Taxes, Risk-Aversion, and the Size of the Underground Economy:
A Nonparametric Analysis With New Zealand Data

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Abstract:
We use nonparametric regression analysis to investigate the relationship between the effective tax rate and the relative size of the underground economy, using New Zealand data. The underlying theoretical framework is established, and it suggests an ambiguous prediction regarding the sign of the relationship we are studying. However, our nonparametric empirical analysis, which also allows for the non-stationarity of the time-series data, produces a positive and "S-shaped" relationship, and this supports earlier empirical studies that imposed such functional forms. The estimated model is used to simulate the effects of hypothetical tax changes on the size of the New Zealand underground economy, and to draw policy conclusions.

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1. Introduction

There is a long-standing hypothesis that there is a relationship between taxes and the degree of tax evasion, or the size of the "underground economy". Various theoretical models have been proposed in support of this hypothesis, but the associated empirical literature is relatively sparse. In part this is due to the difficulty of obtaining meaningful time-series data for the size of the underground workforce or underground output. In this paper, based on recent developments in the theoretical literature on tax evasion, we examine in some detail the empirical relationship between the relative size of the New Zealand underground economy and the effective tax rate in that country, using annual time-series data for the period 1968 to 1994. The relative size of the underground economy (UE) is measured as (UE/GDP), and the data are those generated by Giles (1999) using a structural Multiple Indicator Multiple Causes (MIMIC) model. The (aggregate) effective tax rate is defined as (TR/GDP), where TR is total tax revenue.

As is outlined below, there is theoretical justification for the hypothesis of a positive relationship between the size of the underground economy and the tax rate, but this prediction is not readily testable directly due to the form of the data that are generally available. In this paper we examine the nature of this relationship empirically, making proper allowance for the non-stationarity of our time-series data, and using nonparametric estimation in order to avoid distorting the conclusions by "imposing" an assumed functional form on the analysis. Accordingly, this study extends and corroborates that of Giles and Caragata (1999) and Caragata and Giles (2000) for New Zealand - those authors adopted an explicit parametric analysis in their investigation of the tax burden-underground economy relationship. Specifically, they considered various simple polynomial and "S-shaped functions, and favoured a logistic functional form to "explain" (UE/GDP) as a positive function of (TR/GDP).

Our objective here is to abstract from any such functional constraints, and to investigate some of the practical implications of the recent theoretical model of Trandel and Snow (1999) by estimating this relationship using nonparametric methods. Section 2 provides a general theoretical background; and in Section 3 we re-formulate certain implications of the Trandel-Snow model in terms of an hypothesis that can be tested empirically using macroeconomic data. Section 4 discusses the available data, including issues of non-stationarity and possible cointegration; and Section 5 deals with the estimation issues and results. Some of the economic implications of the estimated model, including some simple simulation results, are described in Section 6; and our concluding comments appear in Section 7.

2. Theoretical considerations

Schneider and Enste (1998) and Giles and Caragata (1999) discuss some of the previous empirical evidence pertaining to the effect of taxes on the underground economy, stemming from early contributions by Clotfelter (1983) and Crane and Nourzad (1987), to the more recent results of Schneider (1994), Johnson *et al.* (1998), Cebula (1997), and Hill and Kabir (1996). This evidence overwhelmingly supports the hypothesis of a positive relationship between taxes and the size of the underground economy, though the data used and the details of the analysis vary enormously from study to study.

Of primary interest here is the extensive theoretical literature that considers the role of taxes in determining the size of the underground economy. In fact, much of this literature deals with theoretical models that are quite narrow in their perspective. In particular, in the spirit of the seminal contribution of Allingham and Sandmo (1972), much of this literature relates to models of "pure tax evasion". In such models, income is earned from only one source, and some of this income is not declared to the taxation authority. Many of these models also assume that the penalties for evasion are imposed as a fraction of undeclared income, rather than as a fraction of the evaded tax, and/or that the tax system is linear[1] (*e.g.*, Yitzhaki, 1974). Allowing for tax progressivity and risk-averse tax-paying agents, the theoretical models of Pencavel (1979) and Koskela (1983) predict that increasing the tax rate reduces the amount of tax evasion. However, this result becomes ambiguous if flexibility is allowed with respect to labour's hours worked, and much of this literature is of questionable interest in terms of empirical verifiability.

On the other hand, there is also a more appealing theoretical literature relating to two-sector "underground economy" models. In these models there are two sources of potential income, and the probability that any evasion will be detected by the authorities differs between the two sectors. In one sector, all earned income is "visible" with respect to taxation liability, while in the other sector the possibility of tax evasion results in lower before-tax wages. Examples of such contributions are those of Watson (1985), Kesselman (1989) and Trandel and Snow (1999). Moreover, the latter model assumes that penalties for detected evasion are imposed in proportion to the amount of evaded tax, *which is exactly the situation in practice* in most Western countries (including New Zealand). Such "underground economy models" more relevant than "pure tax evasion models" from an empirical viewpoint, as their underlying assumptions more closely match reality, and so they provide interesting testable hypotheses.

