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Testing For Convergence in Output and in 'Well-Being' in Industrialized Countries*

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ABSTRACT

There is now an extensive empirical literature relating to tests for various forms of convergence between the real *per capita* outputs of different countries. The evidence from these tests is mixed, and depends upon the type of data used, the countries in question, and the sample period in question. However, very little attention has been paid to the possibility of an associated convergence in "well-being" across countries. Indeed, it is interesting to posit the lack of any connection between convergence in output (income) and convergence in "well-being". In this paper we address this issue in the context of fourteen OECD economies, using various measures of "well-being", and different tests of convergence. The latter include a time-series test recently proposed by Nahar and Inder (2002), and a test based on fuzzy clustering proposed by Giles (2001). Our findings indicate that in general one should *not* expect convergence in output to be associated with convergence in "well-being".

Key Words: Convergence; well-being indicators; time-series analysis; fuzzy clustering

JEL Classifications: C22, C63, O40

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1. Introduction

One of the key contributions of the neo-classical growth model is it that it predicts the conditional convergence of output. Among the various forms of convergence discussed in the literature, two have received particular attention. The first is generally called "β-convergence", where poorer countries grow faster on average than richer ones, and so they "catch up" in terms of the level of *per capita* income or output. Several studies have considered this phenomenon; *e.g.*, Baumol (1986), Barro (1991), Mankiw *et al.* (1992), and Sala-i-Martin (1996). The second type of convergence is the so-called "σ-convergence" (Easterlin, 1960, Borts and Stein, 1964, Barro, 1984, Baumol, 1986, Dowrich and Nguyen, 1989, and Barro and Sala-i-Martin, 1992). Here, one looks for a decrease, over time, in the variability of *per capita* output over time across countries. "σ-convergence" is stronger than "β-convergence" in the sense that an absence of the former can co-exist with the presence of the latter.[1]

It should be noted that the evidence from the convergence literature that is based on cross-section data (*i.e.*, much of the early "β-convergence" literature) is generally in favour of convergence in the case of OECD countries. In contrast, many of the studies that use tests based on time-series data for these countries reject convergence. Moreover, it is noteworthy that this convergence literature focuses almost exclusively on the growth of output (income), and there has been very little empirical research relating to the possibility of convergence in measures of "well-being". In this respect, an interesting question that can be raised is whether or not convergence in the output is also associated with convergence of the "well-being" of the agents in the economies in question.

In this paper we address precisely this question. In a related study, Hobijn and Frances (2000) looked also at this issue by checking 171 countries' real GDP *per capita* convergence against the convergence of four "well-being" indicators: life expectancy at birth, infant mortality, daily protein supply and daily calorie supply. They came to a negative conclusion. In this paper, we focus our attention on fourteen OECD countries. In addition to GDP convergence, we also test for possible convergence in four individual indicators of "well-being": the Gini index, life expectancy at birth, medical expenditure rate with respect to disposable income, and the poverty rate. We employ quite different tests and measures for convergence than those used by Hobijn and Franses (2000).

Our interest in this paper is explicitly to do with testing for convergence using time-series data. However, the result for the "σ-convergence" for each of the case is also provided in this research. What's more, we also employ the fuzzy ratio technique for the convergence analysis in this research.

The rest of the paper is constructed as follows. The next section discusses time-series tests for convergence, including a test proposed very recently by Nahar and Inder (2002). Section 3 provides an overview of ways in which fuzzy clustering analysis may be used to detect convergence, and section 4 describes the data that are used in our analysis. Our empirical results are presented and discussed in the section 5, in terms of both convergence to the mean and also convergence to a leading country. The final section provides some conclusions and suggestions for further research.

2. Time-Series Tests for Convergence

Many different type of tests for output convergence have been used with a variety of data sets. Many of the early studies, such as those of Barro (1991), Barro and Sala-i-Martin (1992) and Sala-i-Martin (1996) used cross-section data, and in general they found evidence of "β-convergence" in output. More recently, attention has turned to tests based on time-series data for output (income), which of course are generally non-stationary. The studies based on time-series data have, in general, returned results that are somewhat less favourable to the convergence hypothesis.[2] One of the most widely used of these tests is that suggested by Bernard and Durlauf (1995), who define *stochastic* convergence in output as follows.

If $y_{i,t}$ is log real per capita output for country i at time t, and I_t is the information set at time t, then countries p = 1, 2, ..., n converge if the long-term forecasts of outputs for all countries are equal at a fixed time t:

$$\underset{kY4}{Limit}\; E(y_{1,\,t+k} - y_{p,t+k} \mid I_t) = 0\ ;\ œ\ p\ \ldots\ 1. \qquad \textbf{(1)}$$

When $p = 2$, for example, this definition of convergence requires that the two countries' outputs must be cointegrated, and specifically with a cointegrating vector [1 , -1]. So, in practice convergence would be *rejected* if the series $(y_i - y_j)$ contains a unit root, and we consider two types of unit root tests in our analysis below. In the multivariate case, convergence requires that there must be (p-1) cointegrating vectors of the form [1 , -1] or one common long-term trend. Accordingly, we can test for convergence by constructing a time-series based on the (p-1) deviations, $Dy_{i,t} = (y_{1,t} - y_{i,t})$, applying Johansen's (1988) multivariate cointegration analysis, and using his likelihood ratio "trace test" to determine the rank of the cointegrating matrix.

Using data for fifteen OECD countries over the period 1900 to 1987, Bernard and Durlauf (1995) find little evidence of convergence, but strong evidence of common trends. Greasley and Oxley (1997) considered pair-wise convergence for a similar set of data, and they allowed for the structural breaks in many of the time-series. Consequently, they found some mild evidence of (pair-wise) stochastic convergence. Evans and Karras (1996) adopted somewhat related definitions of "conditional convergence" and "absolute convergence" of output in their panel-data study of a group of 54 countries, and of the 48 contiguous U.S. states. In contrast to many other authors, they found strong evidence of conditional convergence. St. Aubyn (1999, p.24) offered some explanations for the apparent discrepancy between much of the cross-section and time-series evidence. In addition to cointegration testing he also used a Kalman filtering procedure, and with the latter he found evidence of output convergence between the U.S.A. and every G-7 country except for Canada, after World War 2.

The testing procedures for stochastic convergence proposed by Bernard and Durlauf and by Evans and Karras have been criticized recently by Harvey and Bates (2002), and by Nahar and Inder (2002). The former authors take issue with Bernard and Durlauf's contention that the failure of the time-series tests to reject the null hypothesis of "no convergence" is due to transitional dynamics. They provide Monte Carlo evidence to the contrary, and they focus on a new Lagrange multiplier test for "balanced growth". On the other hand, Nahar and Inder illustrate that there is an inconsistency in the convergence definitions proposed by both Bernard and Durlauf and by Evans and Karras. In both cases, the notion of convergence is linked to the stationarity of output differences, but Nahar and Inder provide counter-examples to show that certain *non-stationary* differences can satisfy these definitions of stochastic convergence. Consequently, Nahar and Inder propose a new procedure for testing for convergence, either towards a single "leading" economy, or towards the mean of a group of economies.

Defining y^*_t to be the mean of the outputs for the group of countries at time "t", their definition of absolute convergence is:

$$\underset{n \to \infty}{Limit}\ E_t\,(y_{it+n} - y^*_{t+n}) = 0 \quad . \qquad \textbf{(2)}$$

If $z_{it} = (y_{it} - y^*_t)$, and $w_{it} = (z_{it})^2$, then convergence requires that w_{it} should always be getting closer to zero. That is, we require that $(dw_{it} / dt) < 0$. To construct a test of convergence, Nahar and Inder represent w_{it} as a simple (stochastic) polynomial in time:

$$w_{it} = f(t) + u_{it} = b_0 + b_1 t + b_2 t^2 + \ldots\ldots + b_k t^k + u_{it} \qquad \textbf{(3)}$$

where the b*'s* are unknown parameters, and u_{it} is a well-behaved random error term.[3] Of course, $(dw_{it} / dt) = f'(t)$, and the sign of this derivative can be examined once the b*'s* are estimated. Nahar and Inder argue that while w_{it} may not decrease *uniformly* over time, it should at least be *generally* decreasing if there is convergence. Accordingly, they focus on the sign of the *average* slope of w_{it} over the sample:

$$T^{-1} \sum_t \,(dw_{it} /dt) = b_1\, r_1 + b_2\, r_2 + \ldots\ldots + b_k\, r_k\ , \qquad \textbf{(4)}$$

where

$$r_i = (i / T) \sum_t t^{\,i-1}\ ;\ i = 2, 3, 4, \ldots., k. \qquad \textbf{(5)}$$

Defining $r = [0, 1, r_2, r_3, \ldots.., r_k]$, and $b = [b_0 , b_1 , b_2 , \ldots\ldots, b_k]$, we can test the null hypothesis of *no convergence*, $H_0 : r'b \geq 0$, against the alternative hypothesis $H_A : r'b < 0$, using OLS estimation of (3) and a simple t-test. A *rejection* of the null hypothesis implies convergence to the group mean.

