ from the model, with the result that this restriction of the parameter space is strongly rejected relative to Model 4: the likelihood ratio statistic is $2(314.216 - 298.324) = 31.784$ ($p$-value=0.000). As well there is a marked deterioration in the $R^2$s of the marginal productivity conditions, and the estimate of the elasticity of substitution of $\sigma = 0.57$, although not implausible given the US and euro-area estimates in Table 2, is well below those of the other models.

On balance, then, consideration of these restricted models confirms the appeal of the KMW generalization of the system that permits general substitution possibilities and factor-specific technical change with differential short-term and long-term effects.

# 5 Conclusions

We have estimated a supply side system for the Canadian business sector—essentially, the private sector aggregate economy—for the half-century 1961-2010. There are reasons to believe that the Canadian data that have recently become available are, for this purpose, superior to the US and European data that have been available to previous researchers.

By using a CES production function the elasticity of substitution is not constrained to unity and factor shares are not fixed, as would be true of a Cobb-Douglas system. The more general substitution possibilities and factor share behaviour that this permits are supported empirically, both by the estimated model and, in the case of factor shares, by the univariate nonstationarity that is evident over the sample period.

By using recent advances in our understanding of how to normalize CES models, the system permits distinct rates of factor-augmenting technical change to be identified joint with the substitution parameter. Consequently hypotheses of classic interest concerning the direction of technical change, such as Harrod neutrality, are testable. As well, the use of Box-Cox specifications for growth rates makes it possible to distinguish between short-term versus long-term biases in technical change. The empirical model is supported by, among other things, yielding plausible implied rates of TFP growth.

In the Canadian data we find an absence of capital-augmenting technical change in both the short and long run, so that all technical change is labour-augmenting, consistent with Harrod neutrality. Furthermore we find that this solely labour-augmenting technology progresses at a rate consistent with constant growth in the long run, rather than accelerating or decelerating growth, and is therefore consistent with balanced growth, the neoclassical growth model, and Uzawa's steady state growth theorem.

Even more surprising, and in contrast to previous results from US and European data, the elasticity of substitution for Canada of 0.9030 yielded by the maintained model is within two standard errors of unity. Our estimation results are therefore consistent with the conception of aggregate production offered by Jones (2005), in which balanced growth requires that local idea-specific production techniques exhibit Harrod neutrality while their aggregation across ideas is Cobb-Douglas. Although our point estimates of the elasticity of substitution are less than one, our model nevertheless successfully captures the modest upward and downward trends, respectively, in the Canadian capital and labour factor shares. This contrasts with recent analyses of similar factor share trends in international data (Karabarbounis and Neiman 2014; Piketty 2014, pp. 215–223; Piketty and Zucman 2014, p. 1271), which tend to argue that these trends imply an elasticity of substitution greater than unity.

Of course, the Jones conception of aggregate production is not explicit (or, indeed, even implicit) in the KMW methodology, and so we do not claim to have offered a direct test of it. It is an open question to what extent the Jones framework may be empirically implementable. But, to the extent that that it is testable at all, the Canadian results seem to be in striking conformity with it.

# Notes

[1]Mention might also be made of KMW (2008), which used quarterly euro-area data 1970–2005. However it calibrates the parameter $\pi$ rather than allowing it to be freely estimated, and so we focus our comparative discussion in Section 3.2 on the euro-area results in KMW (2007b).

[2]Given the internal consistency of the StatCan methodology, we accept its treatment of self-employment income rather than attempting any adjustment of the kind suggested by Gollin (2002) and performed by KMW (2007a, equ. (10)).

[3]This divergent behaviour between the two implied real capital price series is not sensitive to the use of the GDP deflator to obtain the real series. We experimented with the alternative of using a capital deflator constructed from CANSIM Table 031-0002. The behaviour of the resulting real capital prices differs little from Figure 2(d). In any case, the implied real factor prices (of both labour and capital) are not used in the model estimation, only as descriptive evidence justifying our use of the associated factor quantity series.

