Department Discussion Paper DDP2007

ISSN 1914-2838

HAS PUBLIC DEBT BEEN TOO HIGH IN CANADA AND THE U.S.? A QUANTITATIVE ASSESSMENT

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May, 2022

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I quantify the welfare effects of changing the long-run value of public debt using a two-region OLG model with rich income dynamics over the life-cycle, incomplete insurance, and an integrated asset market. I consider two model calibrations, one for Canada and one for the US. In the former case, I find that changes in public debt cause small interest rate effects. To validate the model, I conduct a formal empirical analysis, which does not reject the two-region theoretical framework. The quantitative model is used to perform a welfare analysis of counterfactual debt policies. In the long-run, for both Canada and the US, I find that negative quantities of public debt involve considerable welfare gains. For the US economy, when taking into account the welfare costs of the transitional dynamics, the result is reversed. However, imposing the empirical correlation between changes in public debt and changes in public expenditure found in the OECD data restores the finding that moving to equilibria with public wealth leads to welfare gains.

Keywords: Public debt, Incomplete markets, Welfare.

JEL Classifications: D52, E21, E62, H63.

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Acknowledgments: *I thank both the co-editor and two anonymous referees for their suggestions, which improved the paper. Useful feedback was provided by seminar participants at the University of Saskatchewan, University of Victoria, CEA annual meeting in Banff, and CEF annual meeting in Ottawa.*

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1 Introduction

A rationale for governments to accumulate public debt is the provision of liquidity to borrowing constrained individuals. In an influential paper, Aiyagari and McGrattan (1998) argue that the long-run average of the American public debt is close to the optimal one. They find that the increase in the asset demand brought about by larger values of public debt has important General Equilibrium (GE, hereafter) effects. These lead to positive, yet remarkably small, welfare effects of sizable public debts. In this paper, I undertake an equilibrium analysis examining a two-region model, tailored to the Canadian (CA) and the American (US) cases, in turn. The long-run average of their public debt/GDP ratios has been similar, and the labor income dynamics have some shared characteristics, as shown by Baker and Solon (2003) and Brzozowski, Gervais, Klein and Suzuki (2010). However, one reason for considering these two economies is their drastically different size, with plausibly distinct implications for the magnitude of GE effects in international, but integrated, asset markets.

Some of the main findings in Aiyagari and McGrattan (1998) rest on three important assumptions: a closed economy framework, infinitely-lived households, and wasteful public expenditure.

In a closed economy setup, an increase in public debt is associated with an increase in the aggregate demand for assets, leading to a potentially large response of the equilibrium interest rate. In steady-state comparisons, this can be beneficial for measures of aggregate welfare, as saving in a risk-free asset becomes a more effective tool to smooth income fluctuations. At the same time, higher interest rates crowd out investment in physical capital, reducing aggregate output and worsening social welfare, calling for a quantitative analysis to determine the overall effect.[1] The OECD data plotted in Figure 1 show considerable dispersion in the public debt/GDP ratio for both large and small economies with access to international financial markets. These economies typically display positive and sizable long-run values for this ratio. To better measure the importance of the liquidity provision explanation, an open economy analysis is warranted. I consider a two-region model, with the home economy calibrated to either the Canadian or the American microeconomic and macroeconomic evidence, and the foreign economy representing the Rest-of-the-World (RoW). The two-region model does not rule out a priori the possibility of large public debts to be welfare improving via the improved self-insurance attained by saving in a safe asset. One might expect consistently smaller GE effects on the interest rate, which is what I find. However, these could still be important for the welfare effects, especially in an environment where saving has multiple goals, and the demand for redistribution is driven by several concurrent causes (namely, the degree of market incompleteness, and the nature of labor market shocks).

[1]Such effects would not be present in a basic Small Open Economy framework (SOE). With interest rates determined in international financial markets, and domestic public debt held by foreign savers, welfare improvements induced by higher interest rates would be prevented by assumption.

[**Figure 1 about here**]

A model with infinitely-lived households prevents some key life-cycle dimensions from playing a role in how detrimental incomplete insurance possibilities might be. In particular, young and asset-poor individuals typically prefer more public liquidity, unless this positive effect on their welfare is offset by lower wages (due to a crowding out of investment in physical capital) and more expensive borrowing (due to an increase in the asset demand). Thanks to the availability of both longitudinal surveys and administrative data, the literature on earnings dynamics has provided extensive evidence supporting labor income growth heterogeneity over the life-cycle. Baker (1997), Guvenen (2009), Haider (2001), and Hause (1980) are some influential contributions in this field, while Baker and Solon (2003) is one of the few systematic studies on Canadian administrative data. Recently, Peterman and Sager (2018) have also included life-cycle motives in their analysis of optimal public debt, finding that these have important implications for their welfare measure: the expected utility of the newborns. In particular, and contrary to Aiyagari and McGrattan (1998), they find that large public debts are detrimental for welfare, and the optimal policy is one displaying public savings. Their contribution differs from mine, as I consider two open economies, a richer specification for the income process, with one of the two calibrations targeting the Canadian economy, and with transitional dynamics for the US calibration. I show that the nature of labor market risk is important, as in steady-state comparisons for the closed-economy version of the model, I find large public debts to be optimal.

Regarding the income dynamics, for the Canadian calibration I rely on the state-of-the-art estimates in Baker and Solon (2003), obtained using income tax returns. Their econometric specification nests the standard persistent/permanent decomposition of income shocks into a framework with heterogeneity in the growth rate of labor earnings during the life-cycle. This formulation leads to substantial heterogeneity in income dynamics during the active years in the labor market, together with different pensions upon retirement. To the best of my knowledge, there are no other contributions in the literature of optimal public debt considering such a general specification for the income dynamics. The considerable amount of redistribution present in Canada via the fiscal system might be effective in counteracting the substantial earnings risk that Canadian households have faced. This begs the question of whether the liquidity provision attained with public debt is warranted, as it might inefficiently provide excessive insurance against labor market shocks. A two-region life-cycle model allows accurate quantification of the importance of the GE effects on both wages and the interest rate. This is a critical aspect, as these influence the effectiveness of self-insurance via the accumulation of risk-free assets, versus the lost income and lower pensions resulting from a lower level of aggregate capital. In this setup I find that the quantity of debt maximizing long-run welfare for Canada is large and negative. This implies that the federal and provincial governments should accumulate public wealth. To facilitate the comparison with calibration studies done on the US, I then conduct some quantitative experiments assuming that the Canadian economy is closed. In these counterfactual scenarios, I do find that a sizable public debt maximizes welfare in the long-run. A key mechanism for this result is the considerable decrease in labor market risk faced by the households, which underscores the importance of accurately modeling the income dynamics.