Nahar and Inder adopt the same sort of approach to test for convergence to a leading economy, "*L*", in the group. Define $d_{it} = (y_{it} - y_{Lt})$. If economy "*L*" is a *true* leader at every point in the sample (*e.g.*, if its output *exceeds* that of all of the other countries in the group in every year), then $d_{it} < 0$ for all i and t. In this case convergence is associated with a positive value for the average of slopes of the d_{it}'s, and we can test the null hypothesis of *no convergence*, $H_0 : r'b \leq 0$, against the alternative hypothesis $H_A : r'b > 0$, using OLS estimation of (3) (with d_{it} replacing

w_{it}) and a simple t-test. A *rejection* of the null hypothesis implies convergence to the leading economy.[4]

A further situation, that does not arise in the study of Nahar and Inder, must also be considered. Frequently, as we shall see below, one encounters situations in which there is no *true* leader in a group of economies, in the sense that one country leads the others in some years, but other countries lead in other years. That is, the various time-series for output (or the variable in question) "cross". In this case, one may still choose a nominal "leader", but equation (3) would be re-formulated with $v_{it} = (d_{it})^2$ as the dependent variable. In addition, the null and alternative hypotheses would then be as in the case of convergence to the group mean.

Nahar and Inder find quite strong evidence of convergence in output among the OECD countries over the period 1950 to 1998. Specifically, they find convergence to the mean of the group in the case of all countries except Norway; and convergence to the U.S.A. (the group "leader") in the case of all countries except New Zealand.

3. Fuzzy Clustering Analysis

One of the purposes of this paper is to illustrate the use of "fuzzy sets" to cluster the data (for one series) for the different countries in our sample, with the purpose of measuring the distance between the centres of these clusters at each point in time. If the centres of the fuzzy clusters move towards each other over time, this represents a particular type of convergence in the variable in question (*e.g.*, in output, or in life expectancy). Giles (2001) first suggested this approach, and referred to it as "cluster convergence testing". This type of analysis has also been used recently by Stroomer and Giles (2003), and it relates closely to traditional tests for "σ-convergence". Although Hobijn and Franses (2000) used a type of "clustering analysis" among their tests for convergence, their method is unrelated to the techniques described and used here.

Fuzzy set theory can be traced back to Zadeh (1965). In conventional set theory, elements either belong to some particular set or they do not. We might say that the "degree of membership" of an element with respect to some set is either unity or zero. The boundaries of the sets are "sharp", or "crisp". In the case of fuzzy sets, the degree of membership may be any value between zero and unity, and each element is associated with more than one set. Usually this association will involve different degrees of membership with each of the fuzzy sets.

To apply these ideas to measure convergence, we determine the partitioning of the data (for a given variable) for each country into a fixed number of clusters, year by year. These clusters have "fuzzy" boundaries, in the sense that each datum belongs to each cluster to some degree or other. Following Giles and Draeseke (2003), Stroomer and Giles (2003) and Giles and Mosk (2003), we use the "fuzzy c-means" (FCM) algorithm to determine the cluster mid-points and to evaluate the associated membership functions and degrees of membership for the data-points. The FCM algorithm is attributable to Ruspini (1970).[5]

The FCM algorithm provides a way of dividing up the "n" data-points into "c" fuzzy clusters (where $c < n$), while also determining the locations of these clusters.[6] The metric that forms the basis for the usual FCM algorithm is "squared error distance", and the mathematical basis for this procedure is as follows.[7] Let $\boldsymbol{x}_k$ be the k'th (possibly vector) data-point (k = 1, 2, ..., n). Let $\mathbf{v}_i$ be the center of the i'th. (fuzzy) cluster (i = 1, 2, ..., c). Let $d_{ik} = \| \boldsymbol{x}_k - \boldsymbol{v}_i \|$ be the distance between $\boldsymbol{x}_k$ and $\boldsymbol{v}_i$, and let u_{ik} be the "degree of membership" of data-point "k" in cluster "i", where :

$$\sum_{i=1}^{c} (u_{ik}) = 1.$$

The objective is to partition the data-points into the "c" clusters, locate the cluster centers, and also determine the associated "degrees of membership", so as to minimize the functional

$$J(U,v) = \sum_{i=1}^{c} \sum_{k=1}^{n} (u_{ik})^m (d_{ik})^2 .$$

There is no specific basis for choosing the exponent parameter, "m", which must satisfy $1 < m < \infty$. In practice, $m = 2$ is a common choice, and it is the one that we adopt here.[8] The FCM algorithm involves the following four broad steps:

1. Select the initial locations of the cluster centres.
2. Generate a (new) partition of the data by assigning each data-point to its closest cluster centre.
3. Calculate new cluster centres by using the centroids of the clusters.
4. If the cluster partition is stable then stop. Otherwise go to step 2 above.

In the case of fuzzy memberships, the Lagrange multiplier technique generates the following expression for the membership values to be used at step 2 above:

$$u_{ik} = 1/\left\{\sum_{j=1}^{n} [(d_{ik})^2 / (d_{jk})^2]^{1/(m-1)}\right\}.$$

If the memberships of data-points to clusters are "crisp" then

$$u_{ik} = 0 \; ; \; \forall \; i \neq j,$$
$$u_{jk} = 1 \; ; \; j \;\; \text{s.t.} \; d_{jk} = \min.\{d_{ik}, \; i = 1, 2, ..., c\}.$$

The updating of the cluster centres at step 3 above is obtained via the expression

$$v_i = [\sum_{k=1}^{n} (u_{ik})^m x_k] / [\sum_{k=1}^{n} (u_{ik})^m] \;\; ; i = 1, 2, \ldots., c.$$

The fixed-point nature of this problem ensures the existence of a solution.[9] Once the centres of the fuzzy clusters have been determined, each of the n data-points can be allocated to the cluster whose center it is closest to. There is no generally accepted method for determining the number of clusters, c, that should be used. In our study here we have n = 14 countries, which limits the number of clusters that can usefully be identified. So, we consider c = 2, 3 and 4 and check the sensitivity of our results to this choice.

As was suggested above, the "distance" between the centers of the "lowest" and "highest" clusters for a particular variable can be calculated, year-by-year. To make this distance measure scale-free, we define it as $R_t = (v_{ct} / v_{1t})$; $t = 1, 2, \ldots., T$; where v_{1t} and v_{ct} are the centers of the first and "c$^{\text{th}}$" (ranked) clusters in year t, and T is the number of annual observations in the sample. Following Giles (2201), "cluster convergence" is associated with R_t converging to unity over time. Implicitly, this cluster convergence is with respect to the mean of the data. If we are interested in measuring cluster convergence to a "leader" we can simply as follows. First, for each year, take the ratio of the data for each country to that for the leading country. Then, apply the fuzzy analysis above to each of these (n-1) ratios, and define $R^*_t = (v^*_{ct} / v^*_{1t})$; $t = 1, 2, \ldots.,$ T; where v^*_{1t} and v^*_{ct} are the centers of the first and "c^{th}" (ranked) clusters for the ratio data in year t. Cluster convergence to the leader arises if R^*_t converging to unity over time.

4. Data Issues

Our study uses annual time-series data for the period 1960 to 1996, and we consider the following fourteen countries: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Spain, Sweden, the U.K., and the U.S.A..

The GDP data-set that we have used was constructed by Nahar and Inder (2002). It is based on an updated version of the Penn World Tables (Summers and Heston, 1991). Nahar and Inder (2002, p. 2015) discuss the way in which the more recent values for the GDP data for Germany have been constructed to take account of that country's re-unification. Measuring the level of "well-being" is a difficult task, of course. One of the issues that must be faced is the choice between using one or more individual measures, as opposed to using a single composite index of some sort. Hobijn and Franses (2001) discuss this issue, and provide a compelling case for adopting the former approach. Of course, one then has to decide exactly which individual indicators of "well-being" are to be used. We have considered four measures, these being life expectancy; the Gini index of income inequality; the poverty rate; and the "risk" of medical expenditure. The life expectancy data relate to expectancy at birth, for males and females combined. The poverty rate is the proportion of persons who fall below the poverty line, defined here as one half the median equivalent after-tax family income. The medical expenditure risk is the proportion of disposable income that is assigned to this category of expenditure. So, for our purposes, "well-being" is taken to *increase* with an increase in life expectancy, *ceteris paribus*, but it *decreases* with larger values for the other three indicators. The data for each of these "well-being" indicators are taken from Osberg and Sharpe (2000), which provides the background for Osberg and Sharp (2002a, 2002b).

