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On Statistical Inference for Inequality Measures Calculated from Complex Survey Data

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Abstract

We examine inference for Generalized Entropy and Atkinson inequality measures with complex survey data, using Wald statistics with variance-covariance matrices estimated from a linearization approximation method. Testing the equality of two or more inequality measures, including sub-group decomposition indices and group shares, are covered. We illustrate with Indian data from three surveys, examining pre-school children's height, an anthropometric measure that can indicate long-term malnutrition. Sampling involved an urban/rural stratification with clustering before selection of households. We compare the linearization complex survey outcomes with those from an incorrect independently and identically distributed (iid) assumption and a bootstrap that accounts for the survey design. For our samples, the results from the easy to implement linearization method and the more computationally burdensome bootstrap are in close agreement. This finding is of interest to applied researchers, as bootstrapping is currently the method that is most commonly used for undertaking statistical inference in this literature.

Keywords: complex survey; inequality; Generalized Entropy; Atkinson; decomposition; linearization

JEL Classification : C12, C42, D31

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1. INTRODUCTION

Studying inequality, especially for income, has been regaining attention since the late 1980s, after a period of relative neglect. As Atkinson (1997, p. 297) points out "*For much of this century, it* (the subject of income distribution) *has been very much out in the cold*". This revival has led to a growing literature on measuring inequality, as well as analysis of the statistical properties of such measures. However, gaps still remain between theoretical developments and empirical applications. A key manifestation of this gap is that much of the applied inequality research does not undertake statistical inference. Many such studies use inequality measures to make inter-temporal or inter-regional comparisons and/or for studying policy impacts (e.g., to examine the effect of a tax policy) with conclusions usually based on comparing numerical estimates rather than on formal statistical testing; e.g., Ram (2006).

One argument (see Maasoumi, 1997) used by investigators to support such practices has been that their large samples do not warrant concern about precision but large standard errors can still arise with such data sets. Furthermore, as the majority of the statistical theory in this area is based on large sample or asymptotic approximations, the use of large samples actually makes it more meaningful to report standard errors and undertake statistical tests. So, why the common lack of statistical inference? We believe two factors are perhaps at play – applicability of current theoretical results and ease of use of relevant theory.

On the first factor, many theoretical papers consider approximate asymptotic inference for inequality measures; e.g., to name only a few, Cowell (1989), Binder and Kovačević (1995), Van de gaer et al. (1999), Schluter and Trede (2002), Biewen and Jenkins (2006), Davidson and Flachaire (2007), Bhattacharya (2007) and Davidson (2009). Most focus on an iid framework, whereas data commonly arise from complex surveys. Although some of the iid papers consider weights, these take on a different role than in a complex survey. For instance, Cowell (1989) examines inference for decomposable inequality measures with random household weights that convert the observed household distribution into a personal distribution. Schluter and Trede (2002) allow for contemporaneous dependencies within households, but assume that households are iid. Correlation is also introduced by Van de gaer et al. (1999) but it is temporal dependence rather than correlations arising from the survey design. That a complex sampling design produces the sample data leads to (asymptotic) variances and covariances for inequality measures that differ from those under simple random sampling (SRS) or iid with weights. Consequently, to date, much of the theoretical work does not pertain to the data often used by empirical researchers.

In economics, data obtained under a complex survey design involves both stratification and clustering, undertaken to ensure adequate representation of groups of interest, in addition to minimizing costs. Stratification, which reduces survey costs for a given level of precision, results in the breakdown of the "identical" part of an iid assumption – even when members are independent within a stratum, they are unlikely to come from the same distribution across strata. Moreover, sample observations are likely correlated when the survey design involves clustering, so violating the "independent" part of an iid assumption. Clustering, such as interviewing several households on the same block or from the same village, likely introduces a common unobserved cluster-specific effect, which needs to be taken into account.

Relevant theoretical contributions that do incorporate such effects include Binder and Kovačević (1995), Biewen and Jenkins (2006) and Bhattacharya (2007). Each provides ways to obtain (at least) asymptotic variances for various inequality measures under complex sampling. Binder and Kovačević (1995) use linearization methods based on estimating equations to obtain variance estimators for a few inequality measures (Gini coefficient, coefficient of variation, an "exponential measure" and Lorenz curve ordinates). Asymptotic inference for the Lorenz curve and the Gini coefficient is also considered by Bhattacharya (2007). Our paper extends the work of Biewen and Jenkins (2006) who, based on Woodruff (1971), use a linearization method to obtain asymptotic variances for the Atkinson (1970) and Generalized Entropy measures with complex survey data. This type of linearization method involves using a Taylor series approximation, as is standard, but its novelty is in the reordering of the components of the resulting sums, simplifying evaluations computationally.

This feature of the variance expressions of Biewen and Jenkins (2006) addresses the second factor we identified above for the possible lack of statistical inference in applied research – ease of use. This factor has been identified by others (e.g., Giles, 2004 and Davidson, 2009), in explaining the lack of use of asymptotic variance formulae and subsequent hypothesis testing in applied research. However, Biewen and Jenkins, as well as the other cited references that account for complex sampling, do not consider the elements arising from a decomposition analysis, such as the "between" and "within" components or any share measures that may be generated from these parts. Decomposing inequality measures is standard applied practice. In addition, Biewen and Jenkins do not indicate how to extend the approach to test hypotheses involving two or more inequality measures.

Our goal is to provide these missing pieces. For the Atkinson and Generalized Entropy families, we give linearization variance expressions for sub-group decomposition measures, the between and within components and for sub-group shares of overall inequality. In addition, we extend the method to calculate asymptotic variance-covariance matrices enabling Wald statistics to be formed to test hypotheses involving two or more inequality measures. In particular, we cover testing the equivalence of inequality measures that may be simple inequality indices, sub-group decomposition indices, between and within measures and group shares of overall inequality. As in Biewen and Jenkins (2006), our expressions are applicable to the study of inequality of not just income but many other well-being variables such as wages, years of schooling, heights of children etc. Consequently, the results should be of interest to a wide range of empirical researchers in various fields.

We illustrate using Indian data from three National Family and Health Surveys (1992/93 (NFHS-1), 1998/99 (NFHS-2) and 2005/06 (NFHS-3)), examining children's height, an anthropometric measure that can indicate growth retardation and cumulative growth deficits, suggestive of long-term malnutrition. The sampling design involved an urban/rural stratification with one or two stages of clustering prior to the selection of households. In addition to providing variances for inequality indices, sub-group decomposition measures and sub-group shares of overall inequality, based on the urban/rural split, we test equality of these measures across the three surveys. A brief examination of gender differences in inequality is also provided. India has been experiencing rapid economic growth since the 1990s along with poverty reduction. However, this has been accompanied by rising economic inequality within urban areas and also between urban and rural sectors. But as Deaton and Drèze (2002, p.3744) rightly ask "*What about other types of social inequality, involving other dimensions of well-being........*?". Health is an important dimension of well-being and health inequality among children is a worthy issue to explore.

Others have studied health inequality, convincingly arguing that such measures are important in their own right, not just because of possible correlation between income and health. Ram (2006) estimates cross-country inequality in life expectancy. Gini coefficients using Latin American children's height data and data on years of schooling for women aged 22-30 are calculated by Sahn and Younger (2006). Height inequality among adults in Sub-Saharan Africa is considered by Moradi and Baten (2005). Pradhan et al. (2003) decompose world health inequality, as measured by height inequality among pre-school children, into within-country and between-country inequality using one of Theil's (1967) measures. Neither Ram (2006), Moradi and Baten (2005) nor Pradhan et al. (2003) report standard errors associated with their inequality measures and despite concluding differences in the numerical estimates, they do not undertake formal hypothesis testing. While Sahn and

Younger (2006) report standard errors and undertake significance testing, they do not mention how these have been obtained. Our work is directly relevant to such studies.

As bootstrapping offers a viable alternative, albeit less computationally friendly, we also provide standard error estimates and hypothesis test p-values from a bootstrap experiment that allows for the complex sampling design.[1] Undertaking inference for inequality measures via bootstrapping was proposed by Mills and Zandvakili (1997), who examine the Gini coefficient along with the two Theil (1967) measures, and extended to all Generalized Entropy and Atkinson indices by Biewen (2002). Applications of bootstrapping include Barrett et al. (2000), Gray et al. (2003a), Mills and Zandvakili (2004) and Davidson (2009). Some of these studies use complex survey data that is not accounted for in the bootstrap experiments, leading to incorrect standard errors and p-values. We expect the bootstrap and linearization approaches to provide similar variance estimates, given the large sample size, which makes a case for using the linearization method, given its lower programming demands. Finally, we use the linearization method to calculate variance-covariance matrices (and subsequent test statistics) under a false iid with weights assumption. Although this last case misinterprets the role weights play under an iid assumption for our data, it is a useful illustration of an error that might inadvertently arise in applied research.

This paper is organized as follows. Section 2 reviews our considered inequality measures. Estimators of the variance-covariance matrices are presented in section 3. Section 4 describes our setup for the bootstrap experiment for the application. The setting, data and results from the empirical illustration are detailed in section 5 and section 6 concludes. Relevant formulae are provided in an appendix.

2. INEQUALITY MEASURES

Many measures, or indices, of inequality can be obtained from a population, each with a different sensitivity to inequality in the upper or lower tail of the distribution. We examine the Generalized Entropy (GE), I_{GE}^{α}, and Atkinson (A), I_{A}^{ε}, classes of indices. Theil's (1967) two information indices are special cases: the Theil-1 index, I_{T1}, arises when $\alpha \to 1$, whereas the Theil-2[2] index, I_{T2}, results by letting $\alpha \to 0$. In addition, setting $\alpha = 2$ gives half of the coefficient of variation squared. Accordingly, α determines the sensitivity of the index to inequality; changes in the distribution's upper tail are more important for larger positive α

[1] A novel feature of our work compared with that of Biewen and Jenkins (2006).
[2] Also often termed the mean logarithmic deviation.

while a greater response to inequality in the lower tail occurs when α becomes more negative. The parameter ε (≥ 0) for the Atkinson indices is often called the inequality aversion parameter (or preference for equality parameter), as larger values lead to greater sensitivity to inequality in the lower tail (or more aversion to inequality). Each member of the Atkinson family has an ordinally equivalent member of the GE family (but not *vice versa*).

We adopt the basic setup of Biewen and Jenkins (2006), where each inequality measure is written in terms of population totals of the variable of interest (denoted as y) that captures some aspect of well-being: $U_\theta = \sum_{h=1}^{L}\sum_{i=1}^{N_h}\sum_{j=1}^{M_i}(y_{hij})^\theta$ and $T_\theta = \sum_{h=1}^{L}\sum_{i=1}^{N_h}\sum_{j=1}^{M_i}(y_{hij})^\theta(\log y_{hij})$, summed over the stages of the complex survey sampling design, assumed to involve $h=1,\ldots,L$ strata, $i=1,\ldots,N_h$ clusters in stratum h and $j=1,\ldots,M_i$ individuals in cluster i. The parameter θ is predetermined by which particular index is adopted, being either 0 or 1 for the T totals and 0,1, α or $(1-\varepsilon)$ for the U totals. Note that U_0 is the finite population size. Further stages of sampling beyond the initial stratification and clustering do not matter, as the nonparametric variance estimator is computed from the quantities from the N_h clusters; e.g., Skinner et al. (1989).