To ascertain whether the small interest rate response found in the Canadian calibration of the quantitative model is consistent with the empirical evidence, I undertake an econometric investigation on the interest rate effects of public debt. I extend the analysis proposed by Engen and Hubbard (2005) and Laubach (2009) to the Canadian case. Following the literature, the dependent variable in the regressions is the forward interest rate computed from zero-coupon yield curves, which I derive. The evidence is somewhat mixed, but the estimated cointegration relationship from a vector error-correction specification, arguably preferable because of the presence of unit roots, entails a positive and statistically significant response of the real interest rate to changes in Canadian public debt. These findings provide some evidence validating the two-region economic model, but data limitations suggest caution in their interpretation.

Considering the dynamics outside of the steady state is desirable because including the transition towards the new long-run equilibrium can generate important short-run costs. Desbonnet and Weitzenblum (2012) and Rohrs and Winter (2017) find these costs to be particularly large. The computational burden of my framework makes the study of the transition between steady states feasible only for the US calibration, because I postulate a slightly simpler stochastic process that retains all of the elements but one, which is generally omitted in the empirical studies on US data. The results for the US show that the long-run welfare analysis is qualitatively similar to that of the Canadian case. However, I find high transitional welfare costs in the analysis that takes into account the short-run effects, leading to welfare improvements induced by a rise in public debt.

A model with fixed and wasteful public expenditure has some appealing features. However, in a scenario where an increase in public debt is found to be welfare-improving, the notion of an entirely fixed public expenditure can be problematic. In a model featuring transitional dynamics, a lower public debt can be attained by partially cutting public expenditure, which reduces the short-run welfare costs. I estimate this channel empirically on OECD data, which I then include in another set of counterfactual experiments.[2] I show that the short-run welfare costs are substantially decreased when part of the wasteful consumption of the public sector can be targeted as a source of budget savings. Imposing the empirical correlation between changes in public debt and changes in public expenditure found in the OECD data restores the result that moving to equilibria with public wealth entails large welfare gains.

[2] In Appendix B, I present some evidence on public debt and public expenditure in the Canadian and American contexts.

To conclude with, an important finding is that using closed economy models for the quantitative assessment of the welfare effects of fiscal policies can lead to highly inaccurate conclusions, even when dealing with a large economy such as the US. The results of closed economy models can be unreliable because of artificially strong GE effects, which distort both the sign and the relative strength of the level, inequality and uncertainty effects, whose interaction shapes the behavior of the overall welfare effects.

1.1 Related literature

The literature on public debt has been very active of late, due to the large public debts accumulated by several countries in the aftermath of the great recession, the recent public debt crises that have been observed in Europe, and the vast budget deficits caused by the on-going COVID-19 pandemic. Alesina and Passalacqua (2016) and Yared (2019) are two comprehensive surveys reviewing both the classical and the more recent explanations for public debt. Both contributions point out the need for methodical quantitative work, especially in models with a political economy explanation for public debt.

Rohrs and Winter (2015) and Rohrs and Winter (2017) are two related papers on the optimal quantity of public debt. The first contribution focuses on the distinction between private and public provision of liquidity, in an environment with limited contract enforceability and endogenous borrowing limits, a margin that is often overlooked in the literature. The second contribution undertakes a thorough quantitative analysis, conducted in a more standard model with exogenous borrowing limits that allows for transitional dynamics. They work with a closed economy model and infinitely-lived agents. An important feature of their framework is the formulation of the stochastic process, which is what in the literature is referred to as the "superstar shock". The two papers successfully match the concentration index of both income and wealth. However, they do so with an unrealistic mechanism. Their stochastic process implies a variance of the income shocks that is considerably larger than the available estimates using Panel Study on Income Dynamics data. Therefore, the role of precautionary savings is overemphasized (underplayed) at the top (bottom) of the income distribution. The infinitely-lived individuals that are highly (poorly) productive in a period can save (dissave) a disproportionate amount of resources to achieve consumption smoothing, as eventually they are going to transition to a much lower (higher) productivity. It follows that the saving behavior implied by their formulation is unlikely to be a reliable representation of the American households' saving choices. As a consequence, the response to changes in public debt and the associated welfare effects might be inaccurate, as my analysis indicates.

There is limited work on the role of aggregate shocks and public debt in models with household heterogeneity. Desbonnet and Kankanamge (2017) consider a standard version of the RBC model with incomplete markets, finding that the optimal amount of public debt is negative. In their paper, they also perform a comprehensive decomposition analysis for the steady-state version of the model. However, they do not allow for an open economy analysis or for a life-cycle component, which is found by Peterman and Sager (2018) to be the most important element in their decomposition of the long-run welfare effects. Since the income processes these two contributions are working with are quite parsimonious, it is not straightforward that similar quantitative effects would be obtained in a model with more general –and empirically grounded– income dynamics.

Most papers in the literature on public debt with incomplete markets interpret public spending as consumption of the public sector, assuming that it is both wasteful and fixed. Unlike public transfers, which can be beneficial, as stressed by Floden (2001), and that are included in my framework. Governments do provide valuable services, either in terms of public infrastructure, a hypothesis considered in the literature by Chatterjee, Gibson, and Rioja (2017), or in terms of publicly provided goods that directly affect the welfare of the households, as in Chatterjee, Gibson, and Rioja (2018). Chatterjee, Gibson, and Rioja (2017) argue that the role of public investment is quantitatively important in the determination of the optimal debt/GDP ratio.

Although understanding how the Canadian public debt has reached what I will find to be an inefficiently high level is beyond the scope of this paper, Desbonnet and Weitzenblum (2012) argue that the short-run gains can create a temptation to deviate toward higher levels of debt. Another possible argument is that short-lived governments use generous public transfers for electoral purposes, an instance that is discussed extensively in Yared (2019). For tractability, I do not consider a model with endogenous sovereign default, an element included in D'Erasmo and Mendoza (2016).

The rest of the paper is organized as follows. Section 2 presents an open economy model with two regions. Section 3 discusses the calibration of the model. Sections 4 and 5 present the results of the quantitative analysis, for Canada and the US, respectively. Section 6 concludes. Several Appendices provide an in-depth discussion of the theoretical model, further information on the data used for the quantitative analysis, and additional results.

2 A two-economy model

In the benchmark model there are two asymmetric economies, the home economy and the RoW, which share a fully integrated asset market. In a first case, the home economy represents Canada, while in a second one the US. There are no restrictions to capital movements, but the economies in both regions have incomplete markets, whereby the workers cannot buy insurance for the labor market risks they face. For tractability, there is no international mobility of workers, also because changes in public debt policies do not seem to drive migration flows. The home economy is smaller than the RoW, and the calibration is designed to also match the domestic economy's share in global GDP.

The two-region framework is a natural one when considering the US, while for Canada one might prefer the simpler SOE setup. One reason to use the two-region model also for Canada is that its share in global GDP, and in capital markets, is not trivially small, being approximately 3%. Although it is plausible to expect a limited interest rate response arising from different public debt policies, under incomplete markets and with rich income dynamics, this could still lead to positive welfare effects of public debt.[3] In Section 4.2, I present a model validation based on a formal econometric analysis. Albeit somewhat mixed, the empirical evidence does not reject the two-region theoretical framework.