[4]At a more fundamental level, neutrality concepts are defined in terms of relationships between marginal products and factor ratios as technology advances. Technological progress is Harrod-neutral if relative input shares remain unchanged for a given capital-output ratio, and analogously for Solow neutrality. Such time-invariance presumably rules out accelerating growth, or at least makes it problematic.

[5]See, for example, the findings surveyed in Chirinko (2008, Table 1) or KMW (2012, Tables 1 and 2). Estimates using across-country data, such as those of Karabarbounis and Neiman (2014), are another matter.

[6]This corrects a few typesetting errors in the third equation as it appears in KMW (2007a) where, most importantly, the closing brace is misplaced. The system appears correctly in the working paper version (KMW 2004, equs. (9), (10), (11)) and in KMW (2007b, equs. (3), (4), (5)).

[7]KMW (2008) and León-Ledesma, McAdam, and Willman (2014) are examples of analyses that experiment with estimating the remaining parameters using a calibrated $\pi$. The latter paper remarks that this makes "minimal difference" to the estimates.

[8]Past experience with nonlinear systems estimation leads us to favour TSP, the numerical properties of which have been favourably evaluated by McCullough (1999). However we

began by verifying that our TSP routines successfully replicate the estimation results of KMW (2007a). We thank Alpo Willman for providing the data and RATS code that made this possible.

[9]Table 3 reports no Model 1 or Model 5. KMW (2007a, Table 1) Model 1.1 is for a local rather than global maximum of their nonlinear maximum likelihood estimation. We found our estimation results to be insensitive to alternative starting values, and so encountered no such convergence issues. Their Model 1.5 is for a restricted form of the model ($\lambda_K = 0$) that was of natural interest to them, but is plainly rejected by the Canadian data and so we do not consider it.

[10]See, for example, Karabarbounis and Neiman (2014, p. 86), Piketty (2014, p. 217), or Piketty and Zucman (2014, p. 1271).

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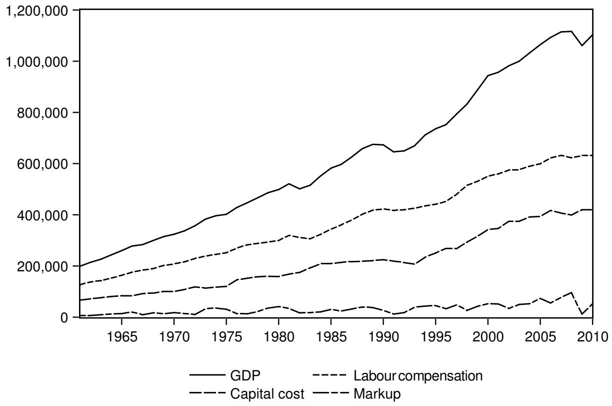
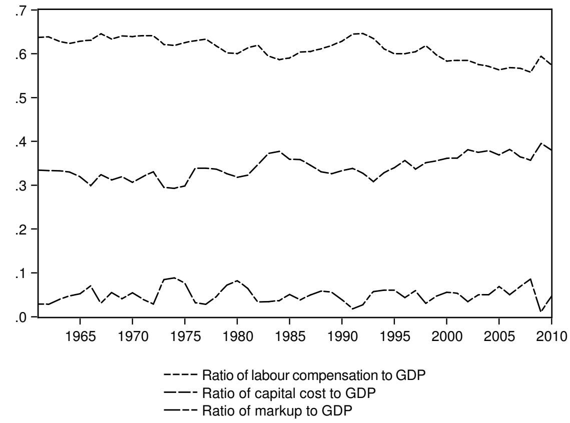
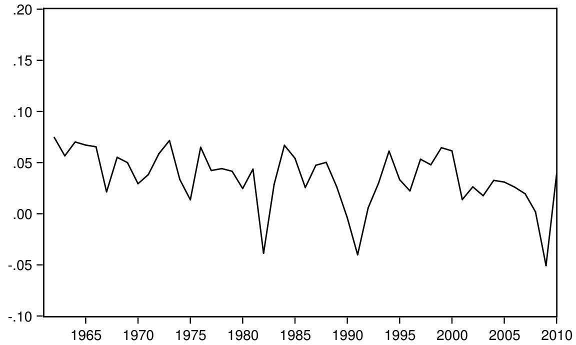
Figure 1: Real GDP and its component factor payments, 1961-2010