5. Empirical Results

5.1 Descriptive Measures

Before considering the formal time-series tests for convergence, and the fuzzy clustering analysis, we begin by presenting some simple descriptive measures for "σ-convergence". In Figure 1 we show the coefficients of variation for GDP and for each of our four well-being indicators, year by year. In each case these are defined as the ratio of the standard deviation to the mean, expressed in percentage terms.[10] We can see that there is clear evidence of a decrease in the variation of the

GDP and life expectancy data, across the fourteen countries over time, and some such evidence in the case of the poverty rate. In contrast, the variability of the data for risk of medical expenditure *increases*, while that of the Gini index data is relatively constant over time. Figure 2 provides similar information, but in this case the measure that is used relates to variation about the leading country.[11] Specifically, if y_{it} denotes the value of the variable y for country i in period t, and y_{Lt} is the value for the "leading country", we can define a (percentage) "coefficient of variation" relative to the leader as:

$$cv_L = 100*\{[\sum_{i=1}^{n-1} (y_{it} - y_{Lt})^2 / (n - 2)]^{0.5}\} / (y_{Lt})$$

where n is the number of countries *including* the leader (n = 14 in our case). As can be seen in Figure 2, the GDP and life expectancy data show the same variability characteristics relative to the leading country as they did with respect to the mean. The variability in the poverty rate data (relative to the leading country) declines somewhat overall, and markedly after 1987. After increasing from 1960 to 1980, the variability in the medical expenditure data (relative to the leading country) declines to approximately its original level by 1996. Finally, in spite of a peak in the middle of the sample period, the variability of the Gini index (relative to the leading country) is fairly constant between 1960 and 1996.

In summary, these descriptive measures suggest rather informally that there may have been "σ-convergence" towards the mean and towards the leading country in the cases of output and life expectancy, and perhaps also in the case of the poverty rate. We will now consider this preliminary evidence in the light of a more rigorous analysis based on time-series and fuzzy clustering methods.

5.2 Convergence of GDP

As was discussed in some detail in Section 2 above, most of the recent empirical studies that have been undertaken using time-series data have failed to find convincing evidence of convergence (to the mean of the countries, at least implicitly) for real *per capita* GDP for 22 OECD countries. A notable exception is the study of Nahar and Inder (2002). We have applied their methodology to their GDP data for our particular 14 countries over our reduced sample period, 1960 to 1996, and we find very strong support for the "β-convergence" hypothesis. This accords well with the above analysis of the coefficients of variation. As may be seen in Table 1, our results are broadly consistent with the ones that Nahar and Inder report, although there are a few differences. For example, they found some evidence that Sweden converged to the leading country, the U.S.A, but we obtain a contrary result. Also, Nahar and Inder found evidence of convergence of the Netherlands' GDP to the group mean, but no such evidence in the case of Germany. Our results for these two countries are the opposite of theirs.

The results of our fuzzy clustering analysis of real *per capita* GDP convergence are summarized in Figures 3 and 4, for convergence to the group mean and to the leading country (the U.S.A.) respectively. In each case we provide results for three choices for the number of clusters, $c = 2, 3$, and 4. In this way we can examine the sensitivity of our results to this choice. In Figure 3 we see that each of the associated fuzzy ratios is decreasing over time towards a value of unity. Although these declines in the ratios are not monotonic, none-the-less they provide quite strong evidence in favor of ("cluster") convergence to the mean GDP for this group of countries. In Figure 4 we see clear evidence of "cluster convergence" to the U.S.A., at least over the sub-periods 1960 to 1975, and 1985 to 1996.

Considering the various results that we have presented with regard to GDP convergence, it is clear that there is very strong time-series evidence of convergence towards the leading country, the U.S.A., for all countries except for Sweden. There is equally strong evidence of convergence to the group mean, the only exceptions being Norway and the Netherlands. This conclusion provides support for the findings of Nahar and Inder (2002), and verifies the robustness of their result to the choices of sample period and countries within the OECD group.

5.3 Convergence of "Well-Being" Indicators

In this paper, we want to test the hypothesis that the convergence of GDP is *not* necessarily associated with convergence of indicators of "well-being", such as the four that we are considering. As has been discussed already, extensive support for this (negative) hypothesis has been provided by Hobijn and Franses (2001). We consider the possibilities of convergence to the mean of the group of countries, and convergence to a leading country, separately in the following discussion.

5.3.1 Convergence to the Group Mean

Table 2 presents our results relating to testing for convergence to the mean for our four "well-being" indicators, using the approach of Nahar and Inder. We use the same sample period and the same fourteen OECD countries as for the GDP convergence that was discussed in Section 5.2. With regard to income inequality, as measured by the Gini coefficient, at the 5% significance level only Belgium and Spain show evidence of "β-convergence" to the mean for our fourteen countries. In the case of the poverty rate, half of the countries (including Belgium and Sweden) exhibit significant evidence of convergence to the mean, but the other half show evidence to the contrary. Of the seven countries that show convergence for this "well-being" indicator, five of these also showed convergence to the mean for GDP. Nine of the fourteen countries (including Belgium) show significant convergence to the mean in the case of life expectancy at birth, and apart from the Netherlands and Norway, all of them showed similar evidence of GDP convergence. In the case of the final "well-being" indicator (medical expenditure "risk") only three countries - Canada, France, and Italy – provide significant support the hypothesis of "β-convergence" to the group mean.[12]

The results of the fuzzy ratio analysis for convergence to the mean are summarized in Figures 5 to 8. As was the case for the fuzzy analysis of the GDP data, we examine the robustness of these results by reporting cluster ratio time-paths for the cases of two, three, or four fuzzy clusters. The results of the fuzzy analysis are strikingly consistent with the time-series results just discussed. In Figures 5 and 8 the slopes of the fuzzy ratio plots suggest a general lack of "cluster convergence" (implicitly to the mean of the group of countries) in the cases of the Gini index and medical expenditure "risk". In Figure 6 we see modest evidence of convergence to the group mean in the case of the poverty rate. From the descriptive coefficients of variation and our application of the Nahar and Inder analysis, the strongest evidence of convergence in "well-being" was in the case of life expectancy at birth, and this is corroborated by the fuzzy results in Figure 7.

What do these results tell us about the relationship between convergence in GDP and convergence in "well-being"? From the above discussion it seems clear that while our analysis supports the existence of convergence to the group mean for GDP, there is much less support for such convergence in "well-being". This is true no matter what type of convergence we choose to focus upon. One way to summarize this finding in a quantitative way is to fit a regression model that explains the presence or absence of GDP convergence in terms of the presence or absence of convergence in each of the four well-being indicators. Using the time-series results in Tables 1 and 2 we have "coded" each of these five variables in a binary manner, with the value unity if the country shows convergence, and zero otherwise.[13] Given that we have a binary dependent variable, we have fitted a probit model to our sample of fourteen observations.[14] The results appear in Table 4. We see that none of the regressors are significant, but in terms of signs, there is a *positive* (partial) relationship between GDP convergence and "well-being" convergence only in the case of the Gini indicator. Moreover, on the basis of the likelihood ratio test (Greene, 2000, p. 826) we cannot reject the null hypothesis of *no relationship* between the dependent variable and the set of regressors, at any reasonable significance level.[15] These results underscore the lack of an association between output convergence and "well-being" convergence, as measured here.

5.3.2 Convergence to a "Leading" Country

Now let us consider the possibility of convergence to a "leader" among the group of countries. It is not easy as simple to find a leading country in the case of the "well-being" indicators as was the case with GDP. Unlike the situation with the latter variable, which country is the well-being leader varies from one measure to another, and for each such measure it also varies over time to some degree. So, we have chosen country "leaders" in an overall (relative) sense, rather than in an absolute sense.[16, 17]

The results of testing based on the approach of Nahar and Inder appear in Table 3. There we see that (at the 5% significance level) there is evidence of "β-convergence" to the leader, Sweden, only for four countries when the Gini index is used to measure "well-being". These countries are Denmark, France, the Netherlands and Spain. Only Canada, Denmark, France and Sweden appear to converge to the leading country (the United Kingdom) in the case of medical expenditure risk. For the poverty rate the leading country is the Netherlands, and for life expectancy it is Sweden. In both of these cases there is more evidence of convergence – for eight of the remaining thirteen countries. For the poverty rate the five non-converging countries are Belgium, Italy, Norway, Sweden, and the United Kingdom; while for life expectancy, they are Denmark, the Netherlands, Norway, the United Kingdom and the U.S.A.. In summary, the evidence regarding convergence to a group leader is quite mixed in the case of these indicators of "well-being". Certainly, this evidence is weak when compared with that for GDP convergence.