The two-economy model is an extension of Huggett (1996), appropriately modified to allow for several sources of heterogeneity in labor income, progressive taxation, public debt, and two open economies producing the same final good.[4] The model has an Overlapping Generations (OLG) structure, to accommodate the empirical evidence on the labor income dynamics of the home economy, which underscore the importance of the life-cycle elements.[5]

The model features two production economies with endogenous asset distributions. In each region, there is a representative firm that supplies final output, relying on an aggregate production function whose inputs are capital and labor. The governments in the two regions collect taxes to finance their pension schemes. The government in the home economy also collects taxes to finance an exogenous stream of public expenditure, public transfers, and the interest rate costs on the accumulated public debt.

Demographics: Time is discrete. The two regions are populated by finitely-lived agents facing an age-dependent death probability π_j^d.[6] Age is denoted by j and there are J overlapping generations, each consisting of a continuum of agents. At age J_R all agents that are still alive retire, and they might live up to age J. Since the average asset holdings of the individuals that die are positive, there are accidental bequests, denoted by b, which are distributed uniformly to both the newborns and all surviving agents.[7]

Preferences: Agents' preferences are assumed to be time-separable and represented by the utility function $U(.)$. Agents' utility is defined over stochastic consumption sequences $\{c_j\}_{j=1}^J$. Their objective is to maximize

[3] Differently, the SOE framework rules out by assumption the possible positive role of public debt through improved self insurance. In a SOE, an increase in public debt is absorbed by the world financial market without any effect on the domestic (and foreign) interest rate. On the one hand, this precludes the crowding out of investment, which would lead to a lower domestic capital stock, output and wages. At the same time, in the SOE scenario, public debt is partially wasteful as the additional asset income is enjoyed by foreigners, leaving the domestic residents to pay higher taxes whose revenues are not fully kept in the home economy. Solving the model with the SOE setup, I found that public debt reductions bring about unambiguously positive and large welfare effects.

[4] For more details on the model, see Appendix C, Rios-Rull (1994), Rios-Rull (1996), Huggett (1996), and Guvenen (2007), among many others.

[5] It is worth mentioning that the welfare effects that will be reported do not stem from the OLG economy being dynamically inefficient, as the calibration guarantees that in the status-quo the economy is well inside the dynamically efficient region.

[6] All endogenous variables, but the interest rate, as well as the parameters are region-dependent. Not to clutter the formulas, here I am omitting the region indicator, which will be made explicit when some confusion might arise.

[7] In a previous version of the paper, I also allowed for a perfectly competitive annuity market, where the agents shared their mortality risk. The results were similar both qualitatively and quantitatively.

their expected lifetime utility $\mathbb{E}U(.)$. To do so, in each period of their lives, they choose optimally how much to consume (c_j) and how much to save in an interest bearing asset (a_{j+1}). The agents' decision problem is:

$$\max_{\{c_j,a_{j+1}\}_{j=1}^{J}} \mathbb{E}_1 U(c_1,...,c_J) = \max_{\{c_j,a_{j+1}\}_{j=1}^{J}} \mathbb{E}_1 \sum_{j=1}^{J} \beta^{j-1} \left[\prod_{h=1}^{j} \left(1-\pi_h^d\right)\right] u(c_j)$$

where $\mathbb{E}_1$ represents the expectation operator at labor market entry over the idiosyncratic sequences of shocks, and $\beta > 0$ is the subjective discount factor. I assume that $u\left(c_j\right) = \frac{c_j^{1-\theta}}{1-\theta}$. The per-period utility function is strictly increasing in consumption, strictly concave, satisfies the Inada conditions, and has a constant relative risk aversion. Labor supply is exogenous and equal to a worker's time-varying productivity.[8]

Labor income dynamics: Given the intricate labor income dynamics observed in both the Canadian and US data, the model includes a rich specification for the labor earnings dynamics. Newborn agents are ex-ante identical, but idiosyncratic realizations of a series of productivity shocks induce substantial ex-post heterogeneity. There are several channels determining the total efficiency units ϵ that the workers supply in the domestic labor market. First, upon entry, workers observe two characteristics that affect their earnings potential: the time-invariant individual effects for both the level (f_α) and the growth rate (f_β) of their labor productivity, the latter effect being an age component that is correlated with the former. These two effects are jointly normally distributed, with m_l and σ_l denoting their means and standard deviations (s.d.), $l = \{f_\alpha, f_\beta\}$, and $\sigma_{f_\alpha,f_\beta}$ denoting their covariance. Furthermore, there are two time-varying shocks, one being a stochastic component ε that follows a stationary AR(1) process with normally distributed shocks ν^ε, and the other one being a stochastic component s that follows a Random Walk (RW) process with normally distributed shocks ν^s. Overall, the income dynamics over the life-cycle for a generic worker i are driven by the following process:

$$\begin{cases} \ln(\epsilon_{i,j}) = f_{\alpha,i} + f_{\beta,i} * (j-1) + \varepsilon_{i,j} + s_{i,j} \\ \varepsilon_{i,j} = \rho_\varepsilon \varepsilon_{i,j-1} + \nu_{i,j}^\varepsilon \\ s_{i,j} = s_{i,j-1} + \nu_{i,j}^s \\ (f_{\alpha,i}, f_{\beta,i}) \overset{iid}{\sim} N(m_{f_\alpha}, m_{f_\beta}; \sigma_{f_\alpha}^2, \sigma_{f_\beta}^2, \sigma_{f_\alpha,f_\beta}), \nu_{i,j}^\varepsilon \overset{iid}{\sim} N(0,\sigma_\varepsilon^2), \nu_{i,j}^s \overset{iid}{\sim} N(0,\sigma_s^2) \end{cases}$$

The estimated processes imply that individuals with high productivity at labor market entry tend to experience slow labor earnings growth over the life-cycle, and vice versa. The top panel in Figure 2 shows these trajectories for three worker types, based on the Canadian estimates.

[8] Since the T-4 tax returns that form the basis for the estimation of the Canadian stochastic income process do not include the number of hours worked, it is not possible to estimate a process for wages or allow for a flexible labor supply. Moreover, Desbonnet and Kankanamge (2017) show that this margin is not the most important channel shaping the welfare effects of public debt policies.

[**Figure 2 about here**]

The average growth type included in the Figure represents 25.4% of the working age population, while both the low and high growth types each represent 11.4% of the working age population.[9] Given the strong negative correlation between the two individual effects, the high growth types tend to start their career with a low level of earnings and vice versa. An interesting feature of the earnings dynamics attributable to the individual effects is that, absent the other shocks, close to the retirement age these three worker types would enjoy virtually the same level of earnings (approximately $\$48,000$).[10] A possible interpretation of these dynamics sees individuals with high educational achievements on one side of the spectrum, who often begin their careers with low paying internships and co-op experiences. On the other side of the spectrum there could be athletes hired by professional teams, young entertainers, and other individuals involved in the show-business or other artistic careers, whose pay can be relatively large from early on, and possibly decline over time. In terms of the consumption smoothing behavior and the latent demand for public liquidity, the low growth types are in a favorable position. Their high entry earnings allow them to start accumulating sizable savings already in the early stages of their life-cycle, providing them with a buffer stock of wealth that can be used against negative income shocks. For these agents, self insurance is extremely effective and not very costly, as they are also saving for retirement and to smooth their initial high labor earnings over their lifetime. Differently, higher growth types can be constrained for two reasons: they would like to borrow both to partially offset negative income shocks and against their substantial future incomes, as their productivity will grow rapidly.