The form of these hypotheses is, however, rather complicated. For example, the sign of the relationship between the tax rate and the degree of tax evasion depends on what is assumed about agents' risk aversion. Trandel and Snow (1999) illustrate this. If taxes are progressive and if the agents' preferences exhibit decreasing absolute risk aversion, and non-decreasing relative risk aversion, then their model predicts a positive relationship between the tax rate and the share of the total labour force in the evasive sector of the economy. A positive relationship is also predicted between the degree of tax-progressivity and the relative size of the underground labour force. We examine aspects of the Trandel-Snow model in our own empirical analysis.

3. Formulating a testable hypothesis

In practice, aggregate data on the size of the underground economy are estimated in terms of the value of "hidden" *output*, rather than the size of the associated labour force. Also, to facilitate international comparisons, these figures are usually reported as a percentage of measured GDP. For example, see Schneider and Enste (1998) for some extensive cross-country comparisons, and Giles (1999) for a complete and recent time-series for the New Zealand underground economy. This form of the data necessitates some manipulations of the predictions of the Trandel-Snow model before they can be tested empirically.

This model is presented briefly in the Appendix. Of course, although its two-sector nature, and the way in which it accounts for evasion penalties, match the New Zealand situation well, the more detailed institutional characteristics of that country's taxation system[2] render this theoretical model somewhat stylized. This is a common situation, of course. However, the quality[3] of the underground economy data, and the fact that they were generated without the use of effective tax rate information (so that the modelling of a relationship between this rate and the underground economy is not a spurious exercise), imply that the New Zealand situation provides a useful basis for testing some of the implications of the Trandel-Snow model[4] empirically.

Within the framework given in the Appendix, Trandel and Snow prove that if agents' preferences exhibit decreasing absolute, and non-decreasing relative, risk aversion, then the size of the underground economy (measured in *employment* terms) grows with the value of the marginal tax rate that is faced in both sectors. Of course, as these preferences are not observable, but are merely revealed through the agents' actions, the sign of the relationship between the marginal tax rate and the employment-size of the underground economy is itself effectively an empirical issue.

We now extend their analysis by reinterpreting this prediction in terms of *average* tax rates (which differ between the evading and non-evading sectors), as well as in aggregate *income* (or output) terms, rather than in employment terms. These extensions are important, empirically. Our first task is to show that, as one would anticipate, the model predicts that the underground economy grows with either an increase in the *average* tax rate in the non-evading sector, or with a decrease in the (expected) *average* tax rate in the evading sector.

Proposition 1. *Suppose that a fixed, non-zero, range of income is untaxed, that preferences exhibit decreasing absolute and non-decreasing relative risk aversion, and that tax evasion is a better-than-fair gamble. Then the size of the underground economy rises, if either the average tax rate in the non-evading sector increases, or if the (expected) average tax rate in the evading sector falls.*

Proof. First, consider the non-evading sector. Using the notation in the Appendix, let $f_n(a^*) = \tau_n - t[y_n(a^*) – b] / y_n(a^*) = 0$. By the implicit function theorem, $(\partial a^*/\partial\tau_n) = - [(\partial f_n/\partial\tau_n) / (\partial f_n/\partial a^*)]$. Now, $(\partial f_n/\partial\tau_n) = 1$, and $(\partial f_n/\partial a^*) = - [tby'_n(a^*) / y_n(a^*)^2]$. So, $(\partial a^*/\partial\tau_n) = [y_n(a^*)]^2 / [tby'_n(a^*)] > 0$, because $y'_n(a) > 0$.

In the evading sector, let $f_e(a^*) = \tau_e - t\{y_e(a^*) - b - [1 - p(1 + m)]x^*\} / y_e(a^*) = 0$. Again, by the implicit function theorem, $(\partial a^*/\partial\tau_e) = - [(\partial f_e/\partial\tau_e) / (\partial f_e/\partial a^*)]$. In this case, $(\partial f_e/\partial\tau_e) = 1$, and $(\partial f_e/\partial a^*) = - [tby'_e(a^*)] / [y_e(a^*)]^2 - [t\{1 - p(1 + m)\}x^*y'_e(a^*)] / [y_e(a^*)]^2$. As $y'_e(a) < 0$, it follows that $(\partial a^*/\partial \tau_e) < 0$ provided that $[1 - p(1 + m)] > 0$. This last condition is simply $(1 - p) > pm$, which holds if tax evasion is a better-than-fair gamble. •

Now consider the implications of the model in terms of hidden *output*, rather than hidden labour. If 'N' is the size of the total labour force, then total *declared* (and measured) equilibrium gross income in the non-evading sector is $[N(1 - a^*)y_n]$. In the evading sector, declared equilibrium income is $[Na^*(y_e - x^*)]$, which equals measured gross income in that sector if evasion is not detected. In the event of detection, measured income from this sector[5] will be $[Na^*y_e]$. Similarly, equilibrium evaded income will be $[Na^*x^*]$ in the absence of detection, and otherwise it will be zero. So, the expected *relative* size[6] of the underground economy in aggregate income (output) terms is:

$$u = [(1 - p)a^*x^*] / [(1 - a^*)y_n + a^*(y_e - x^*)]. \qquad \textbf{(1)}$$

Proposition 2. *When a fixed, non-zero, range of income is untaxed and preferences exhibit decreasing absolute and non-decreasing relative risk aversion, an increase in the marginal tax rate may either increase or decrease the expected relative size of the underground economy, measured in income terms.*

Proof. Denoting $(\partial a^*/\partial t)$ by $a^{*\prime}$, and $(\partial x^*/\partial t)$ by $x^{*\prime}$, it follows from (1) that:

$$(\partial u/\partial t) = (1 - p)\{A(a^*x^{*\prime} + x^*a^{*\prime}) - Ba^*x^*\}/ A^2, \qquad \textbf{(2)}$$

where $A = [(1 - a^*)\, y_n + a^*(y_e - x^*)]$,

and $B = [(1 - a^*)(\partial y_n/\partial t) + a^{*\prime}\, (y_e - x^* - y_n) + a^*\{(\partial y_e/\partial t) - x^{*\prime}\}]$.