Figures 9 to 12 show the results for the fuzzy convergence analysis based on data expressed relative to leading countries. The evidence here is quite consistent with that from the Nahar and Inder analysis. While there is a clear evidence of "cluster convergence" to the leader (Sweden) in the case of life expectancy, and some mixed evidence to this effect in the case of the poverty rate, the results for medical expenditure risk indicate divergence from the leader (U.K.). Finally, the income inequality results in Figure 9 are very similar to those in Figure 5, and they suggest either no convergence or perhaps divergence from the leader (Sweden) over time.

In Table 4 we show the results of a probit regression analysis of the "β-convergence" results for GDP and the four "well-being" measures in Tables 1 and 3, in this case with convergence defined in terms of leading countries. Keeping in mind the relatively small sample size, these results once again point clearly to the *absence* of any significant relationship between convergence in output and convergence in "well-being". Once again, none of the explanatory variables are statistically significant, though in this case the signs of the coefficients are positive for all of the regressors except medical expense risk. The likelihood ratio test once again confirms the *absence* of a relationship between the dependent variable and the set of regressors.

6. Conclusions

In this paper we have addressed the following question: "if there is convergence in output between a group of countries, will there also be a corresponding convergence in 'well-being', with the latter measured via various indicators?" Specifically, we have tested the hypothesis that there is no such connection using data for fourteen industrialized countries, and a range of new tests for convergence.

Our main result may be stated quite simply – convergence in outputs across countries *may* be associated with convergence in certain measures of "well-being", but not necessarily, and not in general. While we find strong evidence of output convergence, using various approaches, generally there is only weak evidence as far as "well-being" convergence is concerned. Moreover, of the four measures of the latter that we have adopted, only life expectancy at birth and the poverty rate exhibit any convincing convergence characteristics.

As an aside, our findings also indicate the robustness of the results of Nahar and Inder (2002), that there is strong evidence of convergence (to the mean and also to the leading country, the

U.S.A.). We corroborate their findings when a sub-set of their countries and a sub-sample of their data are used.

Clearly, there is ample scope for further research into the issues addressed in this paper. For example, work in progress considers the extent to which the results here are applicable to less-industrialized countries. Explicit testing for causality between convergence in different measures would also warrant careful study. Another interesting issue is whether or not sub-sets of the countries that we have considered form "well-being clubs", and how stable the membership of such clubs has been over time. Recent work by Su (2003) relating to "output convergence clubs" is relevant here, and the fuzzy cluster membership analysis provided by Giles (2001) could also be used to determine club membership features.

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Footnotes

* We are extremely grateful to Brett Inder for supplying the data used in the study by Nahar and Inder (2002).

1. For further details, see Maurseth (2001, p.252).

2. See Durlauf and Quah (1999) for an excellent survey of much of the empirical convergence testing literature based on both cross-section and time-series data.
3. This polynomial representation of w_{it} can be justified in terms of a (locally valid) Taylor series approximation. Work in progress investigates the merits of replacing this with various globally valid approximations.
4. If we have a true leader in the context of a variable whose desired long-run value is *low*, then all of the d_{it}'s will be *positive*, and convergence to this leader will be associated with a *negative* average for their slopes. In this case the signs of the parameter restrictions in H_0 and H_A will be reversed.
5. Its development was influenced by the contributions of MacQueen (1967) and it is closely associated with other contributors such as Bezdek (1973) and Dunn (1974, 1977). The FCM algorithm is widely used in such fields as pattern recognition.
6. The rest of the discussion in this section closely follows that of Giles (2001).
7. This metric is not always the most appropriate one. For example, if there are outliers in the data a more robust metric may be needed, and various ways of achieving this have been considered in the literature. For a recent economic application of the FCM algorithm in the context of outliers, see Stroomer and Giles (2003).
8. In the case of crisp (hard) memberships, $m = 1$.
9. See Bezdek (1981, Chapter 3) for complete mathematical details. The FCM algorithm is simple to program, and we have used programming commands in the SHAZAM (2001) package in our application here.
10. For each year, the sample comprises the fourteen country observations. The coefficient of variation is a more appropriate measure of variability than the standard deviation here, as it is unit-less and it is "corrected" to allow for the fact that the mean changes from year to year. This facilitates meaningful comparisons, both across time and across the different variables under consideration. For plotting purposes the coefficients of variation have

been multiplied by the following scale factors: GDP and life expectancy (0.1), Gini index (0.01), poverty rate and risk of medical expenditure (0.001).

11. The leading countries are the U.S.A. (for GDP); Sweden (for the Gini index, and life expectancy); the Netherlands (for the poverty rate); and the United Kingdom (for risk of medical expenditure). For plotting purposes the coefficients of variation have been multiplied by the following scale factors: GDP and life expectancy (0.1), Gini index, poverty rate and risk of medical expenditure (0.01).
12. The results for the Netherlands are significant at the 10% level, but not at the 5% level.
13. We used a 10% significance level to determine the presence of convergence, but the results are not sensitive to this choice.
14. We also fitted four other models, using each of the dummy variables for the well-being indicators in turn as the dependent variable. The results were completely consistent with those reported here.
15. Of course, this test has only asymptotic justification, and we have a very small sample size.
16. In section 2 we foreshadowed this possibility, and we noted there how this affects the application of the Nahar and Inder analysis, and the interpretation of the results with respect to the signs of the test statistics.
17. The countries in question are given in footnote 11.

Table 1.

Convergence of GDP: Time Series Analysis

Country	**Convergence to USA**			**Convergence to Mean**		
	k	**t-statistic**	**p-value**	**k**	**t-statistic**	**p-value**
Australia	8	1.930	0.032	7	-6.023	0.000
Belgium	7	5.085	0.000	7	-2.104	0.022
Canada	8	7.234	0.000	3	-1.761	0.044
Denmark	8	4.585	0.000	5	-2.295	0.014
Finland	8	4.253	0.000	8	-2.020	0.027
France	7	5.093	0.000	4	-5.339	0.000
Germany	6	3.029	0.003	7	-4.260	0.000
Italy	3	12.216	0.000	2	-15.842	0.000
Netherlands	6	4.400	0.000	7	2.433*	
Norway	6	17.704	0.000	8	5.324*	
Spain	7	12.333	0.000	8	-16.748	0.000
Sweden	7	0.709*	0.242	5	-8.388	0.000
UK	3	1.446	0.079	7	-3.580	0.001
USA				8	-8.427	0.000

Note: * indicates no convergence at the 5% significance level.

Table 2.

Convergence to the Mean, Four Well-Being Indicators:

Time-Series Analysis

Country	Gini Coefficient				Poverty Rate			
	k	Average Slope	t-statistic	p-value	k	Average Slope	t-statistic	p-value
Australia	7	0.0000	9.423*		8	0.635	16.1871*	
Belgium	8	0.0000	-2.15	0.020	8	-0.143	-4.019	0
Canada	8	0.0000	-0.581*	0.283	8	-2.691	-18.026	0
Denmark	8	0.0000	8.676*		7	0.129	4.045*	
Finland	8	0.0001	10.931*		8	0.499	8.970*	
France	8	0.0000	6.107*		8	0.176	7.099*	
Germany	8	0.0000	0.483*	0.316	8	-0.196	-3.048	0.002
Italy	7	0.0000	5.666*		8	0.279	23.591*	
Netherland	8	0.0000	3.268*		8	-0.757	-5.939	0
Norway	8	0.0000	1.246*		8	-0.552	-12.003	0
Spain	7	-0.0001	-10.1	0	8	-0.233	-5.618	0
Sweden	8	0.0000	-0.865*	0.197	8	-0.396	-3.768	0
United Kingdom	5	0.0000	22.407*		4	0.231	2.700*	
United states	8	0.0001	7.315*		8	0.211	2.277*	

Country	Life-Expectancy				Medical-Expenditure			
	k	Average Slope	t-statistic	p-value	k	Average slope	T-statistic	p-value
Australia	5	0.01402	1.120*		5	-0.00709	-0.761*	0.226
Belgium	8	-0.00628	-0.289*	0.387	8	0.36849	8.698*	
Canada	2	0.03196	17.454*		8	-0.02527	-1.852	0.037
Denmark	8	-0.01672	-1.442*	0.080	7	0.21408	7.953*	
Finland	6	-0.24507	-13.896	0	7	-0.00370	-0.165*	0.435
France	2	0.00151	0.808*		8	-0.24331	-11.774	0
Germany	5	-0.08381	-5.997	0	8	0.06616	4.944*	
Italy	4	-0.05989	-12.189	0	8	-0.01969	-4.978	0
Netherland	4	-0.18854	-26.930	0	7	-0.08677	-1.578*	0.063
Norway	8	-0.18419	-8.080	0	8	0.14008	7.638*	
Spain	8	-0.03942	-5.090	0	7	0.03680	2.524*	
Sweden	8	-0.04799	-1.817	0.040	8	0.14213	8.693*	
United Kingdom	8	0.00939	0.597*		8	0.14627	6.853*	
United states	6	0.02439	1.242*		8	1.97240	14.141*	

Note: * indicates no convergence at the 5% significance level.