The bottom panel of Figure 2 plots the earnings dynamics over the life-cycle attributable to the RW component. Two patterns are worth pointing out. First, since the variance of the shocks increases linearly, the dispersion of these permanent shocks grows over the life-cycle. Second, since these shocks in logs are symmetric around zero, their average is zero, while the average of the RW component in levels is greater than one, because of Jensen's inequality.[11]

Because of data limitations, I assume that the income dynamics in the RoW region are represented by a parsimonious AR(1) stochastic process, and that all the workers share the same deterministic productivity

[9] I discretize the individual effects with six points in each dimension, for a total of thirty six possible pairs of values (i.e., worker types at labor market entry). Notice that, not to clutter the figure, the average type plotted in the graph corresponds to the four middle types in the quantitative model. In order to maximize the accuracy of the numerical computations, I undertake a form of pruning of the discrete points in the support of the bi-normal density (taking into account that the two random variables are negatively correlated), and the symmetry of the shares at the extreme points of the grid is obtained by construction. By way of comparison, the other two extreme categories, high level-high growth and low level-low growth types, represent less than 0.1% of the labor force.

[10] In terms of dollar values, since the base year is 2010, the deterministic component of earnings might appear low. However, one has to take into consideration that: 1) total labor earnings are on average higher compared to the values depicted in this plot, because the average of the other stochastic components of the income process is larger than one, 2) the model deals with real de-trended earnings.

[11] The theoretical unconditional average of the stationary AR(1) component in levels is one. By construction, the Rouwenhorst method used to discretize it ensures that this property is preserved. See Figure 12 in Appendix F.

profile over the life-cycle.

Pensions: In each region there is a pay-as-you-go pension system, financed by the contributions of the employed workers. The pension benefits are financed with proportional tax rates, τ_R and τ_R^{RoW}, on the current labor earnings, and the equilibrium values of these taxes guarantee balanced budgets of the two pension systems. In the home economy, there is dispersion in the pensions, as the pension benefits $\overline{y}_R\left(f_\alpha, f_\beta\right)$ depend on the two individual effects.[12]

Technology: Production in both the home economy and in the RoW relies on capital and labor to supply a homogeneous good that can be sold internationally. The production functions are Cobb-Douglas, with region-specific inputs and parameters. The interpretation of the RoW region is such that there is a measure $\bar{\mu}^{RoW}$ of foreign economies, each sharing the same preferences and the same technology, a simple Cobb-Douglas production function with labor (L^{RoW}) and capital (K^{RoW}) as inputs.[13] Also in these countries there are incomplete insurance markets and the workers face labor income risk. Given the absence of aggregate shocks, at the aggregate level each foreign country $q \in \mathcal{Q}$ displays the same outcomes and they can be conveniently considered as a unitary region. Both regions share the same exogenous labor-augmenting technological progress, $Z_{t+1} = (1+\gamma)Z_t$, and de-trending the growing variables renders the problem stationary. Variables with a "tilde" are de-trended, e.g. $\widetilde{K_t} \equiv K_t/Z_t$. Along a Balanced Growth Path (BGP), they do not vary over time, their time indexes can be dropped and they are going to be referred to as steady-state outcomes.

$$Y_t = K_t^\alpha \left(Z_t L_t\right)^{1-\alpha} \rightarrow \widetilde{Y} = \widetilde{K}^\alpha L^{1-\alpha}$$
$$\widetilde{Y}_q^{RoW} = (\widetilde{K}_q^{RoW})^{\alpha^{RoW}} (L_q^{RoW})^{1-\alpha^{RoW}}, \forall q \in \mathcal{Q}$$

The input markets are perfectly competitive, and the firms' maximization problem leads to the usual equilibrium conditions equating marginal products to the related input price, with $\widetilde{w}$ and $\widetilde{w}^{RoW}$ denoting the domestic and foreign de-trended wages, respectively.

[12]For computational reasons, I cannot keep track of the actual individual contributions to the system, as it would require an additional continuous state variable. The simpler approach used here assigns pensions on the basis of the average labor market histories, namely it considers the role of both the level and the growth individual effects, while it collapses the effect of the other shocks to their unconditional averages.

[13]I chose this formulation, instead of an alternative one with the RoW using an aggregate production function with RoW labor and capital as inputs, because it is possible to obtain all the relevant parameters by computing cross-country averages.

$$
\begin{aligned}
r &= \alpha \widetilde{K}^{\alpha-1} L^{1-\alpha} - \delta = \alpha^{RoW} (\widetilde{K}_q^{RoW})^{(\alpha^{RoW}-1)} (L_q^{RoW})^{1-\alpha^{RoW}} - \delta^{RoW}, \forall q \in \mathcal{Q} \\
\widetilde{w} &= (1-\alpha) \widetilde{K}^{\alpha} L^{-\alpha} \\
\widetilde{w}^{RoW} &= (1-\alpha^{RoW}) (\widetilde{K}_q^{RoW})^{\alpha^{RoW}} (L_q^{RoW})^{-\alpha^{RoW}}, \forall q \in \mathcal{Q}
\end{aligned}
$$

Since in the global asset market there is only one interest rate r, the symmetry between RoW countries can be imposed, such that $\widetilde{K}_q^{RoW}/L_q^{RoW} \equiv \widetilde{k}_q^{RoW} = \widetilde{k}^{RoW}, \forall q$. It follows that total output in the foreign countries $\widetilde{Y}_{RoW} \equiv \int_{\mathcal{Q}} \widetilde{Y}_q^{RoW} dq$ can be computed as:

$$
\widetilde{Y}^{RoW} = \int_0^{\bar{\mu}^{RoW}} (\widetilde{K}_q^{RoW}/L_q^{RoW})^{\alpha^{RoW}} (L_q^{RoW})^{1-\alpha^{RoW}} dq = \int_0^{\bar{\mu}^{RoW}} \left(\widetilde{k}^{RoW}\right)^{\alpha^{RoW}} dq = \left(\widetilde{k}^{RoW}\right)^{\alpha^{RoW}} \bar{\mu}^{RoW}
$$

Taxes, government outlays and public debt: The fiscal system in the home country entails a flat consumption tax rate τ_c coupled with progressive income taxes. The income tax schedule draws from Heathcote, Storesletten and Violante (2017), and it is characterized by two parameters, λ and ψ. After-tax income is given by $(1+\gamma)^{\psi t} \lambda y_t^{1-\psi}$, which is a monotonic transformation of taxable income y_t. ψ is estimated from the data, while λ is set in equilibrium to satisfy the intertemporal budget constraint of the government. The term $(1+\gamma)^{\psi t}$ is needed to ensure the stationarity of de-trended after-tax income in a growing economy. TX_t denotes total tax revenues, obtained from taxing labor income, asset income, pensions, and consumption. The government finances exogenous, and wasteful, public expenditure G_t. Total government outlays are the sum of government purchases G_t and total public transfers TR_t.