Now, A > 0. Also, under the stated conditions, a*' > 0, so $(\partial y_n/\partial t) > 0$, and $(\partial y_e/\partial t) < 0$, by the chain rule[7]. Clearly, from (2), regardless of the sign of x*', the sign of $(\partial u/\partial t)$ is ambiguous. •

So, perhaps not surprisingly, the nature of the effect of a change in the *marginal* tax rate on the expected *relative* size of the underground economy is an empirical issue. It also follows from the definitions of the average tax rates (τ_n and τ_e), that a similar ambiguity arises with respect to the signs of $(\partial u/\partial \tau_n)$ and $(\partial u/\partial \tau_e)$. The model does not provide a strong prediction of these effects.

So, in modelling the relationship between the macroeconomic aggregates (UE/GDP) and (TR/GDP), the Trandel-Snow model does not predict the sign of the partial derivative, and this issue is an empirical one. Their model and its predecessors are also, of course, silent on the matter of the functional form of any such relationship between the tax rate and tax evasion. This underscores the relevance of exploring a nonparametric approach in our empirical analysis below, notwithstanding the fact that we have only a relatively small sample of data.

4. Data Issues

As noted already, we use two aggregate ratios, (UE/GDP) and (TR/GDP). Annual data for the period 1968 to 1994 for the former variable are taken from Giles (1999), while the latter data are compiled from official data released by Statistics New Zealand and Revenue New Zealand. Both series are available on the web at **www.uvic.ca/econ/uedata.html**, and are displayed[8] in Figure 1. As can be seen from the results in Table 1, we have tested each series for non-stationarity, allowing for the possibilities of I(2), I(1) or I(0) data. We have used both the "augmented" Dickey-Fuller (ADF) tests, in which the null hypothesis is non-stationarity, as well as the tests of Kwiatowski *et al.* (KPSS) (1993) in which the null hypothesis is stationarity of the data. A 10%

significance level has been adopted to deal with the well-known low powers of these tests, although the results are not sensitive to this choice.

In the case of the ADF tests, the augmentation level (p) has been chosen by the default method in the SHAZAM (1997) package, as Dods and Giles (1995) show that this approach leads to low size-distortion in the presence of moving-average errors with samples of the size being used here. We have followed the sequential strategy of Dolado *et al.* (1990) to deal with the issue of the inclusion/exclusion of drift and trend terms in the Dickey-Fuller regressions. So, in Table 1, t_{dt} denotes the ADF unit root "t-test" with drift and trend terms included in the fitted regression; F_{ut} is the corresponding ADF "F-test" for a unit root and zero trend; t_d is the unit root "t-test" with a drift but no trend in the fitted regression; F_{ud} is the corresponding "F-test" for a unit root and a zero drift; and t is the ADF unit root test when the fitted regression has no drift or trend term included. Finite-sample critical values for our "t-tests" come from MacKinnon (1991), and those for the "F-tests" are given by Dickey and Fuller (1979, 1981).

In the case of the KPSS tests, where the null is stationarity, and the alternative hypothesis is non-stationarity, we have used both a zero value for the Bartlett window parameter, l, as well as $l = 5$. The latter value is implied by the KPSS "*l8* rule" for our sample size[9]. KPSS provide asymptotic critical values for the test with null hypotheses of both level-stationarity and trend-stationarity. Cheung *et al.* (1995) provide response-surface information that facilitates finite-sample critical values in the trend-stationary case[10].

The results in Table 1 indicate that both (UE/GDP) and (TR/GDP) are I(1), and hence are non-stationary[11]. Accordingly, it is meaningful to test for possible cointegration between the two series, and in Table 2 we show the results of applying the cointegrating regression ADF (CRADF) test, in which the null is "no cointegration", using MacKinnon's (1991) exact critical values. We see that there is modest evidence of cointegration at the 10% significance level. Johansen's (1988) likelihood ratio "trace test" was also used to test the null of no cointegration in the context of a bivariate VAR model. With respect to the inclusion of drift and/or trend terms in the the cointegating equations and/or the fitted VAR's, the five possibilities suggested by Johansen (1995) were all considered. Asymptotic critical values are given by Osterwald-Lenum (1992), and in the usual case where one allows for a drift (but no trend) in both the data and the cointegrating equations, a finite-sample correction to the critical values is suggested by Cheung and Lai (1993). Using the usual information criteria we arrived at a lag length of k=4 for the VAR models associated with the testing. We see that we clearly reject the null of zero cointegrating vectors, but cannot reject the null of one cointegrating vector. Finally, we have used the Leybourne-McCabe (1994) test, in which the null is "cointegration". We generated finite-sample critical values for this test by Monte Carlo simulation, using the experimental design described by Leybourne and McCabe (1994, p.98), and SHAZAM (1998) code. Leybourne and McCabe's (1994, p.101) critical values for T=500 were reproduced exactly. We clearly cannot reject the null of cointegration at any reasonable significance level[12].

