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Calculating a Standard Error for the Gini Coefficient: Some Further Results

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Abstract

Various authors have proposed using the jackknife technique to approximate a standard error for the Gini coefficient. It has also been shown that the Gini measure can be obtained simply from an artificial OLS regression based on the data and their ranks. Accordingly, we show that obtaining an exact analytical expression for the standard error is a trivial matter. In addition, by extending the regression framework to one involving Seemingly Unrelated Regressions, several interesting hypotheses regarding the sensitivity of the Gini coefficient to changes in the data are readily tested in a formal manner.

Keywords: Gini coefficient; income inequality; standard error; jackknife; SUR model

JEL Classifications: C3; C43; D31; I31;

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Introduction

Although the Gini coefficient is probably the most widely used empirical measure of income inequality, it is usually reported without any acknowledgement of the fact that it is simply a sample statistic. As such, it has a sampling variance, and ideally a standard error should be reported. This has long been understood (*e.g.*, Hoeffding, 1948), but the standard error associated with the Gini coefficient has been reported only rarely in practice. The reason for this is that most of the formulations of this standard error that have been proposed in the literature are mathematically complex, or they require a considerable amount of numerical computation[1]. The latter disadvantage applies, in particular, to the application of the jackknife technique[2] to simulate a variance for the Gini coefficient, as suggested by Sandstrom, Wretman and Walden (1985, 1988) and others.

Recently, Karagiannis and Kovacevic (2000) and Ogwang (2000) have re-considered this issue. In particular they discuss ways in which the computational burden associated with the jackknife approximation of the Gini coefficient's variance can be reduced to a level where this method can be applied even when realistically large data samples are involved. In addition, Ogwang provides a particular regression-based interpretation of the Gini coefficient that not only forms the basis of his approach, but unwittingly exposes the fact that there is really no need to resort to the jackknife technique at all in this context! The purposes of this note are to expose the latter point, and to show how this regression-based interpretation is also helpful with regard to various hypothesis tests that are of practical interest. We illustrate our results with empirical applications.

Basic Results

Let y be a vector of incomes, with extreme values $y_{\min}$ and $y_{\max}$, mean m, and cumulative distribution function $F(y)$. It is well known that the Gini coefficient of inequality is:

$$G = \{\int_{y_{\min}}^{y_{\max}} F(y)[1 - F(y)]\} / m \quad . \tag{1}$$

Suppose that the observed data are in increasing order, with i'th. value y_i . Then Ogwang (2000, p.124) notes that the Gini coefficient can also be expressed as[3]:

$$G = [(n^2 - 1) / (6n)](\hat{b} / \bar{y}) , \tag{2}$$

where $\bar{y}$ is the sample arithmetic mean of y , $\hat{b}$ is the OLS estimator of b in the model

$$y_i = a + b\,i + e_i , \tag{3}$$

and the e_i's are zero-mean, independent, and homoskedastic errors. He also shows that G can be written as:

$$G = (2\hat{q} / n) - 1 - (1/n) , \tag{4}$$

where $\hat{q}$ is the weighted least squares (WLS) estimator of 2 in the model

$$i = q + n_i , \tag{5}$$

where the n_i's are heteroskedastic errors with variances of the form (s^2 / y_i) . So, in the formulation of the Gini coefficient in (4), we have:

$$\hat{q} = [(\sum_{i=1}^{n} i\, y_i) / (\sum_{i=1}^{n} y_i)] . \tag{6}$$

Ogwang's (2000) principal contribution is to use equation (4) as the basis for applying the jackknife principle to develop a standard error for G. His innovation dramatically reduces the computational burden of using the jackknife in this context, as it usually involves computing G from every possible sub-sample that is created by dropping one observation. The key to his result is that the data are first ranked in the construction of (4) from (5) and (6).