The evolution of public debt D_t over time is obtained by considering the intertemporal budget constraint of the government:

$$
D_{t+1} = (1+r_t) D_t + G_t + TR_t - TX_t
$$

The postulated fiscal system implies that $\widetilde{TX} - \widetilde{TR} = \tau_c \widetilde{C} + \int \widetilde{y}_i di - \lambda \int \widetilde{y}_i^{1-\psi} di$, where $\widetilde{y}_i$ is agent's i de-trended taxable income. In a balanced growth path, de-trended public debt is constant over time, and it is such that the related interest costs, adjusted for the growth rate, are equal to the excess of taxes beyond total government outlays:

$$
(r-\gamma)\widetilde{D} = \widetilde{TX} - \widetilde{TR} - \widetilde{G}
$$

In a BGP, de-trended aggregate output $\widetilde{Y}$ is constant, and the previous relationship can be considered relative to de-trended GDP, where $d \equiv \widetilde{D}/\widetilde{Y}$ and $g \equiv \widetilde{G}/\widetilde{Y}$:

$$(r-\gamma)\frac{\widetilde{D}}{\widetilde{Y}} = \frac{\tau_c \widetilde{C} + \int \widetilde{y}_i di - \lambda \int \widetilde{y}_i^{1-\psi} di - \widetilde{G}}{\widetilde{Y}} \rightarrow \lambda = \frac{\tau_c \widetilde{C}/\widetilde{Y} + \int \widetilde{y}_i di/\widetilde{Y} - g - (r-\gamma)d}{\int \widetilde{y}_i^{1-\psi} di/\widetilde{Y}}$$

For given public debt, government expenditure, and consumption tax rate, the last version of the formula represents the condition used to obtain the equilibrium value of λ.

[**Figure 3 about here**]

Equilibrium: Figure 3 depicts the determination of the equilibrium in the two-region economy. In the plot, the label H stands for the Home economy, and RoW for the Rest-of-the-World. As in the textbook two-large-economies model, the equilibrium interest rate is found when the current accounts of the two regions are consistent with each other. In the initial steady-state (namely, under the status quo Home public debt), the current accounts of both regions are forced by the calibration to be approximately zero. Notice that in the home economy there are two variables determining the total asset demand: the demand for capital by firms ($\widetilde{K}^{d,H}$) coupled with the demand for savings to hold public debt ($\widetilde{D}^{H}$). In the global economy the total asset supply comes from the aggregation of the saving decisions of the households in both the home economy ($\widetilde{A}^{s,H}$) and in the foreign one ($\widetilde{A}^{s,RoW}$). Any increase in the domestic public debt/GDP ratio brings about an outward shift in the domestic asset demand. This causes an imbalance in the current accounts, which triggers an interest rate adjustment, until the general equilibrium is restored.

3 Calibration

In order to assign parameter values, I rely on a mix of (reduced-form) estimation and calibration (in equilibrium) methods. The initial steady-state is calibrated to reproduce selected long-run features of both the home economy and the RoW. I begin by presenting the calibration for the Canadian economy, then I move to present the parameterization for the RoW region.

3.1 Home economy parameters

Table 1 reports the list of the calibrated parameters with their values and associated empirical targets. The parameters in the top part of the Table are the ones that apply to Canada, and that can be set externally, without solving the model.

[Table 1 about here]

Demographics: The following parameters are related to the demographic aspects: J_R, J, π_j^d. Agents become economically active at age 26, a choice dictated by the lowest age group included in the labor earnings sample. They retire at age $J_R = 66$, and they can live up to $J = 101$ years. These are conventional assumptions, also made in other studies calibrating OLG models. The survival probabilities π_j^d are obtained from the Canadian Life Tables. In particular, I average the values of each age-dependent survival rate reported for every year of the 1990-2007 period.[14]

Technology: The Canadian labor share is computed from the Penn World Tables 9.0 (PWT9.0), and its long-run average implies a capital share $\alpha = 34.5\%$. The growth rate of technological change γ is chosen to match the average long-run growth of per-capita income for the Canadian economy, leading to $\gamma = 2.0\%$.[15]

Preferences: The relative risk aversion is set to $\theta = 1.50$, which matches an elasticity of intertemporal substitution of 0.75, a common value in the literature.

Income process and pensions: The baseline parameterization of the exogenous stochastic income process mainly relies on the values reported by Baker and Solon (2003). Their estimates, based on the T-4 tax return data, are $\sigma_s = 0.084$ for the s.d. of the permanent (RW) income shocks, while $\rho_\varepsilon = 0.54$ and $\sigma_\varepsilon = 0.358$ are the estimates of the autocorrelation and s.d. of the innovations for the persistent (AR1) component. The heterogeneity at labor market entry is captured by the dispersion of the two individual effects: $\sigma_{f_\alpha} = 0.366$ is the s.d. of the level individual effect, while $\sigma_{f_\beta} = 0.009$ is the s.d. of the slope individual effect. Finally, the covariance between the level and slope individual effects is $\sigma_{f_\alpha, f_\beta} = -0.003$. Notice that the resulting correlation index between the two individual effects is negative and very high, being -0.89.[16]

[14] The two top panels in Figure 12 in Appendix F plot the survival probabilities over the life-cycle together with the resulting age distribution.

[15] The RoW economy grows at the same rate as the home economy, and the world economy grows along a BGP.

[16] Figure 10 in Appendix F plots the joint bi-normal density implied by these estimates.

Baker and Solon (2003) do not provide estimates for the average growth rate of labor earnings during the life-cycle and the average intercept. These two parameters are needed in the quantitative model for the computation of the labor efficiency profiles. Since the tax return data are confidential, I compute these two parameters estimating with OLS a simple log-wage regression on Labour Force Survey (LFS) data. The LFS is a representative sample of the Canadian workforce, and the estimates obtained in the two samples should be similar. The average of the level individual effect is $m_{f_\alpha} = 10.10$ and the average of the slope fixed effect is $m_{f_\beta} = 0.013$.[17]

The pension replacement rate is taken from OECD (2015), and its value is $\phi_R = 0.479$.