We have also tested for causality between (UE/GDP) and (TR/GDP), using the Toda-Yamamoto (1995) approach, and the results[13] are summarised in Table 2. There is evidence of causality from the latter variable to the former one, but only limited evidence of reverse causality. Taken in the context of the theoretical discussion above, this supports an empirical model with (UE/GDP) as the dependent variable.

5. Nonparametric estimation

In view of the above cointegration results, we may legitimately model the long-run relationship between (UE/GDP) and (TR/GDP) by using the "levels" of these two ratio variables. No differencing of the data is needed. As another option, we could construct an error-correction model, which would be appropriate if we were interested in the short-run dynamics of the relationship. Consistent with the theoretical literature, our estimation deals only with the first of these possibilities.

Giles and Caragata (2000) considered some simple polynomial and Fourier series models of the relationship between (UE/GDP) and (TR/GDP), before focussing on several "S-shaped" functional forms such as the Gompertz, cumulative Normal, extreme-value, and logistic models. The last of these was preferred by those authors in terms of overall data "fit" and statistical significance. Before proceeding further, let us briefly elaborate on some aspects of these non-linear parametric results. Fitting an OLS cubic relationship between these variables results in totally insignificant parameter estimates. A quadratic model generates reasonably significant parameter estimates with a Durbin-Watson statistic of 1.53, and with DeBenedictis-Giles (1998) FRESET test results that strongly indicate no mis-specification. The latter model is, of course, extremely restrictive in terms of the curvature properties that it can capture. The parameter estimates imply a positive but decreasing relationship between the underground economy ratio and the tax rate for values of the latter less than about 33%. At the latter value there is a turning point in the relationship. A simple linear relationship was inadequate whether OLS or instrumental variables estimation was adopted[14].

We have also explored the Giles and Caragata (2000) results further by conducting a series of Davidson and MacKinnon (1981) "J-tests" to discriminate between the polynomial models and their preferred logistic model, these models being "non-nested". These tests suggest the rejection of the logistic model against linear, quadratic, and cubic alternatives. However, they also suggest the rejection of the linear and quadratic models against the logistic model. The unsatisfactory nature of the cubic results has been noted already.

These inconclusive results support the use of a flexible nonparametric approach to estimating this relationship, and our model takes the simple form:

$$(UE/GDP)_t = m\{(TR/GDP)_t\} + \varepsilon_t \ , \qquad (3)$$

where the function "m" is the conditional mean of the dependent variable and the ε_t's are Normal, independent and homoskedastic.

In (3), the data for the dependent variable have been *estimated* by Giles (1999) using MIMIC model analysis. That is, the dependent variable is random for reasons other than that allowed for with the inclusion of the usual error term in the model. In fact, this causes no problem for the properties of our estimates, or their interpretation. This additional source of randomness can be assumed to be independent of ε_τ, and our nonparametric estimator will be consistent, anyway, under very weak conditions.

Estimation of (3) was undertaken with the NONPAR routine in the SHAZAM (1997) econometrics package, using the Nadaraya-Watson estimator with a Normal kernel, and the bandwidth parameter was chosen by Silverman's (1986, p.45) "optimal" method. Some experimentation verified that the results were not particularly sensitive to the choice of kernel or bandwidth. We also paid special attention to "edge effect" associated with nonparametric regression, especially when the sample is relatively small. In particular, we re-estimated our nonparametric model using Rice's (1984) boundary modified estimator, with a range of values for his "ρ" parameter. We found that the results reported below were quite *insensitive* to this refinement, unless "ρ" is close to its minimum value of zero. Even in this case the general non-linear shape of the estimated relationship between (UE/GDP) and (TR/GDP) was not affected. This robustness of the results is encouraging, especially given that our nonparametric estimation is based on quite a small-sized sample. We also considered versions of (3) that included the growth of real GDP, and/or one-period or two-period lagged values of (UE/GDP) or (TR/GDP) as additional explanatory variables. However, none of these were supported by the usual information criteria, and our preferred simple estimates are given in Table 3. The LM tests for autocorrelation have their usual asymptotic interpretation[15], and are generally quite satisfactory. The FRESETS results support the form of the estimated model when interpreted in terms of the associated information measures.

The estimated model's within-sample "predictions", with the (TR/GDP) variable sorted into ascending order, appear in Figure 2. The general shape of this relationship supports the earlier use of a logistic function by Giles and Caragata (1999) and by Caragata and Giles (2000). Also shown in Figure 2, for comparative purposes, are the predictions corresponding to the use of Rice's (1984) estimator, with two different choices of his "ρ". Interestingly, the slight downturn in the fitted relationship at historically high tax rates is consistent with the curvature result noted above with respect to a naive quadratic model.

Table 4 shows the corresponding predicted values for (UE/GDP), together with lower and upper limits for the corresponding 95% prediction interval, and the estimated elasticities. The predictions range in value from 7.52% to 9.51% of GDP. These values should be compared with the "actual" sample values for (UE/GDP) given by Giles (1999), which range from 6.84% to 11.31%.