Table 3.

Convergence to the Leader, Four Well-Being Indicators:

Time-Series Analysis

Country	Gini Index (Sweden#)				Poverty Rate (Netherlands #)			
	k	Average Slope	t-statistic	p-value	k	Average Slope	t-statistic	p-value
Australia	8	0.0000	1.021*		8	-0.461	-1.885	0.035
Belgium	8	0.0001	3.940*		8	-0.015	-0.686*	0.249
Canada	8	0.0001	1.525*		8	-6.732	-25.133	0
Denmark	8	-0.0001	-3.947	0	8	-0.790	-5.512	0
Finland	8	0.0000	-1.444*	0.080	8	-0.377	-8.966	0
France	8	-0.0001	-4.239	0	7	-0.812	-6.727	0
Germany	7	0.0000	0.428*		8	-2.148	-10.430	0
Italy	8	0.0000	0.579*		8	0.059	0.412*	
Netherlands	8	-0.0001	-2.521	0.009				
Norway	7	0.0000	-1.383*	0.089	8	0.399	1.601*	
Spain	8	-0.0002	-4.628	0	8	-2.081	-15.688	0
Sweden					8	0.128	0.601*	
UK	8	0.0001	3.396*		8	0.667	1.703*	
USA	8	0.0001	4.318*		7	-1.445	-3.259	0.001

Country	Life Expectancy (Sweden#)				Medical Expenditure (U.K#)			
	k	Average Slope	t-statistic	p-value	k	Average Slope	t-statistic	p-value
Australia	6	-0.096	-1.770	0.043	6	0.191	4.011*	
Belgium	8	-0.151	-1.992	0.028	8	1.148	9.502*	
Canada	4	-0.122	-9.383	0	8	-0.464	-12.737	0
Denmark	8	0.343	9.624*		6	-0.025	-11.690	0
Finland	6	-0.620	-10.284	0	8	0.064	2.376*	
France	8	-0.187	-4.437	0	6	-0.170	-3.643	0.001
Germany	8	-0.293	-4.395	0	7	0.011	1.416*	
Italy	2	-0.354	-32.059	0	6	0.209	8.317*	
Netherlands	8	0.042	3.334*		7	0.169	1.558*	
Norway	5	0.003	0.325*		8	-0.009	-0.460*	0.325
Spain	3	-0.166	-9.602	0	8	0.104	0.890*	
Sweden					7	-0.004	-2.235	0.017
UK	7	0.080	1.655*					
USA	6	-0.025	-0.422*	0.338	8	3.582	16.038*	

Note: * Denotes no convergence on the 5% significance level.

# Denotes the selected "leading" country in each category.

Table 4

Probit Model Results

	Convergence to the Mean*			Convergence to a Leader#		
Regressor	**Coefficient (t-Value)**	**R^2_M (R^2_{CU})**	**LRT**	**Coefficient (t-Value)**	**R^2_M (R^2_{CU})**	**LRT**
GI	5.795			7.124		
	(0.001)			(0.000)		
PR	-11.835			7.240		
	(-0.003)			(0.000)		
LE	-11.886			7.141		
	(-0.003)			(0.000)		
MER	-6.322			-7.289		
	(-0.003)	0.422	7.644	(0.000)	0.419	7.051
Constant	24.152	(0.753)		0.152	(1.000)	
	(0.004)			(0.000)		

Notes: * n = 14

n = 13

GI = Gini Index; PR = Poverty Rate; LE = Life Expectancy at Birth; MER = Medical Expenses Risk; R^2_M = Maddala (1983) R-squared; R^2_{CU} =Cragg and Uhler (1970) R-squared; LRT = Likelihood Ratio Test Statistic (asymptotically ÷^{2} with 4 degrees of freedom).

Figure 1. Scaled Coefficients of Variation about the Mean

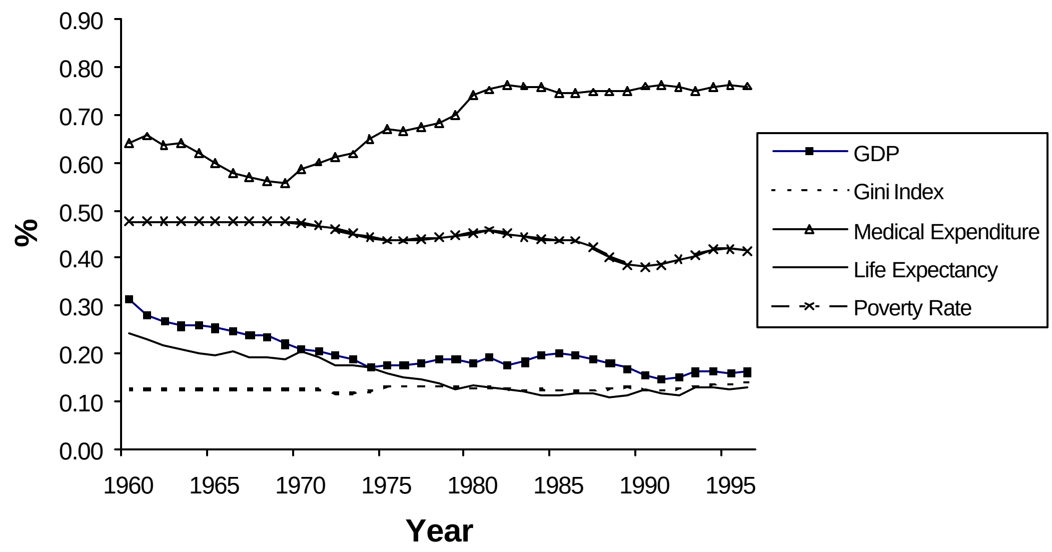


Figure 2. Scaled Coefficients of Variation about the Leader

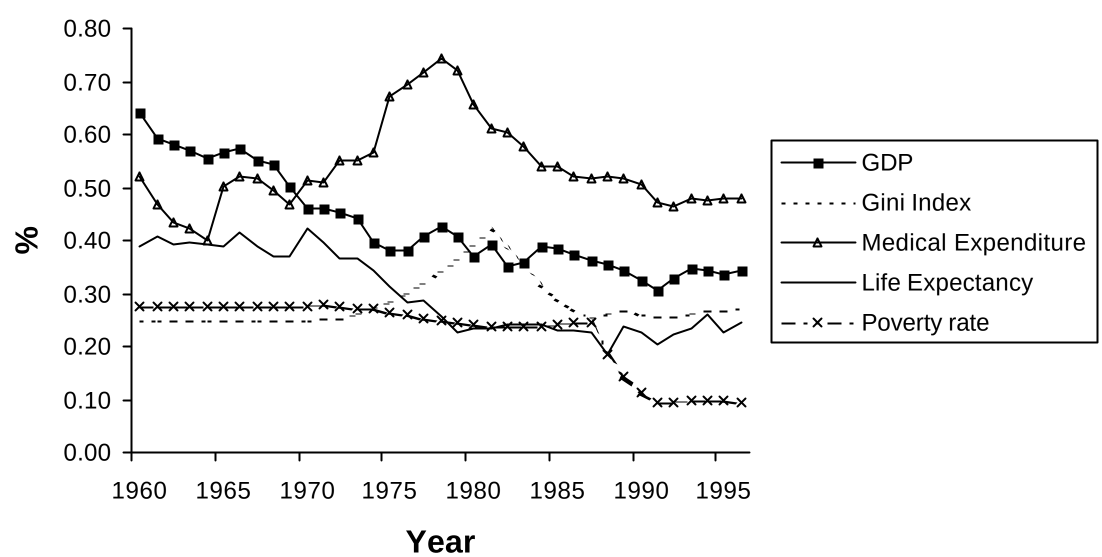


Figure 3. Fuzzy Ratios: GDP Convergence to the Mean (c=2,3,4)