Taxes and government: The consumption tax rate is $\tau_c = 0.124$, which is the average of the estimates reported by Mendoza, Razin and Tesar (1994). This includes both the Provincial Sales Tax (PST) and the Goods and Services Tax (GST). The degree of progressivity parameter ψ is estimated using the 1972, 1986, and 1998 waves of the Survey of Consumer Finances, and the 2007 wave of the Survey of Labour and Income Dynamics. With each sample, I run a regression of log-after-tax-income (which is reported directly by the respondents, unlike in US survey data, where it needs to be inferred) on log-pre-tax-income. Since the model captures the long-run, I compute the average of the four point estimates, which gives $\psi = 0.09$.[18]

The wasteful government consumption is set to match the long-run average over the 1990-2007 period of the public expenditure/GDP ratio, which is $g = 0.214$. Over the same time horizon, the public debt/GDP ratio was $d = 0.685$, which is the value I use to calibrate the status quo.

Asset markets: The borrowing limit is set to $\underline{a} = 0$, such that households cannot borrow.[19]

3.2 Rest of the world parameters

A number of statistics used in the calibration of the RoW economy can be estimated using the PWT9.0.[20] The middle portion of Table 1 reports the full list of the calibrated RoW parameters with their values and associated

[17] I rely on the monthly LFS from January 1997 to December 2007. The wage data are adjusted for inflation using the monthly CPI index. Notice that the LFS waves before 1997 cannot be used, because they did not collect wage data. Because of measurement error concerns, in each wave of the dataset I trim both the top and bottom 5% of the individual wages. I have also performed the estimation on the whole sample, without trimming, and the results are similar. The model is yearly and the Baker and Solon (2003) T-4 data refer to yearly earnings. To make the two datasets consistent with each other I multiply the hourly wages in the LFS by the usual number of weekly hours worked. I then multiply the weekly earnings by an estimate of the number of weeks worked in a year consistent with the data on aggregate number of hours worked available for the Canadian economy. In the log-wage regression I include time dummies, and the time average of their estimated coefficients is added to the intercept. More details are reported in Appendix D.

[18] For more details, see Appendix D.

[19] This is done to ensure that, in a growing economy, households will not eventually borrow an infinite amount of resources. If the de-trended borrowing limit were to be negative, this would happen in the limit.

[20] The model considers two regions that trade assets and a final good. When computing the RoW statistics, I impose that the countries in the sample entertain international economic relationships with the home economy. I select only the top 30 trade partners, which account for more than 90% of total trade flows for Canada (or the US). The RoW definition changes between the Canadian and US calibrations, because it includes the US in the former, and Canada in the latter.

empirical targets.

Demographics: The survival rates are constructed using OECD data. The resulting profiles match the 1960-2007 cross-country average in the life expectancy at birth (72 years) and at age 65 (14 years).[21]

Preferences: Since there is no evidence that Canadian and foreign households differ systematically in their attitudes towards risk, I set the relative risk aversion parameter in the two economies to the same value, $\theta = \theta^{RoW} = 1.50$.

Income process and labor input: The life-cycle profile of the deterministic component of labor productivity is obtained by computing the cross-country average of the profiles reported in Lagakos et al. (2018). Since many countries are lacking good-quality panel data with household-level information on labor income, for the stochastic component of labor productivity in the RoW, I postulate a simple AR(1) process. The persistence and the standard deviation of the autocorrelated income shocks are obtained averaging the available estimates in the literature on income dynamics.[22] This procedure gives $\rho_{\varepsilon}^{RoW} = 0.949$ and $\sigma_{\varepsilon}^{RoW} = 0.152$. By construction, in the RoW the labor input averages to 1.

Asset markets: The borrowing limit is set to $\underline{a}^{RoW} = 0$, such that foreign households cannot borrow.

3.3 Parameters set in equilibrium

Five parameters do not have a direct empirical counterpart, and need to be set in equilibrium. They are reported in the bottom part of Table 1, and they are selected to match the statistics explained next.

From the PWT9.0, I compute the long-run average of the capital/GDP ratio for Canada, which is $K/Y = 2.7$. This value represents one of the calibration targets. Since capital and public debt are both risk-free, a no arbitrage condition forces their rate of return to be equal. Similarly, in a perfectly integrated financial market the interest rate for the home economy and the RoW must be the same. The interest rate is another empirical target, and I use a value of $r^* = 3\%$.[23] The two parameters affecting these moments the most are the discount factors, and the values matching the calibration targets in the initial steady state are $\beta = 1.031$ and $\beta^{RoW} = 1.098$. From the PWT9.0, I compute the ratio between Canada's GDP and the World's GDP, which is equal to

[21] See Figure 13 in Appendix F. In a previous version of the paper, the agents in the RoW region were assumed to be infinitely-lived, and I obtained similar results.

[22] In particular, I consider the estimates for the US reported by Guvenen (2009), for Sweden reported by Domeij and Floden (2010), for Russia reported by Gorodnichenko, Peter and Stolyarov (2010), for France, Germany, Italy, Spain, and the U.K. reported by Le Blanc and Georgarakos (2013), and for Japan reported by Lise et al. (2014). See Figure 13 in Appendix F.

[23] This target is close to the average real interest rate paid by the Canadian Treasury bills, which for the period 1990-2007 was equal to 2.8% (see Appendix D). I used this alternative value in a previous version of the paper, and the results were very similar.

$Y/(Y+Y^{RoW}) = 2.6\%$. This identifies the measure of the RoW region, with $\mu^{RoW} = 30.71$ allowing to match this target. The Canadian data on the national net lending/net borrowing position (Cansim series v62305900) show that the long-run average is virtually zero. This statistic implies that the long-run average of the Canadian current account is approximately equal to zero. The full calibration treats α^{RoW} as a free parameter that is mostly identified by this target. This leads to $\alpha^{RoW} = 0.357$. However, in order to work with a parameterization that is consistent with the calibration for the US economy presented later, I opted to fix this parameter to $\alpha^{RoW} = 0.364$, which represents the average of the two capital shares (home and foreign) obtained in the US calibration.[24]

The last two parameters, δ and δ^{RoW}, are chosen to match the investment share of GDP in the two regions. The calibration targets, computed from the PWT9.0, are $I/Y = 25.5\%$ and $I^{RoW}/Y^{RoW} = 26.1\%$. Given these targets, the target for the interest rate, and values for both the growth rate and the capital shares, the equations identifying the depreciation parameters admit a closed form representation. Solving the two formulas for δ and δ^{RoW} gets: $\delta = [r^*(I/Y) - \alpha\gamma]/[\alpha - (I/Y)] = 0.008$ and $\delta^{RoW} = [r^*(I^{RoW}/Y^{RoW}) - \alpha^{RoW}\gamma]/[\alpha^{RoW} - (I^{RoW}/Y^{RoW})] = 0.005$.

4 Quantitative analysis for the Canadian economy

In this section, I present the results of the quantitative analysis for the Canadian calibration. The assessment of the welfare effects of public debt is based on considering a series of counterfactual policies. I start by discussing the steady state implications for welfare, prices, aggregate variables and taxes for the Canadian economy, with public debt policies ranging from $d = -2$ to $d = 3$.[25] Next, I present an empirical investigation studying the interest rate response to changes in Canadian public debt.