The population inequality indices we examine[3] are then

$$I_{GE}^{\alpha} = (\alpha^2-\alpha)^{-1}\left(U_0^{\alpha-1}U_1^{-\alpha}U_\alpha - 1\right),\ \alpha \in \Re \setminus \{0,1\} \qquad (1)$$

$$I_{T1} = T_1U_1^{-1} - \log\left(U_1U_0^{-1}\right),\ \alpha \to 1 \qquad (2)$$

$$I_{T2} = -T_0U_0^{-1} + \log\left(U_1U_0^{-1}\right),\ \alpha \to 0 \qquad (3)$$

$$I_A^{\varepsilon} = 1 - U_0^{-\varepsilon/(1-\varepsilon)}U_1^{-1}U_{1-\varepsilon}^{1/(1-\varepsilon)},\ \ \varepsilon \geq 0, \varepsilon \neq 1 \qquad (4)$$

$$I_A^1 = 1 - U_0U_1^{-1}\exp\left(T_0U_0^{-1}\right),\ \varepsilon \to 1. \qquad (5)$$

Estimators, $\hat{I}_{GE}^{\alpha}$, $\hat{I}_{T1}$, $\hat{I}_{T2}$, $\hat{I}_A^{\varepsilon}$ and $\hat{I}_A^1$, are generated by using the complex survey sample totals: $\hat{U}_\theta = \sum_{h=1}^{L}\sum_{i=1}^{n_h}\sum_{j=1}^{m_i} w_{hij}(y_{hij})^\theta$ and $\hat{T}_\theta = \sum_{h=1}^{L}\sum_{i=1}^{n_h}\sum_{j=1}^{m_i} w_{hij}(y_{hij})^\theta(\log y_{hij})$ where n_h is the number of sampled first stage clusters and m_i is the number of sampled units in cluster i. As the complex survey design results in units with (usually) different probabilities of being sampled, the weight, w_{hij}, is included to account for such differential sampling rates, in addition to any adjustments for non-response and inadequate frame coverage.

[3] See, for instance, Cowell (1989).

Our focus, as is the case with most empirical applications exploring inequality, is on decomposing a population's overall inequality into that for sub-groups (e.g., age, region, gender, regions, educational levels and so on). Total inequality then arises from that "between" the sub-groups and that "within" each sub-group. Being able to quantify the relative contributions of population characteristics contributing to inequality (and changes in) is informative and may aid the design of policy seeking to address inequality concerns. Specifically, we suppose the population comprises G mutually exclusive and exhaustive sub-groups (g=1,…,G) with sub-group population totals: ${}_g U_\theta = \sum_{h=1}^{L} \sum_{i=1}^{N_h} \sum_{j=1}^{M_i} {}_g D_{hij} (y_{hij})^\theta$ and ${}_g T_\theta = \sum_{h=1}^{L} \sum_{i=1}^{N_h} \sum_{j=1}^{M_i} {}_g D_{hij} (y_{hij})^\theta (\log y_{hij})$ where ${}_g D_{hij}$ is a dummy variable that is 1 when unit hij belongs to sub-group g, 0 otherwise. Then, the sub-group inequality measures are

$$ {}_g I_{GE}^{\alpha} = (\alpha^2 - \alpha)^{-1} \left(({}_g U_0^{\alpha-1})({}_g U_1^{-\alpha})({}_g U_\alpha) - 1 \right), \ \alpha \in \Re \setminus \{0,1\} \tag{6} $$

$$ {}_g I_{T1} = ({}_g T_1)({}_g U_1^{-1}) - \log\left(({}_g U_1)({}_g U_0^{-1}) \right), \ \alpha \to 1 \tag{7} $$

$$ {}_g I_{T2} = -({}_g T_0)({}_g U_0^{-1}) + \log\left(({}_g U_1)({}_g U_0^{-1}) \right), \ \alpha \to 0 \tag{8} $$

$$ {}_g I_A^{\varepsilon} = 1 - ({}_g U_0^{-\varepsilon/(1-\varepsilon)})({}_g U_1^{-1})({}_g U_{1-\varepsilon}^{1/(1-\varepsilon)}), \ \varepsilon \ge 0, \varepsilon \ne 1 \tag{9} $$

$$ {}_g I_A^{1} = 1 - ({}_g U_0)({}_g U_1^{-1}) \exp\left(({}_g T_0)({}_g U_0^{-1}) \right), \ \varepsilon \to 1. \tag{10} $$

To examine how sub-group inequality contributes to overall inequality, we need to define the between-group and within-group contributions and how these two parts combine. Addressing these issues has led to a lengthy literature, as the method adopted to breakdown inequality into its components may well alter the relative significance of the between- and within-group parts; see, among many others, Bourguignon (1979), Cowell (1980), Shorrocks (1980, 1984), Blackorby et al. (1981), Das and Parikh (1981), Cowell and Jenkins (1995), Foster and Shneyerov (1999), Lasso de la Vega and Urrutia (2008). Our goal is not to add to this debate, but merely to provide straightforward formulae for obtaining variance-covariance estimators for two inequality-decompositions used in empirical research.

One popular decomposition, for GE indices, is to hypothetically view the population as one whereby each sub-group member has the same well-being (the arithmetic mean), leading to the between-group component (say B_{GE}) measuring the inequality across sub-group means; i.e., the between-group component is the value of the inequality index when there is no within-group inequalities. The within-group component (say W_{GE}) is a weighted sum of the sub-group inequality measures, with the weights depending only on sub-group and population totals.[4] Total inequality is then the sum of the between-group and within-group inequality measures; i.e., for a generic GE inequality index I_{GE}, this results in $I_{GE} = W_{GE} + B_{GE} = \sum_g \omega_g \ {}_gI_{GE} + B_{GE}$, where for the g'th sub-group (g=1,…,G): ${}_gI_{GE}$ is a GE sub-group inequality index and ω_g is the weight. Inequality indices that decompose this way are *additively decomposable*. GE measures fall into this class, whereas A indices do not.[5]

Although Atkinson indices are not additively decomposable, they do belong to the class of *generally decomposable* or *aggregative* indices (Shorrocks, 1984), for which total inequality only depends on the sizes, group means and inequality values of the sub-groups. Then, a suitable transformation can convert such indices into an additively decomposable inequality measure, with the feature that the transformed measure and the original index will be ordinally equivalent in that they result in the same rankings of distributions. Various studies propose *multiplicative decompositions* of A indices, with the measure of total equality (one minus the inequality index) being the product of the within- and between-group equality measures; i.e., for a generic A index I_A, we have $(1 - I_A) = (1 - W_A)(1 - B_A)$; e.g., Blackorby et al. (1981), de la Vega and Urrutia (2003, 2008). Here, we consider a multiplicative decomposition proposed by Blackorby et al. (1981), for which the between-group inequality is that arising when each unit has the sub-group's equally-distributed-equivalent income, as opposed to the sub-group's mean well-being.

For the decompositions we consider, Table 1 provides the weights for the within component and the between measure for each of our considered inequality indices. Here on, we denote the within components as W_{GE}^{α}, W_{T1}, W_{T2}, W_A^{ε} and W_A^1, and the between components as B_{GE}^{α}, B_{T1}, B_{T2}, B_A^{ε} and B_A^1. Point estimates are obtained by replacing the population totals with their sample counterparts – we denote the estimators as $\hat{W}_{GE}^{\alpha}$, $\hat{W}_{T1}$, $\hat{W}_{T2}$, $\hat{W}_A^{\varepsilon}$, $\hat{W}_A^1$, $\hat{B}_{GE}^{\alpha}$, $\hat{B}_{T1}$, $\hat{B}_{T2}$, $\hat{B}_A^{\varepsilon}$ and $\hat{B}_A^1$. Empirically, interest also lies with the shares of these components to total inequality. Specifically: the contribution of the

[4] This breakdown does not imply independence of between-group and within-group terms because, conditional on index, the weights for the within-group part can be affected by the change in group means. Independence of B and W (as with I_{T2}) leads to the property that elimination of between-group inequality will reduce total inequality by the same amount. This is not the case otherwise (e.g., for I_{T1}). See, for instance, Shorrocks (1980).
[5] Decomposing using the arithmetic mean has been criticized and extended; e.g., Foster and Shneyerov (1999). We leave the examination of other such inequality-decomposition methods for future work.

between component to overall inequality ($S^{\alpha}_{GE,B}$, $S_{T1,B}$, $S_{T2,B}$, $S^{\varepsilon}_{A,B}$ and $S^{1}_{A,B}$) where, generically, $S_B = B/I$; the share of the within component to overall inequality ($S^{\alpha}_{GE,W}$, $S_{T1,W}$, $S_{T2,W}$, $S^{\varepsilon}_{A,W}$ and $S^{1}_{A,W}$) where, generically, $S_W = W/I$; and the proportion of total inequality taken by the within-group component of sub-group g (${}_gS^{\alpha}_{GE,W}$, ${}_gS_{T1,W}$, ${}_gS_{T2,W}$, ${}_gS^{\varepsilon}_{A,W}$ and ${}_gS^{1}_{A,W}$) where, generically, ${}_gS_W = {}_gW/I$ with $W = \sum_{g=1}^{G} {}_gW$.

Estimators of these shares, denoted with a circumflex, are formed using the relevant sample counterparts. Note that $S_B+S_W = 1$ for the GE indices, but not for the A measures. Despite this shortcoming for the latter indices, the share information still provides guidance on how inequality is changing from, for example, one survey to another.

INSERT TABLE 1 HERE

We now obtain variance estimators for the inequality statistics, along with an estimator of the variance-covariance matrix of any linear combination of two or more of these statistics, as needed for hypothesis tests.

3. LINEARIZATION VARIANCE ESTIMATORS

Having estimated sample inequality measures, we turn to estimating sampling variability along with undertaking hypothesis tests involving two or more inequality measures, allowing for the complex survey design. Questions might include: Are sub-group indices equal? Has inequality changed across surveys? Do the sub-groups have equal within shares? Are the shares equal across two or more surveys? To address such questions, we make use of a linearization estimator, formed via a first-order Taylor series approximation, of the relevant variance-covariance matrix. This method, which straightforwardly accommodates the complex survey design, avoids complicated covariance calculations (e.g., Cowell, 1989; Schluter and Trede, 2002; Van de gaer et al., 1999; Bhattacharya, 2007).