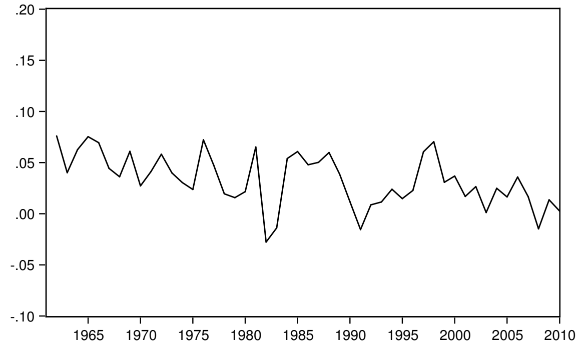

(a) GDP and its component factor payments (2007 dollars)



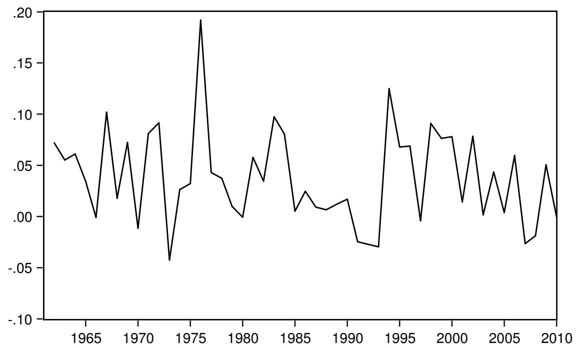

(b) Factor shares



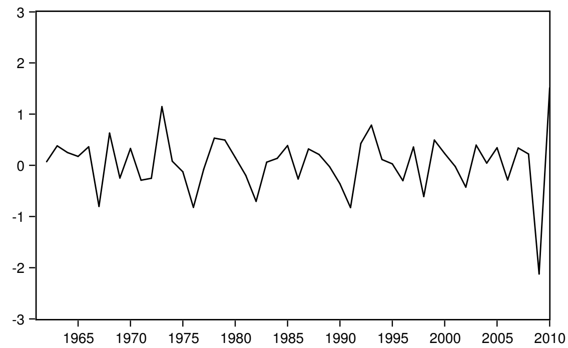

(c) Log difference of real GDP




(d) Log difference of real labour compensation

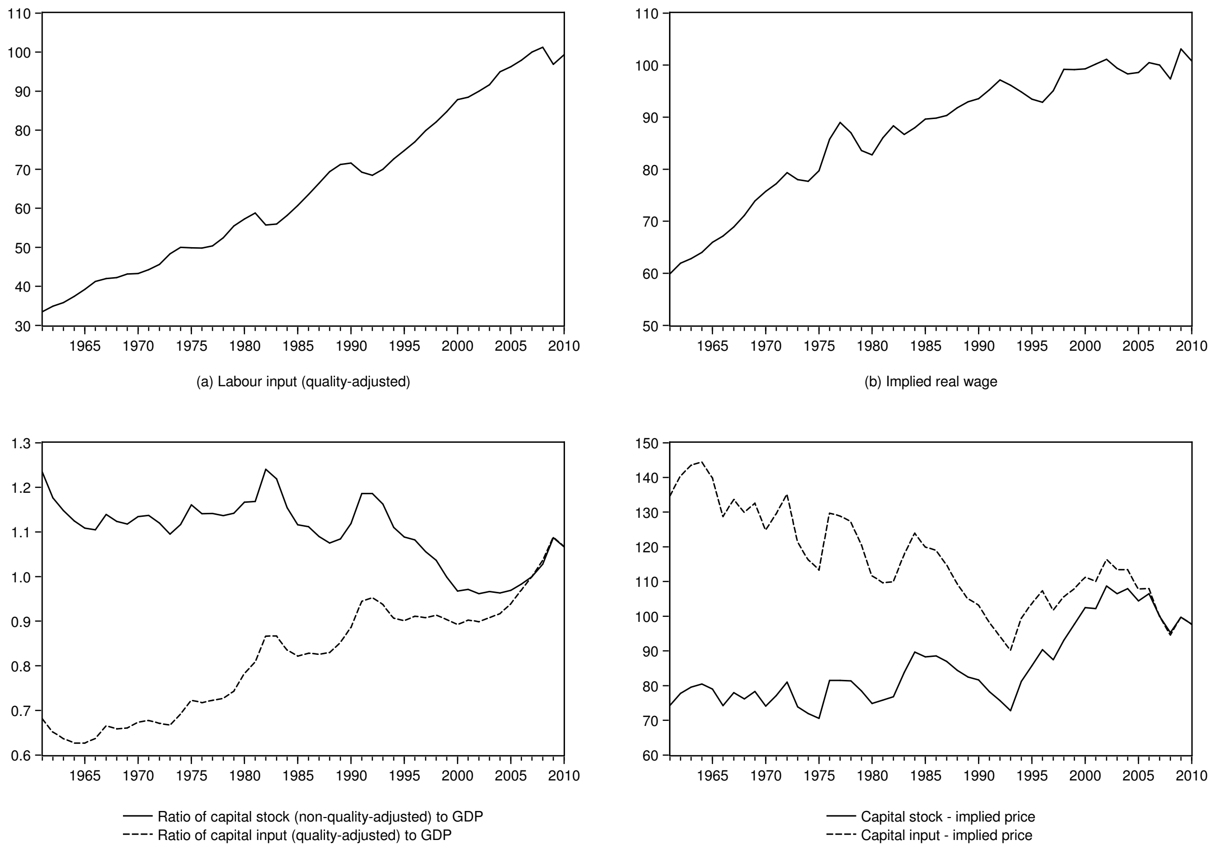



(e) Log difference of real capital cost




(f) Log difference of real markup

Figure 2: Labour, capital, and implied real factor prices




(a) Labour input (quality-adjusted)

(b) Implied real wage

(c) Capital: alternative measures relative to GDP

(d) Real price of capital services: alternative measures

Figure 3: Actual and fitted factor shares

Table 1: Univariate Statistics

| Variable | Mean | ADF tests ($p$-values)[a] constant | constant+trend |
|---|---|---|---|
| Log-difference of: | | | |
| real GDP | 0.0350 | 0.003 | 0.003 |
| real labour compensation | 0.0328 | 0.007 | 0.002 |
| real capital cost | 0.0376 | 0.001 | 0.004 |
| real markup | 0.0452 | 0.000 | 0.000 |
| Share in GDP of: | | | |
| labour compensation | 0.6103 | 0.479 | 0.173 |