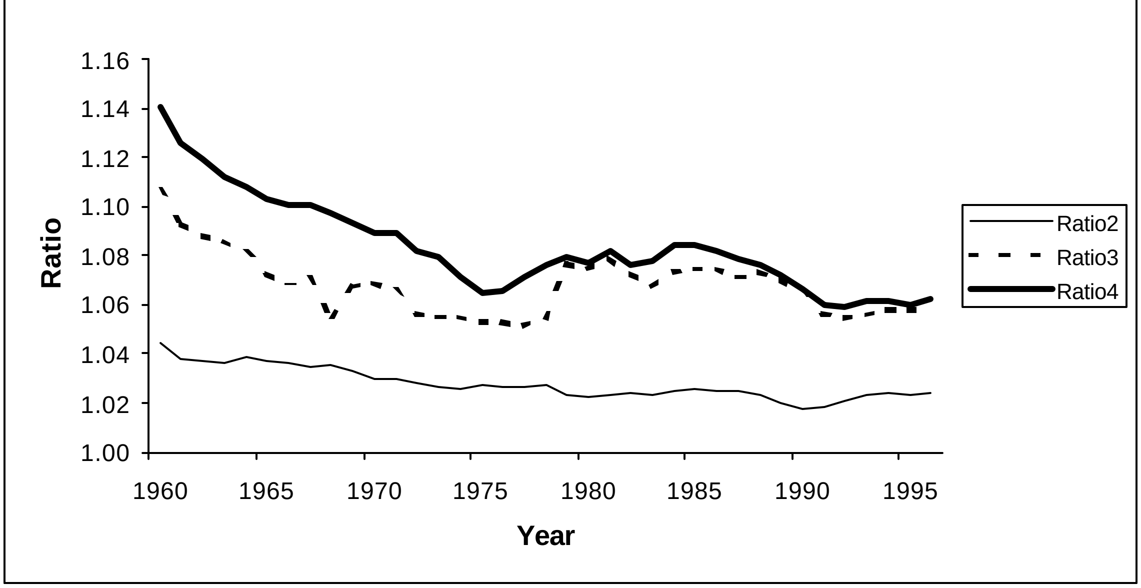
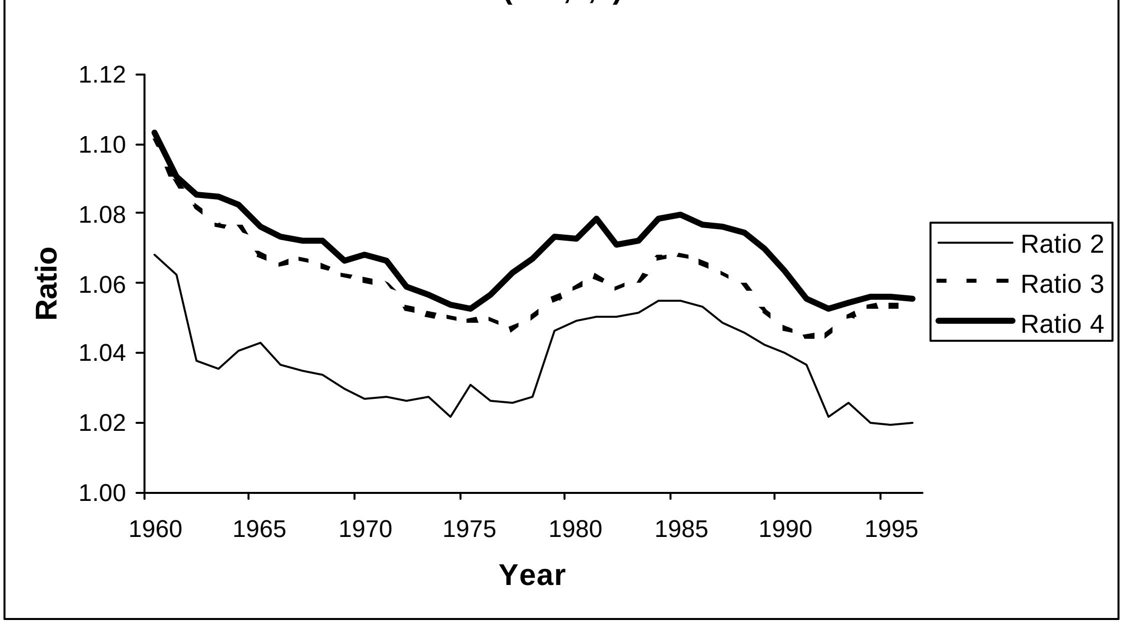


Figure 4. Fuzzy Ratios: GDP Convergence to the Leader (c=2,3,4)



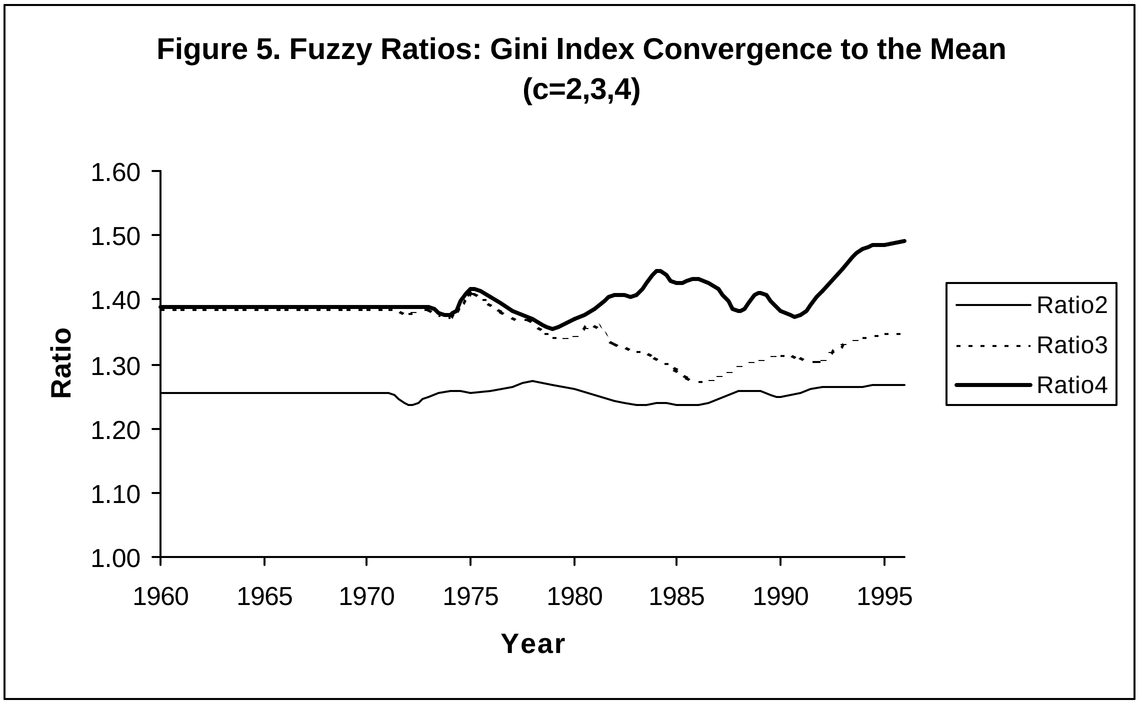


Figure 5. Fuzzy Ratios: Gini Index Convergence to the Mean
(c=2,3,4)
1.60
1.50
1.40
1.30
1.20
1.10
1.00
Ratio
1960
1965
1970
1975
1980
1985
1990
1995
Year
Ratio2
Ratio3
Ratio4

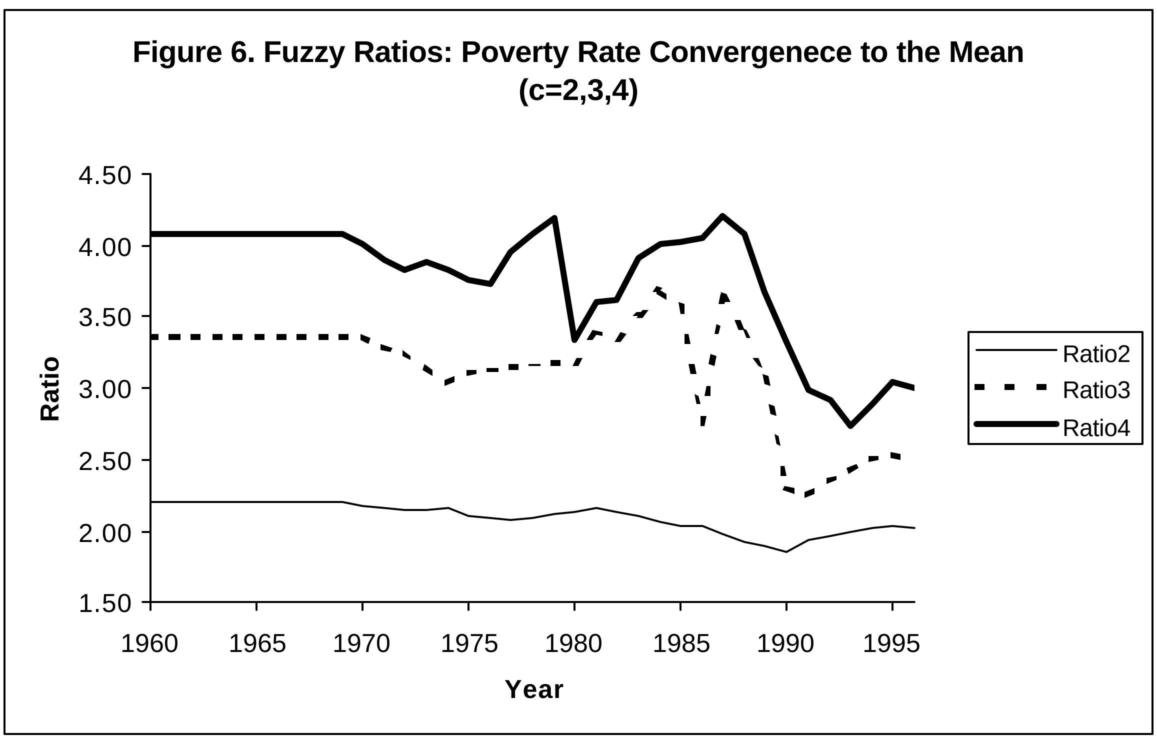


Figure 6. Fuzzy Ratios: Poverty Rate Convergenece to the Mean
(c=2,3,4)
4.50
4.00
3.50
3.00
2.50
2.00
1.50
Ratio
1960
1965
1970
1975
1980
1985
1990
1995
Year
Ratio2
Ratio3
Ratio4