To be general, let $\Omega = [\vartheta_1,...,\vartheta_K]'$ be a K-dimensional vector of inequality quantities; e.g., Ω might contain sub-group inequality measures for a survey or consist of inequality indices for several surveys. We consider testing the null hypothesis $H_0: R\Omega = r$ against a two-sided alternative hypothesis using a Wald statistic, where R is a nonstochastic q×K matrix and r is a nonstochastic q-dimensional vector. Let $\hat{\Omega} = [\hat{\vartheta}_1,....,\hat{\vartheta}_K]'$ be the estimator of Ω formed using the relevant inequality estimators defined in section 2. The Wald statistic is:

$WT = \left(R\hat{\Omega} - r\right)' \left[V\hat{a}r(R\hat{\Omega} - r)\right]^{-1} \left(R\hat{\Omega} - r\right)$, where $V\hat{a}r(R\hat{\Omega} - r) = RV\hat{a}r(\hat{\Omega})R'$ is an estimator of the asymptotic variance-covariance matrix of $(R\hat{\Omega} - r)$. Obtaining the linearization rule for $V\hat{a}r(\hat{\Omega})$ is our focus; we denote this estimator as $V\hat{a}r_L(\hat{\Omega})$. Our results extend those of Biewen and Jenkins (2006) to the vector case. It is based on Woodruff (1971), who extended some results due to Keyfitz (1957). Each inequality measure in Ω is a nonlinear function of relevant population totals U_θ, T_θ, ${}_gU_\theta$ and ${}_gT_\theta$ - we suppose there are P distinct population totals used in forming Ω, placed in a vector $\Phi = [\phi_1, \ldots, \phi_P]'$; i.e., $\Omega = f(\Phi) = [f_1(\Phi), \ldots, f_K(\Phi)]'$. To make this notation concrete, as an illustration, suppose we wish to test the equality of three sub-group Theil-1 indices; i.e., $H_0 : {}_1I_{T1} = {}_2I_{T1} = {}_3I_{T1}$. Then, K=3, q=2, P=9, $\Omega = [{}_1I_{T1} \; {}_2I_{T1} \; {}_3I_{T1}]'$, $\hat{\Omega} = [{}_1\hat{I}_{T1} \; {}_2\hat{I}_{T1} \; {}_3\hat{I}_{T1}]'$, $R = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$,

$r = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Phi = [{}_1T_1 \quad {}_2T_1 \quad {}_3T_1 \quad {}_1U_1 \quad {}_2U_1 \quad {}_3U_1 \quad {}_1U_0 \quad {}_2U_0 \quad {}_3U_0]'$. Let $\hat{\Phi}$ be a consistent estimator of Φ and assume f(.) is appropriately differentiable. A first-order Taylor series approximation is $\hat{\Omega} = f(\hat{\Phi}) \cong f(\Phi) + \sum_{p=1}^{P} f^p(\Phi)(\hat{\phi}_p - \phi_p)$, where $f^p(\Phi) = \partial f(\Phi)/\partial \phi_p$ is a K-dimensional vector of partial derivatives. The asymptotic variance-covariance matrix of $\hat{\Omega}$ is approximated by $\mathrm{Var}\left(\sum_{p=1}^{P} f^p(\Phi)\hat{\phi}_p\right) = \mathrm{Var}\left(F(\Phi)\hat{\Phi}\right)$ where $F(\Phi)$ is the K×P derivative matrix. Noting that $\mathrm{Var}(F(\Phi)\hat{\Phi}) = F(\Phi)\mathrm{Var}(\hat{\Phi})F(\Phi)'$ leads to an estimator that requires evaluation of the PxP variance-covariance matrix of $\hat{\Phi}$, which can be difficult with a complex survey design. Woodruff's approach avoids calculating this matrix. Specifically, as $\Phi = \sum_{h=1}^{L} \sum_{i=1}^{N_h} \sum_{j=1}^{M_i} t_{hij}$, where t_{hij} is a P-dimensional vector with p'th element $t_{p,hij}$, we have[6]

$$\mathrm{Var}\left(\sum_{p=1}^{P} f^p(\Phi)\hat{\phi}_p\right) = \mathrm{Var}\left(\sum_{p=1}^{P} f^p(\Phi) \sum_{h=1}^{L} \sum_{i=1}^{n_h} \sum_{j=1}^{m_i} w_{hij} t_{p,hij}\right)$$

[6] For our illustration, $t_{hij} = [{}_1D_{hij}(y_{hij})(\log y_{hij}) \;\; {}_2D_{hij}(y_{hij})(\log y_{hij}) \;\; {}_3D_{hij}(y_{hij})(\log y_{hij}) \;\; {}_1D_{hij}(y_{hij}) \;\; {}_2D_{hij}(y_{hij}) \;\; {}_2D_{hij}(y_{hij}) \;\; {}_1D_{hij} \;\; {}_2D_{hij} \;\; {}_3D_{hij}]'$.

$$= \mathrm{Var}\left(\sum_{h=1}^{L}\sum_{i=1}^{n_h}\sum_{j=1}^{m_i} w_{hij}\left[\sum_{p=1}^{P} f^p(\Phi)\, t_{p,hij}\right]\right) = \mathrm{Var}\left(\sum_{h=1}^{L}\sum_{i=1}^{n_h}\left[\sum_{j=1}^{m_i} w_{hij}\gamma_{hij}\right]\right)$$

where $\gamma_{hij} = \sum_{p=1}^{P} f^p(\Phi) t_{p,hij}$. Within each stratum, we assume (e.g., Skinner et al., 1989, pp 46-48; Williams, 2000): (i) initial clusters selected are uncorrelated, but there may be heteroskedasticity, both between and within clusters, and there may be arbitrary dependence among observations within a cluster (permitting other layers of sampling design); (ii) with replacement cluster sampling;[7] and (iii) $n_h \geq 2$. Then, the linearization estimator is

$$\hat{\mathrm{Var}}_L(\hat{\Omega}) = \hat{\mathrm{Var}}\left(\sum_{h=1}^{L}\sum_{i=1}^{n_h}\left[\sum_{j=1}^{m_i} w_{hij}\hat{\gamma}_{hij}\right]\right) = \sum_{h=1}^{L}\sum_{i=1}^{n_h}\left(\hat{\mathrm{Var}}\left[\sum_{j=1}^{m_i} w_{hij}\hat{\gamma}_{hij}\right]\right) \qquad (11)$$

where we use $\hat{\Phi}$ to replace Φ in the formulae for γ_{hij} to give $\hat{\gamma}_{hij}$; we denote the k'th element of $\hat{\gamma}_{hij}$ as $\hat{\gamma}_{k,hij}$, k=1,…,K. Switching the summation order has reduced the problem to one of obtaining a variance-covariance matrix for a survey total. We use standard survey literature formulae to estimate the between-cluster variance (e.g., Skinner et al., 1989, p47) to give the linearization method estimator of the variance-covariance matrix

$$\hat{\mathrm{Var}}_L(\hat{\Omega}) =$$

$$\sum_{h=1}^{L}\frac{n_h}{(n_h-1)}\sum_{i=1}^{n_h}\left[\left(\sum_{j=1}^{m_i} w_{hij}\hat{\gamma}_{hij} - \frac{\sum_{i=1}^{n_h}\sum_{j=1}^{m_i} w_{hij}\hat{\gamma}_{hij}}{n_h}\right)\left(\sum_{j=1}^{m_i} w_{hij}\hat{\gamma}_{hij} - \frac{\sum_{i=1}^{n_h}\sum_{j=1}^{m_i} w_{hij}\hat{\gamma}_{hij}}{n_h}\right)'\right] \qquad (12)$$

The elements of this matrix provide variances for each inequality measure on the diagonal and covariances on the off-diagonals. Standard software packages can be used; for instance, with Stata (StataCorp., 2005) after defining the survey's strata, ultimate cluster and

[7] This assumption is usually always violated with surveys, in which case the formula generally leads to overestimation. An alternative assumption is that the n_h clusters within stratum h form a simple random sample without replacement from the stratum, h=1,…,L; see, e.g., Kalton (1977). This results in adding a finite population correction involving the factor (n_h/N_h). The correction adds little when this factor is small, as is often the case. The critical assumption is that the observations between clusters are uncorrelated.

weight variables, along with providing each element of the vector $\hat{\gamma}_{hij}$, use of the *svy: total* command generates the matrix. To use this, we need formulae to create $\hat{\gamma}_{k,hij}$. We provide these in the appendix. For consistency we also report the formulae for the total inequality indices obtained by Biewens and Jenkins (2006). The rest, to our knowledge, are new. Note that we do not provide explicit formulae for generating the linearization method variances for the estimated between component shares for members of the GE family because they are the same as for the within components. This is not so for Atkinson indices.

A substantial body of research exists on asymptotic properties of complex survey estimators of totals, smooth/nonsmooth linear/nonlinear functions of totals, and the corresponding linearization variance/covariance estimators; e.g., Krewski and Rao (1981) and Rao and Wu (1988). Unlike classical iid asymptotics, however, there is no single appropriate framework with a complex survey design, as the analysis requires specifying not just what is happening as a sequence of samples increases in size, but also a sequence of finite populations, in addition to sizes of strata, clusters and so on. Different setups may lead to different results.[8] For instance, Krewski and Rao (1981) examine the consistency of, amongst other things, linearization variance estimators of statistics that are nonlinear functions of population totals, as $L \to \infty$, assuming a sequence of finite populations with fixed stratum sample sizes. Their results are valid for any stratified multistage design in which the primary sampling units are selected with replacement and independent subsampling within those units selected more than once. Key is that there are no isolated, influential, values in the clusters. Applying these results to without replacement sampling typically follows directly when the sampling fraction $f_h=n_h/N_h$ is small, as is usual with the surveys used to generate inequality measures. Assuming that the number of strata L is fixed with $n_h \to \infty$ is considered by, for instance, Dippo and Walter (1984), Williams (2000) and Bhattacharya (2005, 2007). Accordingly, we assume that our inequality estimators are consistent and asymptotically normal with the linearization method variance estimator also consistent. So, WT using $\hat{Var}_L(\hat{\Omega})$, denoted as WT_L, is approximately χ^2_q under its null hypothesis.

To end this section, we comment on the applicability of our results to two other situations often met when inequality measures are considered. The first is a sampling scheme that draws units from the finite population using *simple random sampling with replacement*

[8] This is analogous to asymptotic theory in a panel framework, where asymptotics may consider the time dimension increasing, while fixing the number of cross-sectional units, or the converse case, or allow both dimensions to increase without bound.

(SRSWR).[9] The second case assumes an underlying infinite population under an iid approach with, for the i'th unit, the variable of well-being, y_i, and weight, w_i, viewed as iid draws from a population (y, w). Our results are easily modified to handle both cases. Under SRSWR, a sampling design that assigns equal probability to each possible sample with each population unit having an equal probability of being selected into the sample, the formulae in the appendix apply with the sampling weights either ignored or set to 1 for all units, and the summation is over i=1,….,n rather than over the various stages of the complex survey design. The iid framework regards the inequality measure as a function of population moments rather than population totals, with the treatment of the weights being different than under a complex survey design; e.g., Cowell (1989), Van de gaer et al. (1999), Biewen and Jenkins (2006). Despite these disparities, the numerical estimates of linearization variance-covariance matrices can be obtained using the formulae in the appendix with the summation being over i=1,…,n rather than over the stages of the complex survey design.

4. INFERENCE USING BOOTSTRAPPING

Maintaining the general notation from the previous section, we first outline how we obtained a bootstrap variance-covariance estimator for the nonlinear estimator $\hat{\Omega} = f(\hat{\Phi})$; we denote this estimator as $\hat{Var}_{BT}(\hat{\Omega})$. The method, which involves the following steps for each bootstrap sample, guarantees that the replicate sample has the same sampling design as the parent sample; see e.g., Rao and Wu, 1988; Rao et al. (1992), Rust and Rao, 1996 and Shao and Tu (1995, chapter 6) for discussion on the properties of variance estimators obtained from this, so-called, rescaling bootstrapping. The steps, V1 through V5, are:

V1. Draw a simple random sample of n_h clusters with replacement from the clusters within stratum h independently for each stratum (h=1,…,L).[10]

V2. When a cluster is selected into the bootstrap replicate, all secondary and successive units from the selected cluster are retained, along with their corresponding sampling weights.