| capital cost | 0.3404 | 0.424 | 0.061 |
| markup | 0.0492 | 0.000 | 0.000 |

[a] Schwartz's Bayesian information criterion generally suggested either 0 or 1 augmenting lags in the ADF regressions and so, in addition to the indicated specifications of the deterministic component, we used a single lag in all regressions.

Table 2: Comparison of Key Parameter Estimates

| Parameter | Data source: US 1953–1998[a] (KMW 2007a) | Euro area 1970–2003[b] (KMW 2007b) | Canada 1961-2010[c] |
|---|---|---|---|
| $\gamma_N$ | 0.015 | 0.003 | 0.0296 |
| | (0.000) | (0.001) | (0.0126) |
| $\lambda_N$ | 0.439 | 1.184 | 0.7965 |
| | (0.076) | (0.330) | (0.1389) |
| $\gamma_K$ | 0.004 | 0.002 | −0.0307 |
| | (0.001) | (0.002) | (0.0223) |
| $\lambda_K$ | −0.018 | 0.376 | 0.9870 |
| | (0.336) | (0.234) | (0.2664) |
| $\sigma$ | 0.556 | 0.669 | 0.9030 |
| | (0.018) | (0.065) | (0.0587) |

[a] *Source:* KMW (2007a, Table 1, Model 1.4). Annual data.
[b] *Source:* KMW (2007b, Table 4, Model (1)). Quarterly data.
[c] From Model 4 of Table 3, which is the system (3) of the text. Annual data.