6. Further economic implications

The corresponding estimated elasticities between the effective tax rate and the (UE/GDP) ratio also appear in Table 4. As these two variables are already expressed in percentage terms, these elasticities must be interpreted with care. For example, 1986 was an interesting year in the history of New Zealand taxation policy, with the introduction of the Goods and Services Tax (GST) in October, and the simultaneous major changes to sales taxes and to the personal income tax and corporate income tax schedules[16]. In 1986 the effective tax rate was 29.85% and the estimated underground economy elasticity was 0.303. This implies that a 10% cut in the effective tax rate would lead (with some unspecified delay) to a 3.03% drop in the %(UE/GDP). A 10% cut in the effective tax rate means reducing it from 29.85% to 26.87%. The %(UE/GDP) in New Zealand in 1986 was 9.23%. So, it is predicted that the size of the underground economy would have dropped by 3.03% (that is, it would have dropped from 9.23% of GDP to 8.95% of GDP) to reach a new equilibrium[17] had the tax burden been reduced by 10% in that year, without any change in the "tax-mix". Of course, as this analysis is based on a cointegrating relationship, the above discussion sheds no light on the speed of this adjustment to the new equilibrium position.

In Table 5 we see the results of "simulating" the estimated relationship in (3) to get predicted values for (UE/GDP) as the effective tax rate ranges *hypothetically* in value from 17% to 38%. It is especially interesting to note that as the tax rate is decreased, the effect on the underground economy begins to "flattens out" at a tax rate below 25%. Although nonparametric estimation does not allow us to extrapolate below the range of our sample data, this observation accords remarkably well with the conclusions of Caragata and Giles (2000) and of Scully (1996). Using the criterion of maximizing the responsiveness of the underground economy to tax rate changes, the former authors concluded that the "optimal" effective tax rate is approximately 21%. Scully's results suggested a growth-maximizing effective tax rate of 20% in the case of New Zealand.

Interestingly, therefore, there is a close consistency between the tax rate that needs to be targeted from an economic growth viewpoint, and the one that needs to be targeted from a compliance viewpoint.

7. Conclusions

In this paper we have used nonparametric time-series regression to examine some of the predictions of a class of theoretical models of the underground economy. In particular, we have interpreted the recent model of Trandel and Snow (1999) in aggregate terms, and have provided empirical evidence concerning the partial relationship between the relative (output) size of the underground economy and the effective tax rate, using New Zealand data. This theoretical model predicts an ambiguous sign for the above relationship, but empirically we find a positive, "S-shaped" relationship that fits the data well over our sample period, 1968 to 1994.

When we simulate the model over a plausible range of tax rate values we obtain results that accord closely with other related results by Caragata and Giles (2000) and Scully (1996), each of whom address the issue of an "optimal effective tax rate" for New Zealand from different perspectives. In particular, our nonparametric model suggests that the responsiveness of the underground economy to simple changes in the tax burden drops markedly when the effective tax rate drops below about 25%.

The possibility that the adjustment of the underground economy to changes in the tax burden is asymmetric (in the upward and downward directions) is an important issue that is not explored here. This is discussed in the New Zealand context by Giles *et al.* (1999). The authors are also analyzing such asymmetry issues in the context of the Canadian underground economy data derived by Giles and Tedds (2000).

Appendix – The Trandel and Snow Model

There are two sectors, workers are identical, and each supplies a unit of labour either to sector 'e' (or to sector 'n'), in which evasion is (or is not) undertaken. Income per worker in sector 'i' is denoted 'y_i'; 'a' is the share of the workforce in sector 'e'; and 't' is the constant marginal tax rate (above a positive threshold level of income, 'b'). Faced with a probability 'p' that evaded income will be detected, and penalised in proportion (m) to the additional tax owed, workers in the evading sector choose a level of undeclared income, 'x', to maximise expected utility:

$$(1 - p)U[y_e(1 - t) + bt + xt] + pU[y_e(1 - t) + bt - mxt]. \qquad \textbf{(A.1)}$$

As labour is mobile, an equilibrium is achieved by finding optimal a* and x* values to equate the expected utility of working in each sector. This requires:

$$U[y_n(a^*)(1 - t) + bt] = (1 - p)U[y_e(a^*)(1 - t) + bt + x^*t] + pU[y_e(a^*)(1 - t) + bt - mx^*t]. \qquad \textbf{(A.2)}$$

Assuming that workers' preferences exhibit decreasing absolute and non-decreasing relative, risk aversion, (A.2) is used by Trandel and Snow to establish several results relating the (labour) size of the underground economy to changes in the marginal tax rate, and in tax progressivity.

To derive our results in section 3, we use the fact that the (actual) average tax rate faced by workers in the non-evading sector is $\tau_n = t\,[y_n(a^*) - b] / y_n(a^*)$. Similarly the (expected) average tax rate in the evading sector is $\tau_e = t\,\{ y_e(a^*) - b - [1 - p(1 + m)]x^*\} / y_e(a^*)$. As Trandel and Snow (1999; p 221) note, $\tau_n > \tau_e$. This is because for tax evasion is to be a better-than-fair gamble, we require $(1 - p) > pm$.

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Footnotes

1. A "linear" tax system is one in which the tax rate is a fixed proportion of income - it is a "flat tax" system with a zero exemption-threshold. If the tax system is linear then any fraction of undeclared income can also be represented as a fraction of evaded tax.