Figure 7. Fuzzy Ratios: Life Expectency Convergence to the Mean (c=2,3,4)

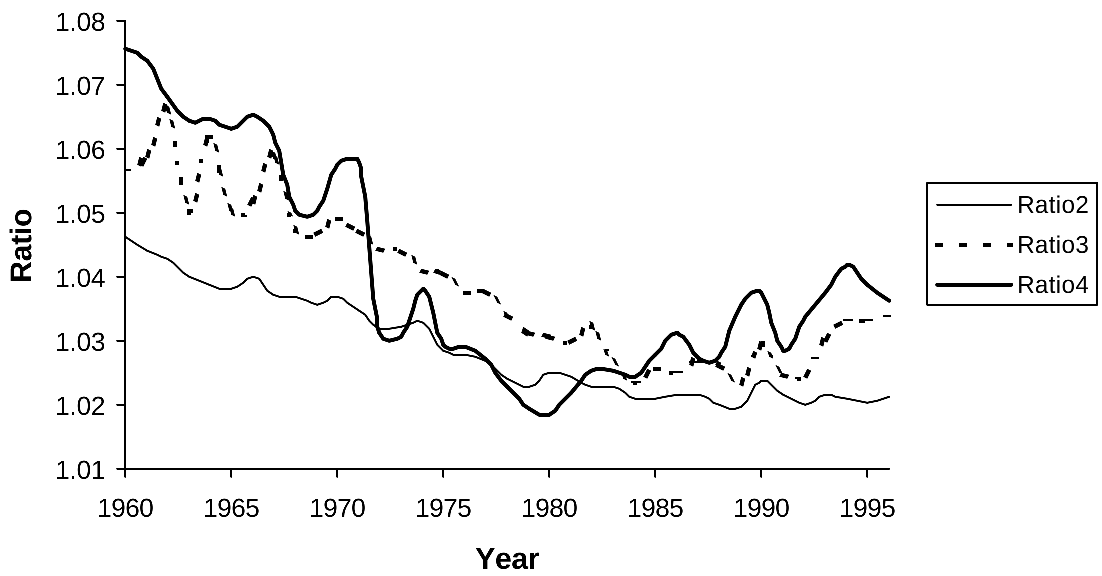
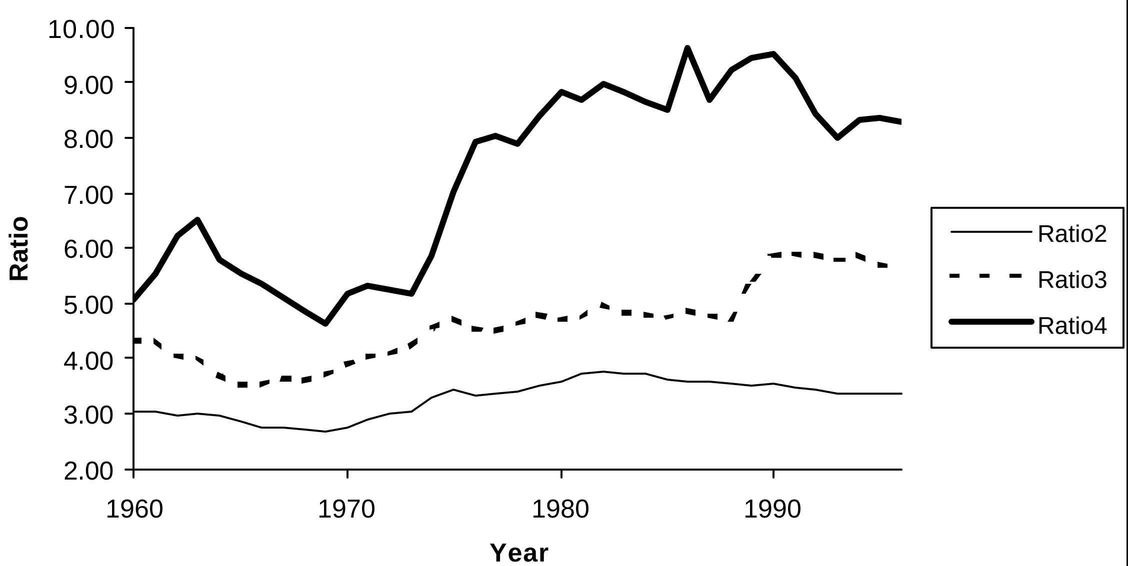


Figure 8. Fuzzy Ratios: Medical Expenditure Convergence to the Mean (c=2,3,4)



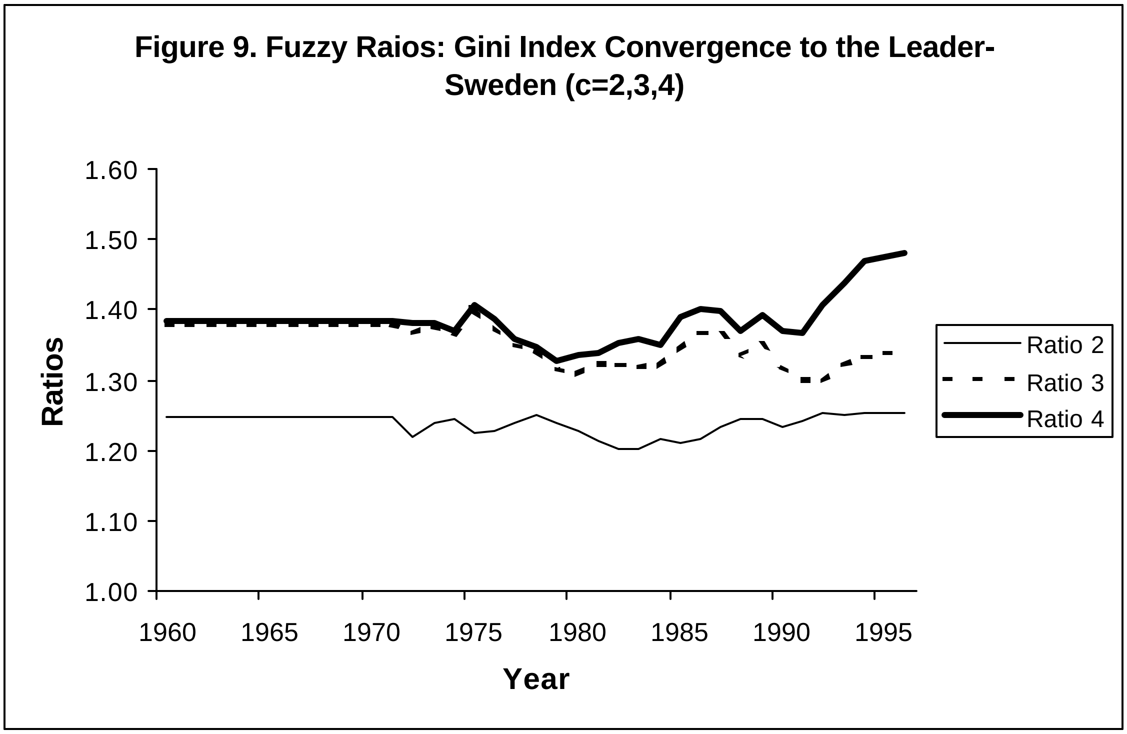


Figure 9. Fuzzy Raios: Gini Index Convergence to the Leader-Sweden (c=2,3,4)
1.60
1.50
1.40
1.30
1.20
1.10
1.00
Ratios
1960
1965
1970
1975
1980
1985
1990
1995
Year
Ratio 2
Ratio 3
Ratio 4