As useful as the proposals made by Karagiannis and Kovacevic (2000) and by Ogwang (2000) are, in fact a closer examination of the latter's approach reveals that the adoption of the jackknife technique is actually unnecessary, and the construction of an appropriate standard error for the Gini coefficient is trivial. To see this, note that from (4):

$$\text{var.}(G) = 4\,\text{var.}(\hat{q}) / n^2 \tag{7}$$

and so the standard error of G is:

$$s.e.(G) = 2[s.e.(\hat{q})] / n \quad . \tag{8}$$

Of course, $s.e.(\hat{q})$ comes directly from the WLS estimation of (5), or equivalently from the OLS estimation of the regression model:

$$(i\sqrt{y_i}) = q\sqrt{y_i} + u_i\,, \tag{9}$$

where $u_i = (\sqrt{y_i})n_i$. In other words, the desired standard error can be obtained directly from standard OLS regression output! Precisely this approach has been used by Selvanathan (1991), Giles and McCann (1994), Crompton (2000) and others to calculate standard errors for Laspeyres, Paasche, and other types of price indices[4]. It should also be stressed that resampling procedures such as the jackknife are justified only in terms of their asymptotic properties. For instance, Shao (1991) provides a detailed analysis of these properties, and establishes the weak consistency of the jackknife variance estimator. This estimator is not necessarily appealing in finite samples - for instance, Efron and Stein (1981) prove that it is biased upwards in small samples, so at least it provides a conservative measure.

Numerical Illustrations

First, we illustrate the relationship between the exact standard error given by (8), and its jackknife counterpart, using an artificial data-set[5]. Table 1 shows the Gini coefficient and its standard error for various sample sizes, and the corresponding jackknife calculations. The asymptotic convergence of the latter to the former is evident, as is the upward bias in the jackknife Gini estimate and its standard error in finite samples. Ogwang (2000), and the other associated authors noted above, propose that the "exact" Gini coefficient should be used with the "jackknife" standard error. The percentage distortion in [G / s.e.(G)] that would be associated with this approach is just the percentage distortion in s.e.(G). This is also shown in Table 1 for our artificial data-set.

Next, we consider a small application using data from the *Penn World Tables*[6]. Our data measure real *per capita* consumption, in internationally comparable terms, for 133 countries in the years[7] 1970, 1975, 1980 and 1985. In Table 2 we again compare the Gini coefficients and their standard errors obtained by the simple regression approach described above, and by using the jackknife. As before, the finite-sample bias of the latter measures is obvious, and in fact is much more pronounced (in percentage terms) than in Table 1. To construct a 95% confidence interval for the Gini coefficient based on the OLS/WLS results we can use the critical t-value[8] of 1.978 and the standard errors. For each year this confidence interval easily covers the associated jackknife Gini estimate. The inter-temporal pattern in consumption inequality implied by the various measures is also interesting. Both sets of Gini coefficient estimates exhibit an increase in value (and hence in consumption inequality) from 1970 to 1975, a small decrease in 1980, and an increase to a maximum value in 1985. Interestingly, if consumption inequality is measured by the coefficient of variation ("c.v." in Table 2), a different picture emerges. By this measure, inequality declines from 1970 to 1975, and to 1980. It then increases to its maximum value in 1985. The regression model (5) that these results are based upon has an error term that is assumed to exhibit a particular form of heteroskedasticity. Accordingly, we have used Harvey's (1976) test to test the hypothesis of homoskedastic errors against the alternative hypothesis that the error variance is proportional to $(1/y_i)$. In each case the null hypothesis is rejected, lending support to the assumptions underlying the calculation of our Gini coefficients[9].

The OLS/WLS approach to calculating the standard errors for the Gini coefficient also facilitates various interesting hypothesis tests that cannot be conducted readily if the jackknife approximation is used. For example, we can test the hypothesis that the Gini coefficient is the same in different years by stacking up the single-year regressions of the form (5) or (9), using Seemingly Unrelated Regressions (SUR) estimation, and testing the equality of the appropriate coefficients across the equations. Part (a) of Table 3 shows the SUR estimates of the Gini coefficient and the standard errors for our consumption data. The coefficients themselves are smaller than those obtained year by year (as in Table 1), and the gain in asymptotic efficiency associated with SUR estimation is reflected in the smaller standard errors. The latter, of course mean that the percentage distortion in the jackknife standard errors is even greater than the Table 2 results suggest.