[9] Although most survey sampling is undertaken without replacement, sampling without replacement is similar to sampling with replacement when the population is very large. Sampling with replacement leads to a sample that is close to the iid approach situation in large data sets; e.g., Lehtonen and Pahkinen (1995).

[10] Undertaking the resampling with replacement simplifies the procedure and should not be an issue with most surveys used to generate inequality measures. Although the number of clusters to be resampled is often chosen to be (n_h-1) to ensure unbiased estimation (at least asymptotically), it is computationally easier with Stata to select n_h clusters from each strata. The effect of this is minimal in our case given the large number of clusters in the surveys.

V3. Let r_{hi}^{b} $(0 \le r_{hi}^{b} \le n_{h})$ be the number of times that cluster i from stratum j is included in bootstrap replicate b. The bootstrap sampling weight is then $w_{hij}^{b} = w_{hij} r_{hi}^{b}$ so that $w_{hij}^{b} = 0$ if cluster i is not selected in the b'th bootstrap sample.

V4. Let $\hat{\Phi}^{b}$ be the estimated Φ using the p'th bootstrap totals $\hat{\phi}_{p}^{b}$ formed from replicate b. Specifically, with θ = 0, 1 or α, depending on the term of interest, $\hat{\phi}_{p}^{b}$ will be one of the following totals: $\hat{U}_{\theta}^{b} = \sum_{h=1}^{L} \sum_{i=1}^{n_h} \sum_{j=1}^{m_i} w_{hij}^{b} (y_{hij})^{\theta}$, $\hat{T}_{\theta}^{b} = \sum_{h=1}^{L} \sum_{i=1}^{n_h} \sum_{j=1}^{m_i} w_{hij}^{b} (y_{hij})^{\theta} (\log y_{hij})$,

$${}_{g}\hat{U}_{\theta}^{b} = \sum_{h=1}^{L} \sum_{i=1}^{n_h} \sum_{j=1}^{m_i} w_{hij}^{b} ({}_{g}D_{hij})(y_{hij})^{\theta}, \quad {}_{g}\hat{T}_{\theta}^{b} = \sum_{h=1}^{L} \sum_{i=1}^{n_h} \sum_{j=1}^{m_i} w_{hij}^{b} ({}_{g}D_{hij})(y_{hij})^{\theta} (\log y_{hij}).$$

Then form the bootstrap copy of Ω, $\hat{\Omega}^{b} = f(\hat{\Phi}^{b})$.

V5. Repeat steps V1-V4 B_V times to give B_V bootstrap estimators of Ω, $\hat{\Omega}^{1}, \ldots \hat{\Omega}^{B_V}$, and compute the bootstrap estimator of the variance-covariance matrix:

$\hat{Var}_{BT}(\hat{\Omega}) = \frac{1}{B_V - 1} \sum_{b=1}^{B_V} \left(\hat{\Omega}^{b} - \hat{\Omega}\right)\left(\hat{\Omega}^{b} - \hat{\Omega}\right)'$. We set $B_V = 200$. Aside from providing standard errors, this rule is used to form a bootstrapped sample value of any Wald statistics for inference; specifically, we form $WT_{BT} = \left(R\hat{\Omega} - r\right)' \left[R \hat{Var}_{BT}(\hat{\Omega}) R'\right]^{-1} \left(R\hat{\Omega} - r\right)$

Turning to bootstrapping p-values, we undertake a double bootstrap with the following steps.

W1. Undertake steps V1 to V4 above to give the first (b=1) bootstrap sample with its estimator of Ω, $\hat{\Omega}^{1} = f(\hat{\Phi}^{1})$.

W2. Treating this first bootstrap replicate as the parent sample, repeat steps V1 to V4 again to give B_V estimates of Ω, $\hat{\Omega}_{1}^{1}, \ldots, \hat{\Omega}_{B_V}^{1}$. Form the estimator of the variance-covariance matrix for this first replicate sample: $\hat{Var}_{BT}(\hat{\Omega}^{1}) = \frac{1}{B_V - 1} \sum_{b=1}^{B_V} \left(\hat{\Omega}_{b}^{1} - \hat{\Omega}^{1}\right)\left(\hat{\Omega}_{b}^{1} - \hat{\Omega}^{1}\right)'$.

W3. Form the bootstrap Wald statistic $WT_{BT}^{1} = \left(\hat{\Omega}^{1} - \hat{\Omega}\right)' R' \left[R \hat{Var}_{BT}(\hat{\Omega}^{1}) R'\right]^{-1} R \left(\hat{\Omega}^{1} - \hat{\Omega}\right)$

where, recall, $\hat{\Omega}$ is the original sample's estimate of Ω and is used to so-call centre the statistic because our data may not have been drawn from a population that satisfies H_0; see, e.g., Hall and Wilson (1991).

W4. Repeat steps W1 through W3 B_W times to obtain B_W values of WT: $WT_{BT}^{1}, \ldots WT_{BT}^{B_W}$.

We choose $B_W = 99$ to correspond with a nominal 10% or 5% level.[11] The bootstrapped p-value is $p = \left(\sum_{b=1}^{B_W} I(WT_{BT}^{b} > WT_{BT}) + 1 \right) / (B_W + 1)$.

5. EMPIRICAL EXAMPLE: HEIGHT INEQUALITY AMONG INDIAN CHILDREN

Here we apply some of our results to study health inequality among Indian children, based on the anthropometric measure height. While we recognize that it is not possible to capture the overall health status of an individual by any one indicator of health, height of pre-schoolers has been used by researchers interested in studying health inequality; e.g., Pradhan et al. (2003), Sahn and Younger (2006). The appropriateness of height of young children as one possible but arguably a good measure of their health is based on numerous evidence from medical and public health research; e.g., WHO (1995a,b). Given this, we use the terms "height inequality" and "health inequality" interchangeably in the following discussion. As a detailed analysis of height inequality among Indian children is beyond our scope, our example illustrates the usefulness of statistical testing involving the simple inequality indices or their decompositions, as well as the effectiveness of our proposed methods compared to the computationally burdensome bootstrapping procedure. For space reasons, we only report results using the Theil-1 measures; outcomes for other statistics are available on request.

We provide standard errors for total inequality indices and sub-group decomposition measures based on an urban/rural split, as well as undertake tests for equality of these across two or three surveys. Whether rural and urban regions differ in health inequality is of interest given the strong evidence of varying economic inequality across these regions (e.g., Deaton and Drèze, 2002). We also examine gender differences in height inequality, of concern given the debate on whether girls and boys are equally well cared for due to the preference for sons,

[11] Given a nominal level for the test of α_W, a choice of B_W that leads to $\alpha_W(B_W+1)$ being an integer results in an exact Monte Carlo test when the statistic is pivotal; Dufour and Kiviet (1998). For a nonpivotal statistic (as is ours) it is not necessary to choose B_W in such a way, but, as advocated by (for example) Davidson and MacKinnon (2000), it would seem reasonable to still follow such a practice.

particularly in rural districts[12]. In particular, we test whether the health inequality, arising from non-natural causes, is the same across both groups. It might be the case that there is less variability in the boys' height distribution, given male bias, than the girls' distribution as the degree of care that girls receive may vary more widely than that for boys. For example, a girl in a poor rural family with a number of female siblings may receive very different care compared to a single girl child in an urban household.

For each case, we compare the linearization outcomes with those from two other scenarios: (i) assuming (incorrectly) that the standardized heights and the sample weights are iid draws from a common population; and (ii) from using the bootstrap procedure described in section 4, designed to account for the complex survey design.

5.1 Survey design and data characteristics

Our data are from the three Indian National Family Health Surveys (NFHS), conducted under the agency of the International Institute for Population Sciences (IIPS): NFHS-1 (1992/93), NFHS-2 (1998/99) and NFHS-3 (2005/06). Due to differences across surveys, our sample includes children (i) whose mothers were interviewed with the Women's Questionnaire, (ii) who are less than three years of age and (iii) who lived in states other than Sikkim, Andhra Pradesh, Himachal Pradesh, Madhya Pradesh, Tamil Nadu and West Bengal. This resulted in sample sizes of 20,410 for NFHS-1, 18,520 for NFHS-2 and 18,146 for NFHS-3.

Stratified multi-stage cluster sampling was used with the design being roughly similar for each survey. We sketch out the key stages for NFHS-3; see IIPS (2007a,b; 2000; 1995). Each state was sampled separately with urban and rural areas forming the first stage strata. Two phases of cluster sampling came next for rural regions: random selection of villages followed by households. A three-stage procedure was adopted for urban areas: selection of wards followed by census enumeration blocks followed by households. On average, 30 households were targeted for interviewing from each village or census enumeration block. There are 559 clusters for NFHS-1, 549 for NFHS-2 and 2719 for NFHS-3. The survey method also ensured self-weighting at the domain level (i.e., the urban and rural areas of each state) so that each child in the same domain has a common sampling weight.

Prior to estimating height inequalities, we account for natural/biological median height differences of children across gender and age by converting the individual heights into

[12] For instance, Kadi et al. (1996) and Tarozzi and Mahajan (2007) report that girls are more nutritionally deprived compared to boys. In contrast, Griffiths et al. (2002) and Marcoux (2002), among others, find little evidence of gender differentials in food consumption.

percent-of-median, which is simply the height of an individual child relative to the median height of comparable children in a reference population, expressed as a percentage; e.g., Gershwin et al. (2000, pp. 7-8). Studies that use percent-of-median include Prudhon et al. (1996) and Zainah et al. (2001). The current reference population group is the World Health Organization's (WHO) Child Growth Standards (WHO Multicentre Growth Reference Study Group, 2006; WHO, 2006), formed from a multiethnic sample of healthy children; we refer to this as the WHO-MGRS standard. Specifically, let h_{iga} be the height of the ith Indian child of gender g and aged month a (g=1 when the child is a boy, 2 when the child is a girl; a=0,…,35). Let Md_{ga} be the median from the WHO-MGRS standard for a child of gender g and age a in months. The percent-of-median is $P_{iga} = 100(h_{iga}/Md_{ga})$; when gender and age is not of issue, we denote this as P_i. For example, if a 6-month old boy's height is 65.3cms then his P_i=96.6% as the WHO-MGRS median height is 67.6236cms for a 6-month old boy.

Using percent-of-median does not account for the natural variability in height around the median height that differs across and within age and gender groups. Natural variability refers to variation in height that is not due to environmental reasons (e.g. availability of food, clean water, medical care) but arises only from genetic variation. Not accounting for natural inequality is likely not an issue when examining total inequality over surveys, as it seems reasonable to assume that the natural inequality in the children is fairly stable over the time frame of our study and across regions. However, this is not so when considering gender differences in inequality, as natural inequality varies across boys and girls, dependent on age. For this case, we estimate the natural inequality by taking appropriate draws of children's height from the WHO-MGRS reference growth curves. We now turn to our results.