2. The tax schedule in New Zealand was simplified considerably during our sample period. With respect to corporate taxes, the tax structure assumed in the Trandel-Snow model is closely approximated. In the case of personal income taxes the statutory rate is certainly progressive, with a very simple scale (in recent years), but with a zero "threshold" level.

3. The accuracy of Giles' (1999) aggregate time-series measure of the underground economy in New Zealand is supported by the independent micro-evidence, based on Revenue New Zealand business audit records, discussed by Giles (2000).

4. Figure 1 shows that our data exhibit considerable cyclical variation over our sample, implying that our analysis is based on "informative" empirical evidence.

5. The penalty for evasion is not counted as part of generated "income", but this does not affect the conclusions below.

6. We are considering the expected value of the ratio of underground to measured income. Alternatively, one could consider the ratio of expected underground income to expected measured income. Pursuing this alternative yields the same implications as below.

7. Recall that $y'_n(a) > 0$ and $y'_e(a) < 0$.

8. GDP is conventional "measured" GDP. This is a standard way of defining the effective tax rate, and it makes the UE measure comparable to other international series. Some countries (*not* including New Zealand) are considering "adjusting" GDP for underground activity, so an alternative would be to use UE/(GDP+UE). This was not pursued, as UE incorporates a mixture of both "legally-based" and "illegally-based" underground activity, and only the former is relevant to the measurement of GDP.

9. This rule sets l = INT $[8(T/100)^{1/4}]$.

10. For our sample size the asymptotic and finite-sample KPSS critical values are very similar, so the unavailability of the latter in the case of a level-stationary null should not be of concern.

11. We also tested the logarithms of the two variables for unit roots, and found *log*(UE/GDP) to be I(0) and *log*(TR/GDP) to be I(1). So, there can be no cointegration between these two variables, and it would be inappropriate to model the relationship between them without differencing the latter variable. We have not pursued this possibility here.

12. The tests for cointegration are actually tests for a *linear* cointegrating relationship between (UE/GDP) and (TR/GDP). In our case, the OLS residuals from this cointegrating regression suggest that a *non-linear* relationship may also be worth investigating. (The Durbin-Watson statistic is 1.280, with an exact p-value of 0.015; the LM tests for fifth-order and sixth-order autocorrelation are significant at the 1% level. The RESET and FRESET tests are generally satisfactory, but the FRESETS(3) statistic is 2.588, with a p-value of 0.053, and the RESET(2) statistic is 4.976, with a p-value of 0.035. The robustness of the FRESETS tests to error-term autocorrelation is documented by DeBenedictis and Giles, 2000). This further supports our use of a nonparametric specification below.

13. Taking account that the data are I(1), the testing was undertaken within a two-equation VAR model, in which two "own" lagged variables and one "other" lagged variable appeared as the regressors. The insignificant intercepts were suppressed. The Toda-Yamamoto (1995) methodology ensures the asymptotic validity of the Wald test applied to the one-period lagged value of the "other" variable here. This Wald statistic is just the square of the usual "t-statistic" in this case.

14. As noted in footnote 12, the linear models we estimated exhibited mis-specification of the functional form. In addition, the R^2 values were of the order of only 30%.

15. The asymptotic normality of kernel regression estimators (and associated statistics) is well known. For example, see Hardle (1990, pp.99-100). Robinson (1997) shows that asymptotic normality holds approximately even in the case of dependent errors.

16. Initially the GST was levied at a rate of 10%, wholesale taxes were abolished and the top marginal personal income tax rate was reduced from 66% to 48%. Other major changes to these rates have taken place subsequently.

17. Note that this is a drop of 3.03%, and **not** a drop of 3.03 percentage points.

Table 1. ***Unit root test results***

a. Augmented Dickey-Fuller tests [a]

		T	p	t_{dt}	F_{ut}	t_d	F_{ud}	t	Outcome
					UE/GDP				
H_0 :	I(2)	23	2	-3.44	n.a.	n.a.	n.a.	n.a.	Reject I(2)
[H_A :	I(1)]								
H_0 :	I(1)	24	2	-2.70	3.66	-1.63	2.14	1.06	I(1)
[H_A :	I(0)]								
					TR/GDP				
H_0 :	I(2)	23	2	-2.62	3.43	-2.65	n.a.	n.a.	Reject I(2)
[H_A :	I(1)]								
H_0 :	I(1)	24	2	-2.42	3.21	-1.40	2.73	1.63	I(1)
[H_A :	I(0)]								

b. KPSS tests [b]

		T	Level-Stationary		Trend-Stationary		Outcome
			l=0	***l=5***	***l=0***	***l=5***	
				UE/GDP			
H_0 :	I(0)	27	1.519	0.520	0.131	0.110	I(1)
[H_A :	I(1)]						
				TR/GDP			
H_0 :	I(0)	27	0.099	0.073	0.129	0.081	I(1)
[H_A :	I(1)]						

Notes: **a.** The outcomes are based on finite-sample 10% critical values from MacKinnon (1991).

b. The outcomes are based on finite-sample 10% critical values from Cheung *et al.* (1995), and the KPSS 10% asymptotic critical values.