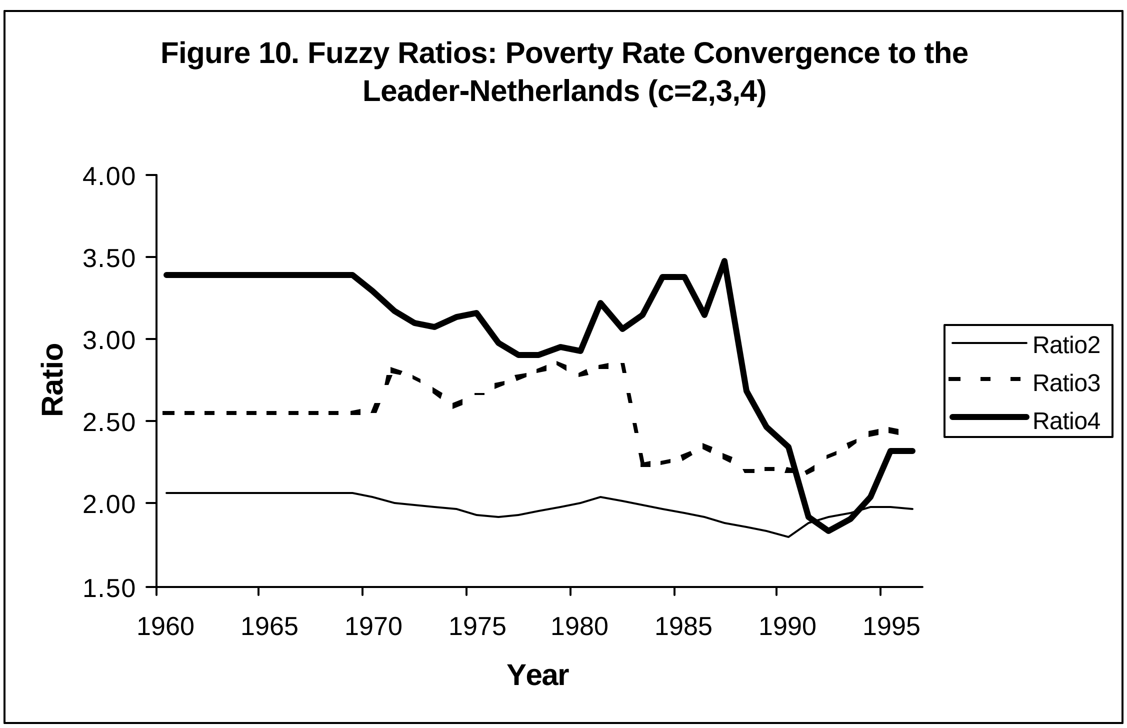


Figure 10. Fuzzy Ratios: Poverty Rate Convergence to the Leader-Netherlands (c=2,3,4)
4.00
3.50
3.00
2.50
2.00
1.50
Ratio
1960
1965
1970
1975
1980
1985
1990
1995
Year
Ratio2
Ratio3
Ratio4

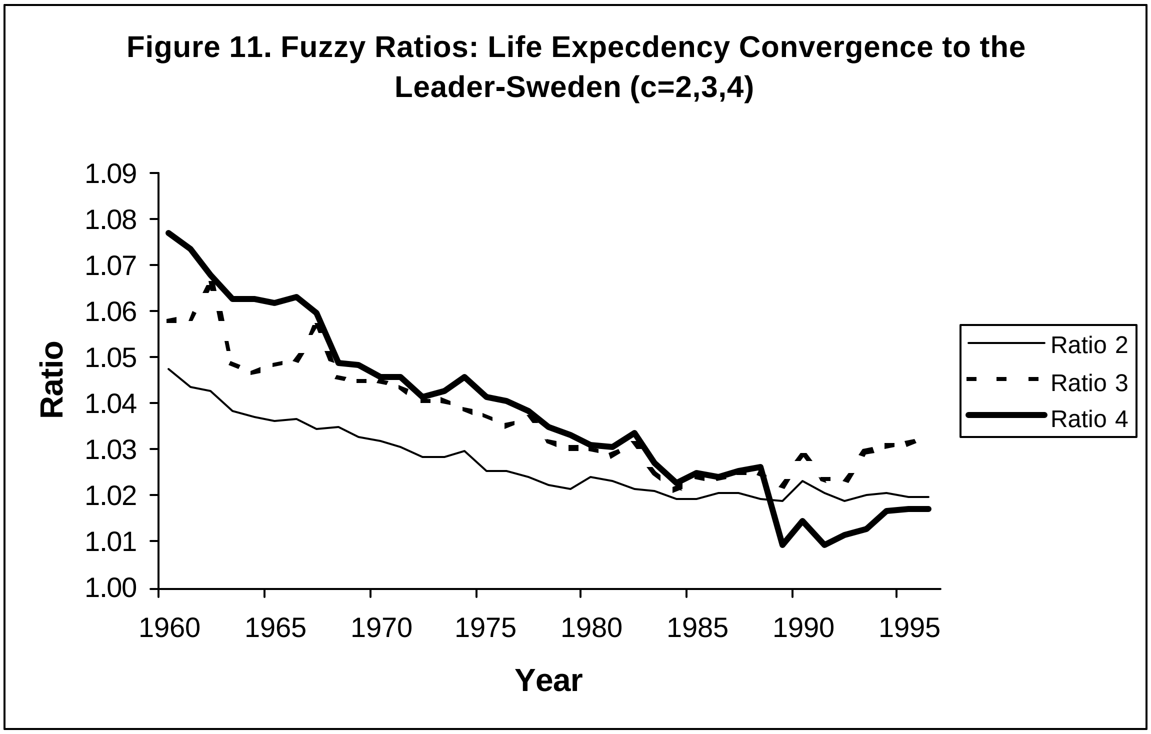


Figure 11. Fuzzy Ratios: Life Expecdency Convergence to the Leader-Sweden (c=2,3,4)
1.09
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1.00
Ratio
1960
1965
1970
1975
1980
1985
1990
1995
Year
Ratio 2
Ratio 3
Ratio 4

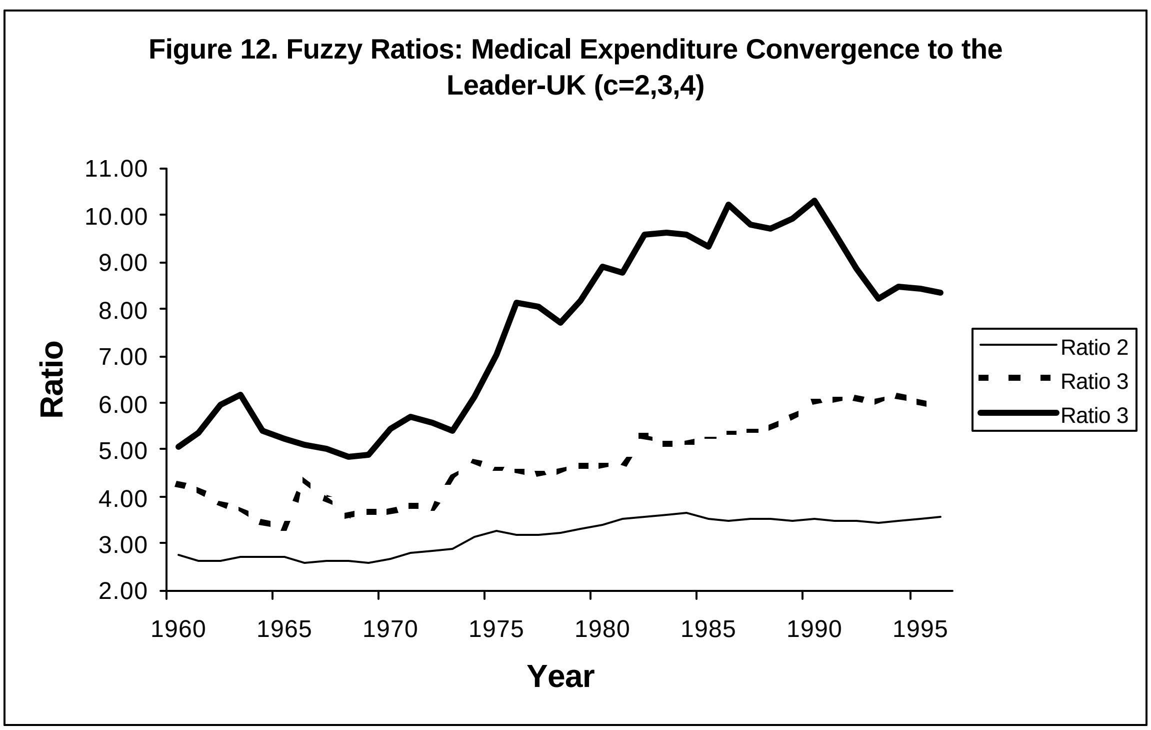


Figure 12. Fuzzy Ratios: Medical Expenditure Convergence to the Leader-UK (c=2,3,4)
11.00
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
Ratio
1960
1965
1970
1975
1980
1985
1990
1995
Year
Ratio 2
Ratio 3
Ratio 3