5.2 Overall inequality

Table 2 provides the estimated inequality indices, I_{T1}, along with standard errors. For each case, we report three standard error estimates based on: the linearization method, a (false) iid assumption with weights and the bootstrap approach; these are denoted as se_L, se_{IID} and se_{BT} respectively. We see that inequality has declined over the three surveys, with the change between NFHS-2 and NFHS-3 being far more than the decline that occurred between NFHS-2 and NFHS-1. That the standard errors allowing for the complex survey design are larger than those under the false iid assumption (by, approximately, 9% to 24%) highlights the importance of taking account of the design when estimating standard errors, the heterogeneity between and within clusters increasing the variance from the iid case. Typically, the linearization and bootstrap standard errors are in close agreement.

INSERT TABLE 2 HERE

How do these outcomes compare with those from a sample of "healthy" children? To examine this, we drew random samples of children's height from the distributions that generated the WHO MGRS growth curves, with samples constructed to match the age/gender structure of each NFHS survey, ensuring that we allow for appropriate natural differences around the median height across age-gender groups. The corresponding NFHS-3 Theil-1 index is 6.169E-04 with that for the other surveys being minimally different. The inequality for Indian children is over four times that of the natural inequality of healthy children, providing an indication of the disparities in health of Indian children and the relevance of examining whether inequality has changed.

Returning back to the results in Table 2, having standard error estimates enables us to ask whether the changes in the indices are statistically significant. Outcomes from hypothesis tests to address this question are given in Table 3. We provide Wald statistics, associated χ^2 and bootstrapped p-values from four tests: the first three test equality of indices across two surveys while the fourth test is for equality of the indices across the three surveys. Results are reported using the three different approaches to estimating variances. We denote Wald statistics by WT_L, W_{IID} and WT_{BT} and associated p-values by p_L, p_{IID} and p_{BT}. We assume that the samples across surveys are independent, allowing the variance of the difference in inequality indices to be the sum of the variances from each individual survey. This is a reasonable assumption, as the clusters are sampled independently from one survey to another.

INSERT TABLE 3 HERE

Turning to the test outcomes, the change between NFHS-1 and NFHS-2 is not statistically significant, while that between NFHS-2 and NFHS-3, and NFHS-1 and NFHS-3 are statistically significant. The bootstrap and linearization methods are in close agreement, again supporting use of the linearization approach over the more computationally intensive bootstrap. Using the iid outcomes does not qualitatively change the results. Our findings perhaps suggest that the high income growth observed in India has taken time to impact health inequality of children, as there is no significant change between NFHS-1 and NFHS-2 but there are strong declines in overall health inequality between NFHS-2 and NFHS-3. One possibility for the delayed impact could be habit persistence in food consumption, as it may take time for any income increase to lead to consumption of more nutritious food.

5.3 Inequality by place of residence and gender

This subsection contains results based on various sub-groups (sector- or gender- specific) and on the decomposition of the overall inequality by place of residence (rural or urban). Turning first to the urban sector, Table 4 provides estimated Theil 1 indices, their percentage changes and standard errors from the three methods, and Table 5 reports on tests across surveys.

INSERT TABLE 4 HERE

The results in Table 4 highlight the importance of sector-specific analysis, as the observed decline in health inequality of children occurred between NFHS-1 and NFHS-2 for this sector, with minimal change (indeed a nominal increase) in inequality between NFHS-2 and NFHS-3. This contrasts with the findings observed for overall inequality. In terms of standard errors, those under the false iid assumption are again smaller than those that account for the complex survey design, more so for NFHS-1 than for the other two surveys. We again observe that the bootstrap and linearization standard errors are in close agreement.

INERT TABLE 5 HERE

The outcomes provided in Table 5 support the statistical significance of the inequality change between the first two surveys but not between NFHS-2 and NFHS-3, with results qualitatively consistent across the three variance methods. Our findings on urban health inequality contrast with those on urban income inequality (at least for the 1990s); e.g., Deaton and Drèze (2002), which highlights the importance of exploring the impact of economic reforms on not just income inequality but also on other social inequality measures such as children's health inequality. Aside from examining sub-group inequality measures by region of residence, it is also of interest to ascertain the contribution of the between component to total inequality and how this share has changed across surveys. We report this information in Tables 6 and 7; Table 6 provides estimates of shares along with standard errors using the three variance methods and Table 7 details outcomes from hypothesis tests.

INSERT TABLE 6 HERE

Around one per cent of total health inequality arises from inequality between the rural and urban sectors, implying that the majority of inequality arises from within each sector. This between component is significantly different from zero. This small between component contrasts with most income inequality studies, where the between component often contributes more to overall inequality than the within component.[13] Our finding is similar to

[13] An example of an exception is Gray et al. (2003b), who report a one per cent between-group inequality share when comparing incomes of those born in Canada, immigrants who arrived before 1981, and immigrants who arrived after 1981.

that of, for instance, Pradhan et al. (2003), who, using children's height data, find that the within country health inequality component dominates world health inequality. Here, we find that this holds even within regions for an intra-country study. Whether this is a common finding for health inequality remains to be seen. At least at the 5% level, there is no evidence that the between group shares differ across surveys when using the complex survey linearization or bootstrap variance estimates, but not so under the false iid assumption in two of the four cases, highlighting, yet again, the importance of allowing for the sampling design.

INSERT TABLE 7 HERE

The final set of results we provide are given in Table 8, based on gender-specific inequality. Health inequality indices for boys and girls are reported, along with outcomes from hypothesis tests that examine whether height inequality for girls is the same as that for boys. Examining the estimated values, inequality for girls is higher than for boys, with the two-sided hypothesis tests indicating that this difference is statistically significant.[14] However, these results do not allow for the natural inequality within each gender group, which varies across groups. To estimate this, we drew random samples of healthy children's height from the distributions used to generate the WHO-MGRS growth curves, with the samples constructed to have the same age/gender structure as our NFHS samples. The boys and girls natural inequality Theil-1 estimates from these simulated samples of healthy children, denoted ${}_B\hat{I}^N_{T1}$ and ${}_G\hat{I}^N_{T1}$ respectively, are reported in the bottom part of Table 8, along with the differences $({}_B\hat{I}_{T1} - {}_B\hat{I}^N_{T1})$ and $({}_G\hat{I}_{T1} - {}_G\hat{I}^N_{T1})$, so-called adjusted inequalities, that estimate the inequality in children's height due to poor health and nutrition. The genetic natural inequality in the heights of healthy girls (as represented by percent-of-median) exceeds that of boys, mitigating much of the observed inequality differences between our Indian boys and girls when we do not account for natural inequality. Indeed, although not reported in the table, tests of the hypothesis that $({}_B\hat{I}_{T1} - {}_B\hat{I}^N_{T1}) = ({}_G\hat{I}_{T1} - {}_G\hat{I}^N_{T1})$ confirm that we cannot reject equality of these adjusted inequality measures, suggesting that these samples do not support gender differences in health inequality, at least at the national level.

INSERT TABLE 8 HERE

[14] Interestingly, we see an example (NFHS-2 boys) where the linearization and bootstrap standard errors are marginally smaller than the iid standard error.

6. CONCLUDING COMMENTS

We have considered undertaking inference on GE and Atkinson inequality indices when data are drawn under a complex survey design. Variance-covariance matrices are obtained via a linearization method that avoids the calculation of often cumbersome covariance expressions. One of our key contributions is to obtain expressions that enable inference for the components of common decompositions of the inequality measures, including "between" and "within" elements and any subsequent share measures generated from these. A key benefit of using these expressions for inference is ease of coding in standard software packages (e.g., Stata), in contrast to the coding that must be undertaken to bootstrap variances and p-values.

Our illustrative application using height data on Indian children highlights the importance of accounting for the stratified multi-stage cluster sampling design and the (typically) similar outcomes obtained using the linearization and bootstrap methods. This latter finding is particularly encouraging for applied researchers. Although it is unclear as to whether our findings can be broadly generalized, it is clear that the linearization approach to inference provides a user friendly way to undertake inference for inequality measures.

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Appendix A: Formulae for $\hat{\gamma}_{k,hij}$ [15]

A.1 Sample total indices

$$\hat{I}_{GE}^{\alpha}: \ \hat{\gamma}_{k,hij} = \alpha^{-1}\hat{U}_{\alpha}\hat{U}_1^{-\alpha}\hat{U}_0^{\alpha-2} - (\alpha-1)^{-1}\hat{U}_{\alpha}\hat{U}_0^{\alpha-1}\hat{U}_1^{-\alpha-1}y_{hij} + (\alpha^2-\alpha)^{-1}\hat{U}_0^{\alpha-1}\hat{U}_1^{-\alpha}y_{hij}^{\alpha};$$

$$\hat{I}_{T1}: \ \hat{\gamma}_{k,hij} = \hat{U}_1^{-1}y_{hij}\log y_{hij} + \hat{U}_0^{-1} - \hat{U}_1^{-1}(\hat{T}_1/\hat{U}_1+1)y_{hij};$$

$$\hat{I}_{T2}: \ \hat{\gamma}_{k,hij} = -\hat{U}_0^{-1}\log y_{hij} + \hat{U}_1^{-1}y_{hij} + \hat{U}_0^{-1}\left(\hat{T}_0/\hat{U}_0 - 1\right);$$

[15] The expressions given in subsections A.2 to A.7 are our contributions whereas those in A.1 are from Biewen and Jenkins (2006).

$$\hat{I}_A^{\varepsilon}: \ \hat{\gamma}_{k,hij} = \left(\varepsilon/(1-\varepsilon)\right)\hat{U}_0^{-1/(1-\varepsilon)}\hat{U}_1^{-1}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)} + \hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_1^{-2}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)} y_{hij}$$
$$- \left(1/(1-\varepsilon)\right)\hat{U}_1^{-1}(\hat{U}_{1-\varepsilon}/\hat{U}_0)^{\varepsilon/(1-\varepsilon)} y_{hij}^{1-\varepsilon};$$
$$\hat{I}_A^{1}: \ \hat{\gamma}_{k,hij} = \hat{U}_0^{-1}\left(\hat{I}_A^1 - 1\right)\log y_{hij} + \hat{U}_0^{-1}\left(\hat{I}_A^1 - 1\right)\left(1 - \hat{T}_0/\hat{U}_0\right) + \hat{U}_1^{-1}\left(1 - \hat{I}_A^1\right) y_{hij}.$$