Table 3: Estimation Results for Canada 1961–2010

| | Constant factor-augmenting growth: $\lambda_N = \lambda_K = 1$ | | Maintained system | | | |
|---|---|---|---|---|---|---|
| Parameter | Model 2[a] | Model 3[b] $\sigma = 1$ | Model 4[c] | Model 4[d] Kmenta | Model 6 $\gamma_K = 0$ | Model 7 $\lambda_K = 1$ |
| $\zeta$ | 1.0261 | 1.0002 | 1.0257 | 1.0262 | 1.0200 | 1.0257 |
| | (0.0056) | (0.0049) | (0.0051) | (0.0049) | (0.0047) | (0.0051) |
| $\pi$ | 0.3576 | 0.3582 | 0.3582 | 0.3579 | 0.3628 | 0.3582 |
| | (0.0022) | (0.0035) | (0.0026) | (0.0025) | (0.0029) | (0.0023) |
| $\gamma_N$ | 0.0779 | 0.0081 | 0.0296 | 0.2050 | 0.0124 | 0.0294 |
| | (0.0120) | (0.0003) | (0.0126) | (1.4024) | (0.0005) | (0.0114) |
| $\lambda_N$ | 1 | 1 | 0.7965 | 0.9041 | 0.5487 | 0.8013 |
| | | | (0.1389) | (0.2391) | (0.0642) | (0.1060) |
| $\gamma_K$ | −0.1153 | 0.0081 | −0.0307 | −0.3458 | 0 | −0.0302 |
| | (0.0218) | (0.0003) | (0.0223) | (2.5162) | | (0.0201) |
| $\lambda_K$ | 1 | 1 | 0.9870 | 0.9391 | | 1 |
| | | | (0.2664) | (0.1968) | | |
| $\sigma$ | 0.9693 | 1 | 0.9030 | 0.9895 | 0.5727 | 0.9017 |
| | (0.0077) | | (0.0587) | (0.0753) | (0.0257) | (0.0537) |
| $1 + \mu$ | 1.0525 | 1.0533 | 1.0526 | 1.0534 | 1.0528 | 1.0526 |
| | (0.0028) | (0.0028) | (0.0028) | (0.0028) | (0.0028) | (0.0028) |
| Average TFP growth rate | 0.0088 | 0.0081 | 0.0094 | 0.0103 | 0.0099 | 0.0027 |
| Loglikelihood | 310.780 | 269.981 | 314.216 | 316.338 | 298.324 | 314.214 |
| $R^2_N$ | 0.593 | 0.000 | 0.552 | 0.588 | 0.128 | 0.552 |
| $R^2_K$ | 0.504 | 0.000 | 0.502 | 0.496 | 0.265 | 0.503 |
| $R^2_Y$ | 0.974 | 0.966 | 0.977 | 0.978 | 0.980 | 0.977 |
| $\text{ADF}_N$ | −2.865 | −1.550 | −2.758 | −2.819 | −2.447 | −2.760 |
| $\text{ADF}_K$ | −3.604 | −1.787 | −3.638 | −3.574 | −3.134 | −3.641 |
| $\text{ADF}_Y$ | −2.879 | −3.005 | −2.996 | −3.018 | −3.157 | −3.000 |

*Note:* ADF regressions include an intercept, no trend, and (in view of the data being annual) one augmenting lag. For 50 observations the associated ADF critical values are −2.93 (5%) and −2.60 (10%).

[a] Corresponds to Model 1.2 of KMW (2007a, Table 1).

[b] Corresponds to Model 1.3 of KMW (2007a, Table 1).

[c] Corresponds to Model 1.4 of KMW (2007a, Table 1), which is the system (3) of the text. See also Model (1) of KMW (2007b, Tables 3, 4).

[d] Corresponds to Model 1.4 Kmenta of KMW (2007a, Table 1).