[**Figure 4 about here**]

4.1 Counterfactual long-run public debt policies for Canada

Figure 4 displays the equilibrium outcomes for the home economy. The plots included in the left (right) portion of the figure, and labeled with the letter A (B), refer to the two-region (closed economy) version of the model. Panel A1 conveys a strong message. According to the model, the Canadian long-run public debt has been

[24] As a consequence, in the Canadian calibration I dropped one calibration target, obtaining a very small, yet positive, current account. The results based on the full calibration are very similar.

[25] The welfare effects represent a Consumption-based Equivalent Variation (CEV), and are expressed as percentage changes in consumption for either the newborns or all agents in the economy (the latter measure posits a utilitarian social welfare function).

inefficiently large. The welfare of both the newborns and households taken as a whole is substantially higher in equilibria characterised by policies with public savings. Differently, policies with a public debt larger than in the initial steady state entail a lower welfare. In all of the counterfactuals considered here, the behavior of both welfare measures is quantitatively similar. A noteworthy result is the size of the welfare effects of changing the long-run public debt/GDP ratio from its status quo value of 68.5%. For instance, a public debt/GDP ratio of -200% features a welfare gain close to 7%, while a public debt/GDP ratio of 200% results in a 3.5% loss. The second panel (A2) displays the response of the interest rate, which only moves from 2.99% to 3.01%. As expected, this is quantitatively small, yet different from a completely flat behavior. In this class of models, the GE response of this price is crucial in shaping the welfare effects. In Section 4.2, I undertake an econometric analysis showing that the Canadian empirical evidence does not reject small interest rate effects driven by changes in public debt. The third panel (A3) plots the response of GDP and aggregate consumption, relative to their values in the initial steady-state. Given the minimal changes in the interest rate, aggregate output declines moderately in d, because of a negligible crowding out of investment in physical capital. Differently, the change in consumption is more pronounced, due to the decrease in disposable income induced by the rising costs of servicing the public debt. In the range for d considered here, the interest payments on public debt move from being an additional source of government income, reducing the burden of taxation, to a disbursement paid mostly to foreigners.[26] Since the interest rate is almost unchanged across equilibria, this effect is monotonic, a behavior that is not shared by the corresponding response in the closed economy version of the model. Another way of looking at these results is to recall that the interest rate is the price of current consumption relative to future consumption. A decrease in public debt triggers an increase in the aggregate demand for consumption. In a closed economy, this additional demand can only be met by domestic producers, which leads to large adjustments in the interest rate. These changes are needed to increase production by using more capital. In the two-region model, smaller adjustments in the interest rate are required to meet the additional demand, as the measure of global producers is substantially larger than the measure of domestic firms. As for consumption, there are both level and dispersion effects. The larger the public debt, the lower the aggregate consumption. At the same time, the variance of log-consumption increases in d by more than two percentage points, due in part to the reduced redistribution that can be attained with the tax revenues. In terms of the welfare effects, these two outcomes reinforce each other. The fourth panel (A4) plots the response of λ, the variable determining the level of the after-tax income, the status quo value being $\lambda = 0.77$ (recall that, from a household's perspective, a higher λ is better, as it affords a higher after-tax income). Overall, this variable is crucial in shaping the welfare effects because it changes by more than ten percentage points. This behavior reflects the need to both finance

[26]It is worth mentioning that the share of public debt held domestically is indeterminate, because both assets are risk free and pay the same rate of return. However, in some counterfactuals, the size of public debt is larger than domestic savings.

higher interest costs on the public debt, and to compensate the fall in revenues from consumption taxes.

By way of comparison, I conduct the same quantitative experiments assuming that the Canadian economy is closed. The plots displayed in the right portion of Figure 4, and labeled with the letter B, refer to this case. In this alternative environment, I do find that large public debts are optimal, reconciling my results with those of Aiyagari and McGrattan (1998). Panel B1 shows that welfare will peak at a value of the public debt/GDP ratio larger than 300%. Compared to the two-region model, the interest rate response is 70 times larger, as it now ranges from 2.38% to 3.79%. This causes a significant fall in output, because capital is crowded out more intensely. This effect is particularly strong, as depicted in panel B3, and it is the one that eventually dominates. This explains the decreasing portion of the welfare effects profile, leading to the inverse-U shape that is typically seen in the literature. The decrease in GDP indirectly dampens the fall in consumption, as across steady-states it is the government share of GDP that is constant. This effect is clearly seen in the response of λ, plotted in panel B4. It is hump-shaped, but remarkably flat for values of $d < 1$, the increasing costs of servicing the public debt notwithstanding. Eventually, its fall accelerates. The consumption dispersion decreases in d, partially compensating the decrease in welfare stemming from a lower aggregate consumption.

[**Figures 5 and 6 about here**]

4.1.1 Welfare effects decomposition

Following Floden (2001), I decompose the overall CEV into level and distributional effects, plotted in Figure 5. The former margin is a combination of changes both in taxes and in the intensity of the crowding out of capital, which affect the aggregate level of consumption. The latter is a combination of uncertainty and inequality channels, which affect the distribution of consumption.

The two top panels in the Figure refer to the Canadian case, but share many characteristics with the US calibration, which corroborate why a two-region model should be used. The welfare effects decomposition is drastically different in the open economy version of the model (labeled Two-region) compared to the closed economy one. The first panel shows that in the Canadian calibration of the two-region model the change in aggregate consumption accounts for most of the overall welfare effects, with the distributional margin playing a limited role. The decomposition behaves very differently in the closed economy, where the distributional effect is considerably stronger and monotonically increasing in d. This monotonicity is not unique to the OLG model considered here, as Floden (2001) shows that it also holds in an infinitely-lived agents model. In general, the distributional channel consists of two factors: an inequality component and an uncertainty one. In the infinitely-lived model in Floden (2001), the inequality and uncertainty components move in opposite directions, and the latter is found to be relatively stronger. In particular, the crowding-out of capital leads to lower wages, which –for a given labor supply– decrease the degree of uncertainty by reducing the average of the stationary stochastic process. In an OLG economy, the distributional effects are also affected by a third component: the life-cycle. In my OLG model with heterogeneous productivity growth over the life-cycle, the risk reduction stemming from lower wages compounds over time, substantially reducing the uncertainty the workers are facing in the labor market. A further decomposition of the distributional effects reveals that in my OLG model the uncertainty component accounts for almost the totality of their value.

In the closed economy version of the model, the level and distribution effects typically have opposite signs. Nevertheless, quantitatively there is no reason why their relative strength should vary in d in a systematic manner. Therefore, the overall welfare gains are not always guaranteed to be hump-shaped.

The age profiles of the welfare effects are depicted in Figure 6. In the closed economy, there can be substantial dispersion in the welfare effects over the life-cycle, unlike in the two-region economy, where they have limited heterogeneity because the GE effects are weak, and they do not spur a generational conflict.