A.2 Sample sub-group indices

$${}_g\hat{I}_{GE}^{\alpha}: \ \hat{\gamma}_{k,hij} = \alpha^{-1}({}_g\hat{U}_\alpha)({}_g\hat{U}_1^{-\alpha})({}_g\hat{U}_0^{\alpha-2})({}_gD_{hij}) - (\alpha-1)^{-1}({}_g\hat{U}_\alpha)({}_g\hat{U}_0^{\alpha-1})({}_g\hat{U}_1^{-\alpha-1})\times$$
$$({}_gD_{hij})y_{hij} + \left(\alpha^2-\alpha\right)^{-1}({}_g\hat{U}_0^{\alpha-1})({}_g\hat{U}_1^{-\alpha})({}_gD_{hij})y_{hij}^{\alpha};$$
$${}_g\hat{I}_{T1}: \ \hat{\gamma}_{k,hij} = ({}_g\hat{U}_1^{-1})({}_gD_{hij})y_{hij}\log y_{hij} + ({}_g\hat{U}_0^{-1})({}_gD_{hij}) - ({}_g\hat{U}_1^{-1})\left({}_g\hat{T}_1/{}_g\hat{U}_1 + 1\right)\times$$
$$({}_gD_{hij})y_{hij};$$
$${}_g\hat{I}_{T2}: \ \hat{\gamma}_{k,hij} = -({}_g\hat{U}_0^{-1})({}_gD_{hij})\log y_{hij} + ({}_g\hat{U}_1^{-1})({}_gD_{hij})y_{hij} + ({}_g\hat{U}_0^{-1})\left({}_g\hat{T}_0/{}_g\hat{U}_0 - 1\right)\times$$
$$({}_gD_{hij});$$
$${}_g\hat{I}_A^{\varepsilon}: \ \hat{\gamma}_{k,hij} = \left(\varepsilon/(1-\varepsilon)\right)\left({}_g\hat{U}_1^{-1}\right)(({}_g\hat{U}_{1-\varepsilon})/({}_g\hat{U}_0))^{1/(1-\varepsilon)}({}_gD_{hij}) + ({}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)})({}_g\hat{U}_1^{-2})\times$$
$$({}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)})({}_gD_{hij})y_{hij} - \left(1/(1-\varepsilon)\right)\left({}_g\hat{U}_1^{-1}\right)(({}_g\hat{U}_{1-\varepsilon})/({}_g\hat{U}_0))^{\varepsilon/(1-\varepsilon)}({}_gD_{hij})y_{hij}^{1-\varepsilon};$$
$${}_g\hat{I}_A^{1}: \ \hat{\gamma}_{k,hij} = ({}_g\hat{U}_0^{-1})\left({}_g\hat{I}_A^1 - 1\right)\left({}_gD_{hij}\right)\log y_{hij} + ({}_g\hat{U}_0^{-1})\left({}_g\hat{I}_A^1 - 1\right)\left(1 - {}_g\hat{T}_0/{}_g\hat{U}_0\right)({}_gD_{hij})$$
$$+ ({}_g\hat{U}_1^{-1})\left(1 - {}_g\hat{I}_A^1\right)({}_gD_{hij})y_{hij}.$$

A.3 Sample within indices

$$\hat{W}_{GE}^{\alpha}: \ \hat{\gamma}_{k,hij} = (\alpha^2-\alpha)^{-1}\hat{U}_0^{\alpha-2}\hat{U}_1^{-\alpha}\left(\sum_{g=1}^{G}\left\{\left[(\alpha-1)\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)^{\alpha} - \alpha\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)^{\alpha-1}y_{hij}\right.\right.\right.$$
$$\left.\left.\left. + y_{hij}^{\alpha}\ \right]\hat{U}_0({}_gD_{hij}) + \left({}_g\hat{U}_\alpha - ({}_g\hat{U}_0^{1-\alpha})({}_g\hat{U}_1^{\alpha})\right)\left[\alpha\left(1 - \left(\hat{U}_0/\hat{U}_1\right)y_{hij}\right) - 1\right]\right\}\right);$$
$$\hat{W}_{T1}: \ \hat{\gamma}_{k,hij} = -\hat{U}_1^{-2}y_{hij}\sum_{g=1}^{G}\left[{}_g\hat{T}_1 - ({}_g\hat{U}_1)\log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)\right] + \hat{U}_1^{-1}\sum_{g=1}^{G}({}_gD_{hij})\left[{}_g\hat{U}_1/{}_g\hat{U}_0\right.$$
$$\left. + y_{hij}\log y_{hij} - \left(1 + \log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)\right)y_{hij}\ \right];$$

$$\hat{W}_{T2}: \ \hat{\gamma}_{k,hij} = \hat{U}_0^{-2}\sum_{g=1}^{G}\left\{\hat{U}_0({}_gD_{hij})\left[\left({}_g\hat{U}_0/{}_g\hat{U}_1\right)y_{hij} + \log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right) - 1 - \log y_{hij}\right]\right.$$
$$\left. + ({}_g\hat{T}_0) - ({}_g\hat{U}_0)\log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)\right\};$$
$$\hat{W}_A^{\varepsilon}: \ \hat{\gamma}_{k,hij} = \hat{U}_1^{-1}\sum_{g=1}^{G}({}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)})({}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)})\left[\hat{U}_1^{-1}y_{hij} + \left(\varepsilon/(1-\varepsilon)\right)\left({}_g\hat{U}_0^{-1}\right)({}_gD_{hij})\right.$$
$$\left. - \left(1/(1-\varepsilon)\right)\left({}_g\hat{U}_{1-\varepsilon}^{-1}\right)({}_gD_{hij})y_{hij}^{1-\varepsilon}\right];$$
$$\hat{W}_A^{1}: \ \hat{\gamma}_{k,hij} = \hat{U}_1^{-1}\sum_{g=1}^{G}\exp\left(({}_g\hat{T}_0)/({}_g\hat{U}_0)\right)\left[({}_g\hat{U}_0/\hat{U}_1)y_{hij} - ({}_gD_{hij})\left(1 - {}_g\hat{T}_0/{}_g\hat{U}_0 + \log y_{hij}\right)\right].$$

A.4 Sample between indices

$$\hat{B}^{\alpha}_{GE}: \ \hat{\gamma}_{k,hij} = (\alpha^2-\alpha)^{-1}\hat{U}_0^{\alpha-2}\hat{U}_1^{-\alpha-1}\left(\sum_{g=1}^{G}\left\{\hat{U}_0\hat{U}_1({}_gD_{hij})\left[\alpha\left({}_g\hat{U}_0/{}_g\hat{U}_1\right)^{1-\alpha}y_{hij}\right.\right.\right.$$

$$\left.\left.\left.-(\alpha-1)\left({}_g\hat{U}_0/{}_g\hat{U}_1\right)^{-\alpha}\right]+({}_g\hat{U}_0^{1-\alpha})({}_g\hat{U}_1^{\alpha})\left[(\alpha-1)\hat{U}_1-\alpha\hat{U}_0\right]\right\}\right);$$

$$\hat{B}_{T1}: \ \hat{\gamma}_{k,hij} = \hat{U}_0^{-1}+\hat{U}_1^{-1}\left\{-y_{hij}+\sum_{g=1}^{G}({}_gD_{hij})\left[\left(1+\log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)\right)y_{hij}-{}_g\hat{U}_1/{}_g\hat{U}_0\right]\right\}$$

$$-\hat{U}_1^{-2}\sum_{g=1}^{G}({}_g\hat{U}_1)\log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)y_{hij};$$

$$\hat{B}_{T2}: \ \hat{\gamma}_{k,hij} = -\hat{U}_0^{-1}\sum_{g=1}^{G}\left({}_g\hat{U}_0/{}_g\hat{U}_1\right)({}_gD_{hij})y_{hij}+\hat{U}_0^{-1}\sum_{g=1}^{G}\left(1+\log\left({}_g\hat{U}_0/{}_g\hat{U}_1\right)\right)({}_gD_{hij})$$

$$+\hat{U}_1^{-1}y_{hij}-\hat{U}_0^{-1}\left(1+\hat{U}_0^{-1}\sum_{g=1}^{G}({}_g\hat{U}_0)\log\left({}_g\hat{U}_0/{}_g\hat{U}_1\right)\right);$$

$$\hat{B}^{\varepsilon}_{A}: \ \hat{\gamma}_{k,hij} = \left(1/(1-\varepsilon)\right)\left(\hat{U}_{1-\varepsilon}/\hat{U}_0\right)^{\varepsilon/(1-\varepsilon)}\left[\sum_{g=1}^{G}({}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)})({}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)})\right]^{-2}\left\{\left(\sum_{g=1}^{G}({}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)})\times\right.\right.$$

$$({}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)})\left(\varepsilon\left(\hat{U}_{1-\varepsilon}/\hat{U}_0\right)-y_{hij}^{1-\varepsilon}\right)+\hat{U}_{1-\varepsilon}\left(\sum_{g=1}^{G}({}_gD_{hij})(({}_g\hat{U}_{1-\varepsilon})/({}_g\hat{U}_0))^{\varepsilon/(1-\varepsilon)}\left[y_{hij}^{1-\varepsilon}\right.\right.$$

$$\left.\left.\left.-\varepsilon\left({}_g\hat{U}_{1-\varepsilon}/{}_g\hat{U}_o\right)\right]\right)\right\};$$

$$\hat{B}^{1}_{A}: \ \hat{\gamma}_{k,hij} = \exp\left(\hat{T}_0/\hat{U}_0\right)\left[\sum_{g=1}^{G}({}_g\hat{U}_0)\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\right]^{-1}\left\{-\log y_{hij}+\left(\hat{T}_0/\hat{U}_0-1\right)\right.$$

$$+\hat{U}_0^{-1}\left[\sum_{g=1}^{G}({}_g\hat{U}_0)\exp\left({}_g\hat{T}_0/{}_g\hat{U}_1\right)\right]^{-1}\left[\sum_{g=1}^{G}\left(\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\right)({}_gD_{hij})\times\right.$$

$$\left.\left.\left(1-{}_g\hat{T}_0/{}_g\hat{U}_0+\log y_{hij}\right)\right]\right\}.$$

A.5 Sample within shares

$$\hat{S}^{\alpha}_{GE,W}: \hat{\gamma}_{k,hij} = \left(\hat{U}_\alpha-\hat{U}_0^{1-\alpha}\hat{U}_1^{\alpha}\right)^{-1}\sum_{g=1}^{G}({}_gD_{hij})\left[-\alpha\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)^{\alpha-1}y_{hij}+(\alpha-1)\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)^{\alpha}\right.$$

$$\left.+y_{hij}^{\alpha}\right]+\left(\hat{U}_\alpha-\hat{U}_0^{1-\alpha}\hat{U}_1^{\alpha}\right)^{-2}\left[(1-\alpha)\left(\hat{U}_1/\hat{U}_0\right)^{\alpha}+\alpha\left(\hat{U}_1/\hat{U}_0\right)^{\alpha-1}y_{hij}-y_{hij}^{\alpha}\right]\times$$

$$\sum_{g=1}^{G}\left(({}_g\hat{U}_\alpha)-({}_g\hat{U}_0^{1-\alpha})({}_g\hat{U}_1^{\alpha})\right);$$

$$\hat{S}_{T1,W}: \hat{\gamma}_{k,hij} = \left(\hat{T}_1-\hat{U}_1\log\left(\hat{U}_1/\hat{U}_0\right)\right)^{-2}\left[\left(1+\log\left(\hat{U}_1/\hat{U}_0\right)\right)y_{hij}-y_{hij}\log y_{hij}-\hat{U}_1/\hat{U}_0\right]\times$$

$$\left[\sum_{g=1}^{G}\left({}_g\hat{T}_1 - ({}_g\hat{U}_1)\log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)\right)\right] + \left(\hat{T}_1 - \hat{U}_1\log\left(\hat{U}_1/\hat{U}_0\right)\right)^{-1} \times$$

$$\sum_{g=1}^{G}({}_g D_{hij})\left[{}_g\hat{U}_1/{}_g\hat{U}_0 - \left(1+\log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)\right)y_{hij} + y_{hij}\log y_{hij}\right];$$