The relevance of using SUR estimation rather than year-by-year OLS is clear when we test the diagonality of the model's error covariance matrix. The Breusch-Pagan Lagrange multiplier test statistic is 796.12, while the corresponding likelihood ratio test statistic is[10] 2787.50. Table 3(a) also shows the results of testing the equality of the slope coefficients (and hence the Gini coefficients) across the equations of the model, and hence across years. With the exception of the 1975/1980 pair, we strongly reject the hypothesis that the Gini coefficient is the same in two different years[11]. Not surprisingly, the Wald statistic for testing equivalence across *all* of the years is 65.10, leading to a clear rejection of this hypothesis[12]. In part (b) of Table 7 we show the results when the 1975 and 1980 coefficients are restricted to be the same. With the inclusion of this additional (data-supported) information, the Gini coefficient standard errors are further reduced, and so the distortions associated with the jackknife approximation are more pronounced.

As a final example of the usefulness of the SUR approach to calculating both the Gini coefficient and its estimated variability, we consider the *significance* of the effect on this measure of international consumption inequality if one or more countries are deleted from the sample. In each year, the U.S.A. has the highest real *per capita* consumption among the countries in our sample, and Ethiopia has the smallest. Table 4 shows the results of testing the robustness of the Gini coefficient estimates, in each year (based on restricted SUR estimation), to the deletion of one or both of these extreme sample values[13].

Comparing Tables 3(b) and 4, we see that the Gini coefficient is slightly more sensitive to the omission of the U.S.A. from the sample than to the omission of Ethiopia. Not surprisingly, it is even more sensitive to the omission of both countries. The Wald statistics relate to the equivalence of the Gini values before and after the various omissions - they are asymptotically chi-square distributed with degrees of freedom equal to the number of countries deleted. Interestingly, when we focus on statistical significance rather then the numerical values of the Gini coefficients, a different picture emerges. When the U.S.A. is dropped from the sample we *cannot* reject the null hypothesis, that the Gini coefficient is unaltered, at the 15% significance level or lower. On the other hand, when Ethiopia is dropped from the sample, we *reject* this null hypothesis at the 5% level, though not at the 2.5% level or lower. Finally, when both countries are dropped, we again *reject* the stability of the Gini coefficient at the 5% level, though not at the 4% level or lower.

Concluding Remarks

The Gini coefficient is the most common economic measure of inequality. A standard error is needed if confidence intervals or tests are to be constructed for this coefficient, and various authors have proposed using the jackknife technique to get a large-sample approximation for this standard error. However, because the Gini coefficient can be obtained from a simple OLS regression-based approach, the exact calculation of its standard error is actually trivial. This insight also provides the basis for constructing various tests of the robustness of the Gini coefficient to changes in the sample of data, using SUR estimation as the basis for this analysis. Such tests are not readily constructed if the jackknife methodology is used.

Table 1

Gini Coefficients and Standard Errors - Artificial Data

n	"Exact" (OLS/WLS)		Jackknife		% Distortion in
	G	s.e.(G)	G	s.e.(G)	Jackknife s.e.(G)
25	0.2291	0.1054	0.2800	0.1125	6.7
50	0.2291	0.0738	0.2541	0.0767	3.9
100	0.2291	0.0520	0.2415	0.0533	2.5
500	0.2291	0.0231	0.2316	0.0235	1.7
1000	0.2291	0.0164	0.2303	0.0166	1.2
5000	0.2291	0.0073	0.2293	0.0074	1.4
10000	0.2291	0.0052	0.2292	0.0052	0.0

Table 2

Gini Coefficients and Standard Errors - PWT Consumption Data

(133 Countries)

	"Exact" (OLS/WLS)		Jackknife		% Distortion in	c.v
Year	G	s.e.(G)	G	s.e.(G)	Jackknife s.e.(G)	(%)
1970	0.4705	0.0417	0.4816	0.0481	15.3	93.16
1975	0.4796	0.0405	0.4908	0.0460	13.6	93.04
1980	0.4785	0.0396	0.4897	0.0448	13.1	91.16
1985	0.4940	0.0391	0.5053	0.0441	12.8	95.20

Table 3

Gini Coefficients and Standard Errors - PWT Consumption Data
(SUR Estimation)