Table 2. ***Cointegration and causality test results***

a. Cointegrating regression augmented Dickey-Fuller "t-tests"

T	**p**	**No Trend**		**Trend**		**Outcome**[a]
		R^2	**t**	R^2	**t**	
27	0	0.40	-3.434 (-3.57) [-3.20]	0.58	-3.856 (-4.15) [-3.77]	Cointegration

b. Johansen's "trace" likelihood ratio tests

	Drift/Trend Case[b]				
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
	Trace Test Statistic, H_0: Zero Cointegrating Vectors				
	27.32	32.02	22.88	31.26	30.38
Asy. 10% crit.	10.47	17.85	13.33[c]	22.76	16.06
Asy. 5% crit.	12.53	19.96	15.41[c]	25.32	18.17
Asy. 1% crit.	16.31	24.60	20.04[c]	30.45	23.46
	Trace Test Statistic, H_0: No More Than One Cointegrating Vector				
	4.45	5.16	0.97	7.83	7.81
Asy. 10% crit.	2.86	7.52	2.69[d]	10.49	2.57
Asy. 5% crit.	3.84	9.24	3.76[d]	12.25	3.74
Asy. 1% crit.	6.52	12.97	6.65[d]	16.26	6.40

Table 2. ***Cointegration and causality test results (continued)***

c. Leybourne-McCabe cointegration tests[e]

T	**h_1**	**Asymptotic Critical Values**	**Finite-Sample Critical Values**	**Outcome**
27	0.122	(0.31)	(0.35)	Cointegration
		[0.23]	[0.25]	

d. Granger causality tests

Causality	**Wald Test ($\chi^2(1)$)**	**Asymptotic p-value**	**Bootstrapped p-value**[f]
(TR/GDP) $\not\Rightarrow$ (UE/GDP)	3.594	5.8%	0.3%
(UE/GDP) $\not\Rightarrow$ (TR/GDP)	3.664	5.6%	23.4%

Notes:

a. MacKinnon's (1991) finite-sample 5% (10%) critical values appear in parentheses (brackets).

b. (1) No drift/no trend in cointegrating equation or fitted VAR.

(2) Drift/no trend in cointegrating equation; no drift in fitted VAR.

(3) Drift/no trend in both cointegrating equation and fitted VAR.

(4) Drift and trend in cointegrating equation; no trend in fitted VAR.

(5) Drift and trend in both cointegrating equation and fitted VAR.

c. The corresponding finite-sample critical values are 20.13, 23.27 and 30.26 respectively.

d. The corresponding finite-sample critical values are 4.06, 5.67, and 10.04 respectively.

e. 5% (10%) critical values appear in parentheses (brackets).

f. These are based on 5,000 bootstrap replications.

Table 3. ***Nonparametric estimation of the relationship between (UE/GDP) & (TR/GDP)***
(Simple nonparametric regression)

Bandwidth Parameter	0.548		
R^2 (Adjusted R^2)	0.492	(0.428)	
Cross-Validation Mean Square Error	0.794		
AIC (SC) [FPE]	0.842	(1.016)	[0.844]
Residuals Analysis			
Durbin-Watson Statistic	1.433		
Runs Test, Normal Statistic (p-value)	-0.975	(0.330)	
Coefficient of Skewness (Standard Deviation)	0.956	(0.448)	
Coefficient of Excess Kurtosis (Standard Deviation)	0.665	(0.872)	
Jarque-Bera, Chi-Square, asy. $\chi^2(2)$ (p-value)	3.681	(0.159)	
Chi-Square Goodness of Fit, asy. $\chi^2(3)$ (p-value)	6.094	(0.107)	
LM(1), asy. Standard Normal (p-value)	1.006	(0.157)	
LM(2), asy. Standard Normal (p-value)	0.208	(0.418)	
LM(3), asy. Standard Normal (p-value)	0.205	(0.419)	
LM(4), asy. Standard Normal (p-value)	0.152	(0.440)	
FRESETS(1)[a]: AIC (SC) [FPE]	0.995	(1.479)	[1.016]
FRESETS(2)[a]: AIC (SC) [FPE]	1.110	(2.086)	[1.215]
FRESETS(3)[a]: AIC (SC) [FPE]	1.330	(2.395)	[1.333]

Note: **a.** FRESETS(i) is the DeBenedictis and Giles (1998) Fourier version of the RESET test with "i" sine and cosine terms. The test statistic cannot be computed in the nonparametric case, but the resulting information criteria that emerge when these extra terms are added into the nonparametric regression can be compared with their counterparts from the original model, given in the first part of this table. As the latter are smaller than those for the FRESETS regressions, this suggests "no mis-specification".

Table 4. ***Ranked nonparametric within-sample predictions and 95% confidence limits***

Lower	Predicted (UE/GDP)%	Upper	(TR/GDP)%	Elasticity
7.080	7.525	7.970	23.643	0.467
7.124	7.559	7.994	23.859	0.531
7.155	7.584	8.013	24.003	0.577
7.510	7.911	8.312	25.287	1.053
7.655	8.053	8.452	25.693	1.178
7.667	8.065	8.464	25.726	1.186
8.163	8.557	8.952	26.996	1.157
8.495	8.888	9.281	28.075	0.754
8.506	8.899	9.292	28.123	0.736
8.620	9.013	9.407	28.696	0.536
8.665	9.059	9.454	28.992	0.453
8.706	9.103	9.499	29.328	0.379
8.716	9.113	9.511	29.423	0.362
8.755	9.157	9.559	29.852	0.303
8.794	9.205	9.616	30.412	0.267
8.804	9.217	9.631	30.563	0.264
8.804	9.217	9.631	30.564	0.264
8.824	9.244	9.665	30.904	0.267
8.832	9.256	9.680	31.047	0.272
8.854	9.286	9.717	31.403	0.290
9.003	9.478	9.952	33.432	0.295
9.017	9.497	9.977	33.681	0.259
9.026	9.511	9.996	33.884	0.224
9.034	9.529	10.024	34.220	0.158
9.033	9.540	10.046	34.527	0.094
8.941	9.525	10.109	35.843	-0.167
8.908	9.513	10.119	36.080	-0.204