$$\hat{S}_{T2,W} : \hat{\gamma}_{k,hij} = \left(-\hat{T}_0 + \hat{U}_0\log\left(\hat{U}_1/\hat{U}_0\right)\right)^{-2}\left[\log y_{hij} + 1 - \log\left(\hat{U}_1/\hat{U}_0\right) - \left(\hat{U}_0/\hat{U}_1\right)y_{hij}\right] \times$$

$$\left[\sum_{g=1}^{G}\left(-({}_g\hat{T}_0) + ({}_g\hat{U}_0)\log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)\right)\right] + (-\hat{T}_0 + \hat{U}_0\log(\hat{U}_1/\hat{U}_0))^{-1} \times$$

$$\sum_{g=1}^{G}({}_g D_{hij})\left[-\log y_{hij} + \log({}_g\hat{U}_1/{}_g\hat{U}_0) - 1 + ({}_g\hat{U}_0/{}_g\hat{U}_1)y_{hij}\right];$$

$$\hat{S}^{\varepsilon}_{A,W} : \hat{\gamma}_{k,hij} = \left(\hat{U}_1 - \hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\right)^{-1}\left\{\left[1 - \left(\hat{U}_1 - \hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\right)^{-1} \times\right.\right.$$

$$\left.\left(\hat{U}_1 - \sum_{g=1}^{G}({}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)})({}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)})\right)y_{hij}\right] - \hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)} \times$$

$$\left(\hat{U}_1 - \hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\right)^{-1}\left(\hat{U}_1 - \sum_{g=1}^{G}({}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)})({}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)})\right) \times$$

$$\left[(\varepsilon/(1-\varepsilon))\hat{U}_0^{-1} - (1/(1-\varepsilon))\hat{U}_{1-\varepsilon}^{-1}y_{hij}^{1-\varepsilon}\right] + \sum_{g=1}^{G}({}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)})({}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}) \times$$

$$\left.({}_g D_{hij})\left[\ (\varepsilon/(1-\varepsilon))({}_g\hat{U}_0^{-1}) - (1/(1-\varepsilon))({}_g\hat{U}_{1-\varepsilon}^{-1})y_{hij}^{1-\varepsilon}\ \right]\right\};$$

$$\hat{S}^{1}_{A,W} : \hat{\gamma}_{k,hij} = \left(\hat{U}_1 - \hat{U}_0\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-1}\left\{y_{hij} - \sum_{g=1}^{G}({}_g D_{hij})\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right) \times\right.$$

$$\left.\left(\log y_{hij} + 1 - {}_g\hat{T}_0/{}_g\hat{U}_0\right)\right\} + \left(\hat{U}_1 - \hat{U}_0\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-2} \times$$

$$\left[\hat{U}_1 - \sum_{g=1}^{G}({}_g\hat{U}_0)\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\right]\left[-y_{hij} + \exp\left(\hat{T}_0/\hat{U}_0\right)\left(1 - \hat{T}_0/\hat{U}_0 + \log y_{hij}\right)\right].$$

A.6 Sample between shares

$\hat{S}^{\alpha}_{GE,B} : \hat{\gamma}_{k,hij}$ is the same as for $\hat{S}^{\alpha}_{GE,W}$;

$\hat{S}_{T1,B} : \hat{\gamma}_{k,hij}$ is the same as for $\hat{S}_{T1,W}$;

$\hat{S}_{T2,B} : \hat{\gamma}_{k,hij}$ is the same as for $\hat{S}_{T2,W}$;

$$\hat{S}^{\varepsilon}_{A,B} : \hat{\gamma}_{k,hij} = (1/(1-\varepsilon))\hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\left(1 - \hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_1^{-1}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\right)^{-1} \times$$

$$\left\{-\varepsilon\hat{U}_0^{-1}\hat{U}_1^{-1}\left(1 - \hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_1^{-1}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\right)^{-1} + \left(\sum_{g=1}^{G}({}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)})({}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)})\right)^{-1} \times\right.$$

$$\left[1+\hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_1^{-1}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\left(1-\hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_1^{-1}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\right)^{-1}\right]\times$$

$$\left(\varepsilon\hat{U}_0^{-1}+\hat{U}_{1-\varepsilon}^{-1}y_{hij}^{1-\varepsilon}\right)+(1-\varepsilon)\hat{U}_1^{-2}\left(1-\hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_1^{-1}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\right)^{-1}\times$$

$$\left[-1+\hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\left(\sum_{g=1}^{G}({}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)})({}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)})\right)^{-1}\right]y_{hij}+$$

$$\hat{U}_{1-\varepsilon}^{-1}\hat{U}_1^{-1}\left(1-\hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_1^{-1}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\right)^{-1}y_{hij}^{1-\varepsilon}-\left(\sum_{g=1}^{G}({}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)})({}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)})\right)^{-2}\times$$

$$\left.\sum_{g=1}^{G}({}_gD_{hij})\left({}_g\hat{U}_{1-\varepsilon}/{}_g\hat{U}_0\right)^{\varepsilon/(1-\varepsilon)}\left(\varepsilon\left({}_g\hat{U}_{1-\varepsilon}/{}_g\hat{U}_0\right)-y_{hij}^{1-\varepsilon}\right)\right\}.$$

$$\hat{S}_{A,B}^{1}:\ \hat{\gamma}_{k,hij}=\ \hat{U}_1^{-1}\exp\left(\hat{T}_0/\hat{U}_0\right)\left(1-(\hat{U}_0/\hat{U}_1)\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-2}\log y_{hij}-\exp\left(\hat{T}_0/\hat{U}_0\right)\times$$

$$\left(1-(\hat{U}_0/\hat{U}_1)\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-1}\left[\sum_{g-1}^{G}({}_g\hat{U}_0)\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\right]^{-1}\times$$

$$\left[(\hat{U}_0/\hat{U}_1)\exp\left(\hat{T}_0/\hat{U}_0\right)\left(1-(\hat{U}_0/\hat{U}_1)\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-1}+1\right]\log y_{hij}$$

$$-\left(1-(\hat{U}_0/\hat{U}_1)\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-2}\hat{U}_1^{-1}\exp\left(\hat{T}_0/\hat{U}_0\right)\left(-1+(\hat{T}_0/\hat{U}_0)\exp\left(\hat{T}_0/\hat{U}_0\right)\right)\times$$

$$\left[1-\hat{U}_0\exp\left(\hat{T}_0/\hat{U}_0\right)\left(\sum_{g=1}^{G}({}_g\hat{U}_0)\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\right)^{-1}\right]+\left(1-(\hat{U}_0/\hat{U}_1)\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-1}$$

$$\times\exp\left(\hat{T}_0/\hat{U}_0\right)\left(\sum_{g=1}^{G}({}_g\hat{U}_0)\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\right)^{-1}\left(\hat{T}_0/\hat{U}_0-1\right)-\hat{U}_0\hat{U}_1^{-2}\times$$

$$\exp\left(\hat{T}_0/\hat{U}_0\right)\left(1-(\hat{U}_0/\hat{U}_1)\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-2}\left[1-\hat{U}_0\exp\left(\hat{T}_0/\hat{U}_0\right)\times\right.$$

$$\left.\left(\sum_{g=1}^{G}({}_g\hat{U}_0)\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\right)^{-1}\right]y_{hij}+\hat{U}_0\exp\left(\hat{T}_0/\hat{U}_0\right)\left(1-(\hat{U}_0/\hat{U}_1)\times\right.$$

$$\left.\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-1}\left(\sum_{g=1}^{G}({}_g\hat{U}_0)\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\right)^{-2}\left[\sum_{g=1}^{G}\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\left(1-\right.\right.$$

$$\left.\left.{}_g\hat{T}_0/{}_g\hat{U}_0\right)({}_gD_{hij})\right]+\hat{U}_0\exp\left(\hat{T}_0/\hat{U}_0\right)\left(1-(\hat{U}_0/\hat{U}_1)\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-1}\times$$

$$\left(\sum_{g=1}^{G}({}_g\hat{U}_0)\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\right)^{-2}\left[\sum_{g=1}^{G}\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)({}_gD_{hij})y_{hij}\right].$$

A.7 Sample sub-group within shares

$$ {}_g\hat{S}^{\alpha}_{GE,W} : \hat{\gamma}_{k,hij} = -\left(\hat{U}_\alpha - \hat{U}_0^{1-\alpha}\hat{U}_1^{\alpha}\right)^{-1}({}_g D_{hij})\left[\alpha\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)^{\alpha-1} y_{hij} + \right.$$

$$\left.(1-\alpha)\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)^{\alpha} - y_{hij}^{\alpha}\right] + \left(\hat{U}_\alpha - \hat{U}_0^{1-\alpha}\hat{U}_1^{\alpha}\right)^{-2}\left(({}_g\hat{U}_\alpha) - ({}_g\hat{U}_0^{1-\alpha})({}_g\hat{U}_1^{\alpha})\right)\times$$

$$\left[\alpha\left(\hat{U}_1/\hat{U}_0\right)^{\alpha-1} y_{hij} + (1-\alpha)\left(\hat{U}_1/\hat{U}_0\right)^{\alpha} - y_{hij}^{\alpha}\right];$$

$$ {}_g\hat{S}_{T1,W} : \hat{\gamma}_{k,hij} = \left(\hat{T}_1 - \hat{U}_1\log\left(\hat{U}_1/\hat{U}_0\right)\right)^{-2}\left({}_g\hat{T}_1 - {}_g\hat{U}_1\log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)\right)\times$$

$$\left[\ -y_{hij}\log y_{hij} - \hat{U}_1/\hat{U}_0 + \left(1+\log\left(\hat{U}_1/\hat{U}_0\right)\right)y_{hij}\ \right] + \left(\hat{T}_1 - \hat{U}_1\log\left(\hat{U}_1/\hat{U}_0\right)\right)^{-1}\times$$

$$({}_g D_{hij})\left[\ ({}_g\hat{U}_1/{}_g\hat{U}_0) - \left(1+\log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)\right)y_{hij} + y_{hij}\log y_{hij}\ \right];$$

$$ {}_g\hat{S}_{T2,W} : \hat{\gamma}_{k,hij} = \left(-\hat{T}_0 + \hat{U}_0\log\left(\hat{U}_1/\hat{U}_0\right)\right)^{-2}\left(\log y_{hij} - \log\left(\hat{U}_1/\hat{U}_0\right) + 1 - \left(\hat{U}_0/\hat{U}_1\right)y_{hij}\right)\times$$

$$\left(-{}_g\hat{T}_0 + ({}_g\hat{U}_0)\log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right)\right) - \left(-\hat{T}_0 + \hat{U}_0\log\left(\hat{U}_1/\hat{U}_0\right)\right)^{-1}\times$$

$$({}_g D_{hij})\left(\log y_{hij} - \log\left({}_g\hat{U}_1/{}_g\hat{U}_0\right) + 1 - \left({}_g\hat{U}_0/{}_g\hat{U}_1\right)y_{hij}\right);$$

$$ {}_g\hat{S}^{\varepsilon}_{A,W} : \hat{\gamma}_{k,hij} = \left(\hat{U}_1 - \hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\right)^{-2}\left\{-\left({}_g\hat{U}_1 - {}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)}\,{}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\right)\times\right.$$

$$\left[\ y_{hij} + \left(1/(1-\varepsilon)\right)\hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\left(\varepsilon\hat{U}_0^{-1} - \hat{U}_{1-\varepsilon}^{-1}y_{hij}^{1-\varepsilon}\right)\ \right] +$$