Year	G	s.e.(G)	% Distortion in Jackknife s.e.(G)	z-tests 1970	1975	1980
(a) Unrestricted Estimation						
1970	0.3369	0.0238	102.1			
1975	0.3454	0.0231	99.1	-5.61		
1980	0.3478	0.0226	75.2	-3.94	-1.43	
1985	0.3575	0.0232	68.5	-6.74	-6.20	-6.56
(b) Restricted Estimation						
1970	0.3478	0.0222	116.7			
1975	0.3552	0.0217	112.0	-5.80		
1980	0.3552	0.0217	106.5	-5.80	n.a.	
1985	0.3653	0.0213	107.0	-8.08	-7.13	-7.13

Table 4

Tests for Robustness of Gini Coefficient

(Restricted SUR Estimation)

Year	**Omit U.S.A.**		**Omit Ethiopia**		**Omit U.S.A. & Ethiopia**	
	G	**Wald**	**G**	**Wald**	**G**	**Wald**
	[s.e.(G)]	**[p-value]**	**[s.e.(G)]**	**[p-value]**	**[s.e.(G)]**	**[p-value]**
1970	0.3505	1.9826	0.3488	4.3938	0.3514	6.3849
	[0.0223]	[0.160]	[0.0222]	[0.038]	[0.0223]	[0.041]
1975	0.3578	2.0266	0.3562	4.3382	0.3588	6.37545
	[0.0218]	[0.155]	[0.0217]	[0.037]	[0.0218]	[0.041]
1980	0.3578	2.0266	0.3562	4.3382	0.3588	6.3745
	[0.0218]	[0.155]	[0.0217]	[0.037]	[0.0218]	[0.041]
1985	0.3679	2.0566	0.3663	4.2339	0.3689	6.3001
	[0.0214]	[0.152]	[0.0213]	[0.040]	[0.0214]	[0.042]

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Footnotes

1. For example, see Glasser (1962), Sendler (1979), Sandstrom, Wretman and Walden (1985, 1998), and other authors cited by Ogwang (2000, p. 123).
2. See Efron (1982), especially Chapter 3, for details of the theoretical justification for the jackknife and other related resampling techniques. The jackknife was first suggested by Quenouille (1949, 1956) as a non-parametric method for estimating bias, and it was extended by Tukey (1958) to the problem of estimating varaince. Yitzhaki (1991) discusses the application of the jackknife to a range of measures related to the Gini index.
3. See, also, Lerman and Yitzhaki (1984) and Shalit (1985).
4. For a general discussion of the stochastic approach to price index construction, see Clements and Izan (1987).
5. The basic data, for n = 25, is:{1 7 6 5 6 7 8 4 3 6 4 2 1 3 4 5 6 7 8 9 8 7 6 5 4}.

   The sample size is increased by assuming that the data are "fixed in repeated samples". That is, if n = 25j, the above sample is repeated "j" times. Accordingly, the "exact" (OLS) Gini coefficient values shown in Table 1are invariant to the sample size. All of the calculations were undertaken with the SHAZAM (2001) econometrics package.
6. See Summers and Heston (1995). The data were extracted using the Windows-based freeware also available at the NBER website at http://www.nber.org/pub/pwt56/.
7. The *Penn World Tables* data-set covers more countries than this, over the period 1950-1992. We have chosen a selection of recent years for which the data of interest are available for a large proportion of the countries. The list of countries and data used in our sample are available at http://web.uvic~dgiles/ewp0202data.xls.
8. The Student-t assumption follows if the errors in (5) or (9) are Normally distributed. Asymptotically this will be a reasonable approximation, but the *exact* finite-sample distribution of G is another matter that we don't pursue in this paper.
9. The t-statistics (and their p-values) for 1970, 1975, 1980 and 1985 are -0.3817 (0.739), -0.3790 (0.741), 0.4232 (0.714) and -0.6615 (0.576) respectively. Any remaining concerns about other types of heteroskedasticity could be addressed by using White's (1980) heteroskedasticity-consistent estimator of the covariance matrix, and hence of the standard errors.

10. Both statistics are asymptotically chi-square with six degrees of freedom under the null hypothesis, so we clearly reject the null of a diagonal covariance matrix.

11. The p-value associated with the z-statistic for 1975/1980 is15.22%.

12. This statistic is asymptotically chi-square with three degrees of freedom under the null hypothesis, so the 1% critical value is 11.3449, and the p-value is essentially zero.

13. These tests are readily implemented through the use of simple dummy variables to isolate the observations (countries) of interest.