Table 5. ***Simulated values of (UE/GDP)% for various effective tax rates***

(TR/GDP) (%)	(UE/GDP) (%)	
	Nonparametric	**Logistic**
17	7.314	6.519
18	7.307	6.668
19	7.306	6.821
20	7.312	6.977
21	7.331	7.136
22	7.370	7.299
23	7.445	7.465
24	7.584	7.634
25	7.821	7.808
26	8.170	7.984
27	8.559	8.164
28	8.870	8.348
29	9.060	8.536
30	9.171	8.728
31	9.252	8.923
32	9.340	9.122
33	9.438	9.326
34	9.518	9.533
35	9.545	9.745
36	9.517	9.960
37	9.451	10.180
38	9.367	10.405

Figure 1. Effective tax rate and underground economy, New Zealand 1968-1994

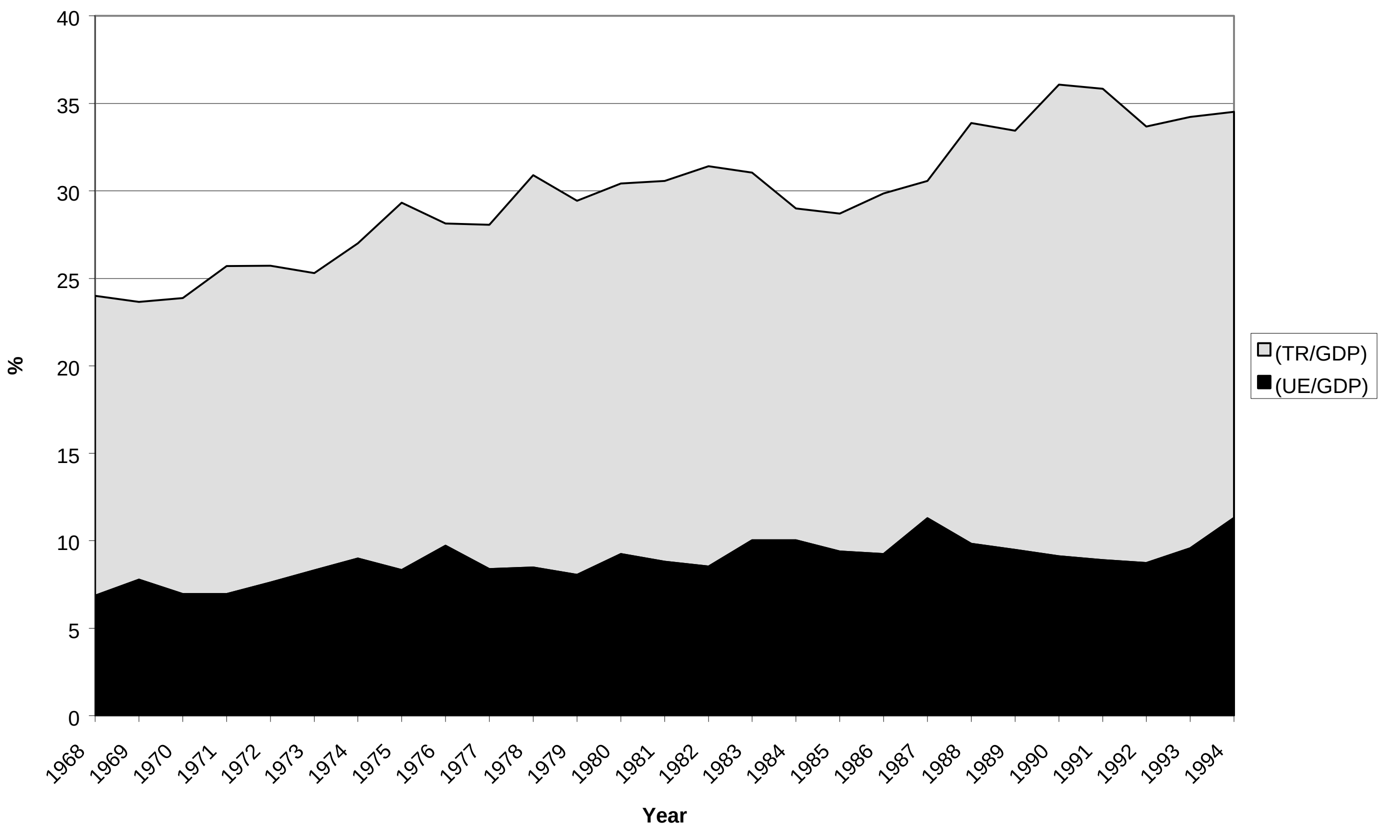

Figure 2. Non-parametric relationship between underground economy and effective tax rate: New Zealand, 1968-1994