$$\left(\hat{U}_1 - \hat{U}_0^{-\varepsilon/(1-\varepsilon)}\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\right)\left({}_g D_{hij}\right)\left[\ y_{hij} + \left(1/(1-\varepsilon)\right){}_g\hat{U}_0^{-\varepsilon/(1-\varepsilon)}\,{}_g\hat{U}_{1-\varepsilon}^{1/(1-\varepsilon)}\times\right.$$

$$\left.\left.\left(\varepsilon({}_g\hat{U}_0^{-1}) - ({}_g\hat{U}_{1-\varepsilon}^{-1})y_{hij}^{1-\varepsilon}\right)\ \right]\ \right\};$$

$$ {}_g\hat{S}^{1}_{A,W} : \hat{\gamma}_{k,hij} = \left(\hat{U}_1 - \hat{U}_0\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-1}\left\{-\hat{U}_0^{-2}({}_g\hat{U}_0)(\hat{U}_1 - \hat{U}_0 y_{hij}) + ({}_g D_{hij})\left[\hat{U}_1/\hat{U}_0 - \right.\right.$$

$$\left.\left.\exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\left(1 - ({}_g\hat{T}_0/{}_g\hat{U}_0) + \log y_{hij}\right)\right]\right\} + {}_g\hat{U}_0\left(\hat{U}_1 - \hat{U}_0\exp\left(\hat{T}_0/\hat{U}_0\right)\right)^{-2}\times$$

$$\left(\hat{U}_1/\hat{U}_0 - \exp\left({}_g\hat{T}_0/{}_g\hat{U}_0\right)\right)\left[\ -y_{hij} + \exp\left(\hat{T}_0/\hat{U}_0\right)\left(1 - \hat{T}_0/\hat{U}_0 + \log y_{hij}\right)\ \right].$$

Table 1. GE and A indices: between component and weights for the within component

Index	*Weight* ω_g	*Between* *B*
I_{GE}^{α}	$\left(\frac{{}_gU_0}{U_0}\right)^{1-\alpha}\left(\frac{{}_gU_1}{U_1}\right)^{\alpha}$	$(\alpha^2-\alpha)^{-1}\left[\sum_{g=1}^{G}\left(\frac{{}_gU_0}{U_0}\right)^{1-\alpha}\left(\frac{{}_gU_1}{U_1}\right)^{\alpha}-1\right]$
I_{T1}	$\left(\frac{{}_gU_1}{U_1}\right)$	$\sum_{g=1}^{G}\left(\frac{{}_gU_1}{U_1}\right)\log\left(\frac{{}_gU_1/U_1}{{}_gU_0/U_0}\right)$
I_{T2}	$\left(\frac{{}_gU_0}{U_0}\right)$	$\sum_{g=1}^{G}\left(\frac{{}_gU_0}{U_0}\right)\log\left(\frac{{}_gU_0/U_0}{{}_gU_1/U_1}\right)$
I_{A}^{ε}	$\left(\frac{{}_gU_1}{U_1}\right)$	$1-\left[\sum_{g=1}^{G}\left(\frac{({}_gU_{1-\varepsilon}/U_{1-\varepsilon})^{1/(1-\varepsilon)}}{({}_gU_0/U_0)^{\varepsilon/(1-\varepsilon)}}\right)\right]^{-1}$
I_{A}^{1}	$\left(\frac{{}_gU_1}{U_1}\right)$	$1\text{-}(\exp(T_0/U_0))\left[\sum_{g=1}^{G}\left(\frac{{}_gU_0}{U_0}\right)\exp({}_gT_0/{}_gU_0)\right]^{-1}$

Note: The table provides the weights used to form the within component for each index; i.e., $W=\sum_{g=1}^{G}\omega_g I_g$, where I_g is the g'th sub-group's inequality index (g=1,…,G).

Table 2. Estimates of overall height inequality using Theil-1 and standard errors

	NFHS-1 (1992/93)	*NFHS-2 (1998/99)*	*NFHS-3 (2005/06)*	*% change NFHS-2 /NFHS-1*	*% change NFHS-3 /NFHS-1*	*% change NFHS-3 /NFHS-2*
$\hat{I}_{T1}$	2.655E-03	2.609E-03	2.263E-03	-1.73%	-14.76%	-13.26%
se_L	4.635E-05	3.919E-05	3.734E-05			
se_{IID}	3.543E-05	3.405E-05	3.414E-05			
se_{BT}	4.772E-05	3.644E-05	3.944E-05			

Note: The standard error obtained via the linearization method is denoted by se_L, that from the linearization approach assuming (falsely) that sampling is iid with weights as se_{IID} and that from the bootstrap method by se_{BT}.

Table 3. Testing whether overall inequality has changed across surveys using Theil-1

	Hypothesis test			
	NFHS-1= NFHS-2	*NFHS-2= NFHS-3*	*NFHS-1= NFHS-3*	*NFHS-1= NFHS-2= NFHS-3*
WT_L (p_L)	0.586 (0.444)	40.956 (0.000)	43.573 (0.000)	59.019 (0.000)
WT_{IID} (p_{IID})	0.894 (0.344)	51.617 (0.000)	63.764 (0.000)	77.785 (0.000)
WT_{BT} (p_{BT})	0.599 (0.500)	41.625 (0.010)	40.281 (0.010)	55.803 (0.010)

Notes: The table reports Wald statistics and associated p-values for equality of indices. The subscripts are: L = complex survey linearization; IID = iid with weights linearization; BT= complex survey bootstrap.

Table 4. Urban sector: estimates of height inequality and standard errors for Theil-1

	NFHS-1 (1992/93)	*NFHS-2 (1998/99)*	*NFHS-3 (2005/06)*	*% change NFHS-2 /NFHS-1*	*% change NFHS-3 /NFHS-1*	*% change NFHS-3 /NFHS-2*
${}_U\hat{I}_{T1}$	2.445E-03	2.140E-03	2.160E-03	-12.47%	-11.66%	0.93%
se_L	8.297E-05	6.444E-05	6.830E-05			
se_{IID}	6.458E-05	5.721E-05	6.268E-05			
se_{BT}	8.953E-05	6.6290E-05	6.430E-05			

Notes: The standard error obtained via the linearization method is denoted by se_L, that from the linearization approach assuming (falsely) that sampling is iid with weights as se_{IID} and that from the bootstrap method by se_{BT}. The subscript U in ${}_U\hat{I}_{T1}$ indicates index estimates for the urban sector.

Table 5. Testing whether height inequality of urban children has changed using Theil-1

	Hypothesis test			
	NFHS-1= NFHS-2	*NFHS-2= NFHS-3*	*NFHS-1= NFHS-3*	*NFHS-1= NFHS-2= NFHS-3*
WT_L (p_L)	8.428 (0.004)	0.047 (0.828)	7.011 (0.008)	9.654 (0.008)
WT_{IID} (p_{IID})	12.496 (0.000)	0.058 (0.810)	9.997 (0.002)	14.738 (0.001)
WT_{BT} (p_{BT})	7.495 (0.040)	0.049 (0.800)	6.664 (0.020)	8.594 (0.030)

Notes: The table reports Wald statistics and associated p-values for equality of urban indices. The subscripts are: L = complex survey linearization; IID = iid with weights linearization; BT= complex survey bootstrap.

Table 6. Urban-rural between component shares and standard errors (%'s)

	NFHS-1 (1992/93)	*NFHS-2 (1998/99)*	*NFHS-3 (2005/06)*
$\hat{S}_{T1,B}$	0.50%	1.04%	0.75%
se_L	0.19%	0.22%	0.18%
se_{IID}	0.11%	0.16%	0.15%
se_{BT}	0.23%	0.22%	0.17%

Note: The standard error obtained via the linearization method is denoted by se_L, that from the linearization approach assuming (falsely) that sampling is iid with weights as se_{IID} and that from the bootstrap method by se_{BT}.

Table 7. Testing whether the Theil-1 between group (urban/rural) share has changed

	Hypothesis test			
	NFHS-1= NFHS-2	*NFHS-2= NFHS-3*	*NFHS-1= NFHS-3*	*NFHS-1= NFHS-2= NFHS-3*
	$S_{T1,B}$			
WT_L (p_L)	3.346 (0.067)	1.023 (0.312)	0.891 (0.345)	3.351 (0.187)
WT_{IID} (p_{IID})	7.328 (0.007)	1.688 (0.194)	1.691 (0.193)	7.465 (0.024)
WT_{BT} (p_{BT})	2.864 (0.100)	1.108 (0.290)	0.754 (0.320)	2.892 (0.240)

Note: The table reports Wald statistics and associated p-values for equality of urban/rural between shares. . The subscripts are: L = complex survey linearization; IID = iid with weights linearization; BT= complex survey bootstrap.

Table 8. Boys and girls: height inequality and standard errors using Theil-1

	NFHS-1 (1992/93)	*NFHS-2 (1998/99)*	*NFHS-3 (2005/06)*	*% change NFHS-2 /NFHS-1*	*% change NFHS-3 /NFHS-1*	*% change NFHS-3 /NFHS-2*
${}_{B}\hat{I}_{T1}$	2.561E-03	2.472E-03	2.153E-03	-3.48%	-15.93%	-12.90%
se_L	5.655E-05	4.278E-05	4.595E-05			
se_{IID}	4.880E-05	4.326E-05	4.323E-05			
se_{BT}	5.696E-05	3.816E-05	4.784E-05			
${}_{G}\hat{I}_{T1}$	2.752E-03	2.758E-03	2.383E-03	0.22%	-13.41%	-13.60%
se_L	5.792E-05	5.956E-05	5.569E-05			
se_{IID}	5.140E-05	5.335E-05	5.363E-05			
se_{BT}	5.935E-05	5.867E-05	5.980E-05			
		boys=girls				
WT_L	8.061	17.984	11.152			
(p_L)	(0.005)	(0.000)	(0.001)			
WT_{IID}	7.311	17.333	11.168			
(p_{IID})	(0.007)	(0.000)	(0.001)			
WT_{BT}	8.248	18.911	9.883			
(p_{BT})	(0.010)	(0.010)	(0.010)			
		natural inequality				
${}_{B}\hat{I}^{N}_{T1}$	5.851E-04	5.788E-04	5.703E-04			
${}_{G}\hat{I}^{N}_{T1}$	6.673E-04	6.706E-04	6.673E-04			
		adjusted inequality				
$({}_{B}\hat{I}_{T1} - {}_{B}\hat{I}^{N}_{T1})$	1.976E-04	1.893E-04	1.583E-04			
$({}_{G}\hat{I}_{T1} - {}_{G}\hat{I}^{N}_{T1})$	2.085E-04	2.087E-04	1.716E-04			

Notes: The subscript L denotes use of the linearization method (accounting for the complex survey) to form variances, IID implies that the variances are obtained using the linearization approach under a false iid assumption with weights, and BT refers to the bootstrap method that accommodates the complex survey design. The subscript B and G in ${}_{B}\hat{I}_{T1}$ and ${}_{G}\hat{I}_{T1}$, and ${}_{B}\hat{I}^{N}_{T1}$ and ${}_{G}\hat{I}^{N}_{T1}$, indicate Theil-1 index estimates for boys and girls respectively.