4.1.2 Discussion

To better understand the results, recall that in a complete markets economy public debt is neutral with respect to the capital allocation, and the interest rate does not vary in d. In that environment, public debt is often justified as a means to reduce the intertemporal distortions caused by fluctuations in aggregate income. This margin is not present in the models considered in this paper because, at the aggregate level, they are deterministic. With complete markets, choosing a public debt policy boils down to choosing a taxation policy, with higher taxes collected in steady-state equilibria characterised by larger public debts. Since consumption tracks disposable income, the response of welfare follows: under complete markets, the welfare profiles are monotonically decreasing in d because they are influenced only by the level effect.

In the closed economy versions of the incomplete markets models considered both in the literature and in my model, which are displayed in panels B1 in Figures 4 and 7, the welfare effects profiles are hump-shaped. The reasons behind this difference are linked to the strength of the GE effects, and the associated level and distributional channels. In general, in the closed economy version of the model, the level and distributional effects have opposite signs (which, in Figure 5, is more apparent in the US calibration), and these two effects change systematically in d. First, the level effect decreases in d because: a) the GE effects on the interest rate are large enough to cause a sizable rise in the cost of servicing the debt, which increases taxes and reduces disposable income (doing so non-linearly, because the interest rate response is convex in d), b) there is a considerable crowding out of capital, which negatively affects both aggregate income and aggregate consumption (doing so non-linearly, because of the concavity of the aggregate production function). Noticeably, these two channels reinforce each other. Second, the distributional effect increases in d because: a) the rise in the interest rate makes saving a more effective tool for self-insurance, b) the fall in wages induced by the decrease in productive capital weakens labor market risk, via a reduction in income uncertainty. These two channels combined lead to a substantial reduction in consumption inequality, which is partially offset by: c) a lower degree of income redistribution that can be attained when the tax revenues have to finance increased costs of servicing the debt, with a concurrent decrease in the proceeds of consumption taxes. Overall, for low values of d, the improvement of the distributional outcomes dominates the worsening of the level outcomes, which explains the increasing portion of the welfare profiles. However, the level outcomes deteriorate quickly, eventually forcing the welfare profiles to reach a maximum and decrease afterwards.

In the two-region version of the model, because of the limited strength of the GE effects, the level effect is always extremely strong. Since the interest rate does not vary considerably, the large increase in taxes substantially reduces disposable income, while the crowding out of capital is minimal. In this environment, the distributional effect has the same sign as the level effect for the following reasons. The small change in the interest rate results in a negligible modification of the insurance properties of saving in a safe asset. Similarly, the modest change in wages implies that the amount of labor market uncertainty remains virtually unchanged. Finally, the decrease of both aggregate income and consumption in d lowers the degree of income redistribution achieved via the fiscal system, which moderately increases consumption inequality.

As for the behavior of either calibration of the two-region version of the model, they are qualitatively closer to the complete markets case. For the Canadian calibration, the response of both the interest rate and wages are so modest that the level effect always dominates. Although there is a non-linear behavior, it is quantitatively small.

4.2 An empirical assessment of the interest rate effects of Canadian public debt

The Canadian calibration of the model predicts that changes in Canadian public debt lead to small and positive effects on the interest rate. Engen and Hubbard (2005) and Laubach (2009) are two influential contributions whose empirical methodology is designed to identify the causal effect of public debt on this variable. Since they focused on the US economy, I adapt their analysis to the Canadian case. A key aspect of their procedure is the choice of the dependent variable: they mostly work with forward interest rates, derived from zero-coupon yield curves. Forward rates help address some of the econometric issues because they embed the private sector's expectations regarding the future state of the economy, thus reducing the importance of current economic conditions in the assessment of the effects of government debt on the interest rate. Engen and Hubbard (2005) worked with inflation-adjusted interest rates, which is the approach that I discuss in this section because it is consistent with the framework of my theoretical model.[27]

For comparability with Engen and Hubbard (2005) and Laubach (2009), I rely on an econometric specification similar to theirs. The control variables (all in real terms) are: GDP growth, the oil price, a measure of the equity premium, and the Bank of Canada (BoC) holdings of government assets.[28] The oil price series, which is available in USD, is converted into CAD using the CAD/USD exchange rate. The steps that I use to obtain the equity premium measure are explained below. The BoC holdings of government assets consists of the sum of Treasury Bills and Government of Canada Bonds.

4.2.1 Compiling the Canadian dataset

There are two noteworthy differences between the Canadian dataset I am using and the American data the literature has relied on: the sample period and the data frequency. The Canadian (American) data are quarterly (yearly) and span the 1990Q1-2007Q4 (1976-2003) period.[29]

Before the estimation of the econometric models can be performed, the empirical analysis requires a number of steps. First, since some series are monthly and others are quarterly, I compute quarterly averages. Second, I follow Gurkaynak, Sack and Wright (2007) to obtain forward interest rates. This is accomplished by fitting a yield curve for every time period in the dataset, using a widely used specification. Third, in the spirit of Engen and Hubbard (2005) and Laubach (2009), I obtain public debt projections. Finally, I need to create the equity premium variable.

Quarterly averages: The oil price in CAD and the BoC holdings of government assets are the two monthly variables for which I have to compute both their quarterly averages and their real counterparts. I first compute the quarterly averages, which I then divide by the quarterly CPI index. From the monthly data on the (annualized) rates of return of the Canadian securities, namely T-bills and government bonds with a given set of maturities, I compute the quarterly averages.

Forward interest rates: With the (annualized) rates of return at the quarterly frequency in hand, I fit a yield curve for Canadian securities in every period between 1990Q1 and 2007Q4. I postulate a parametric yield

[27] Laubach (2009) used nominal interest rates, and I report the analysis for this case in Appendix F. The results are robust.

[28] More details on the data are included in Appendix D. The specification modeling the nominal interest rate has one more control: expected inflation, which consists of the estimates in Yetman (2017).

[29] I do not consider the data after 2007, as these years are likely to represent a different regime. Because of the onset of the great recession, Central Banks interventions soared and the management of monetary policy changed drastically, which had a first order impact on the determination of interest rates in Canada and elsewhere.

function, using the Nelson and Siegel (1987) formulation, as it is known to provide an excellent fit.[30] From the yield function one can easily obtain the discount function, which provides prices of zero-coupon bonds. The discount function, in turn, allows computation of the par-yield curve, which can be compared to the data. A non-linear optimization algorithm finds the fitted yield curve by searching over the parameter space, with the goal of minimizing the distance from the empirical rates of return. As customary in this literature, in the objective function, each security is imputed a weight proportional to its maturity. This is done to reduce the importance of the T-bills, which –given their short-term nature– are less relevant for the forward rates needed in the regressions. With every fitted yield curve, I compute a sequence of n one-year forward rates for each quarter, whose average is the desired n-year-ahead forward rate in that quarter. The final step consists of adjusting the nominal forward rates for inflation. The real n-year-ahead forward rates are obtained by subtracting the annualized inflation rate from their nominal values.

Public debt projections: Another important difference between the Canadian and American data is the lack of credible projections of public debt and deficits for Canada. Engen and Hubbard (2005) and Laubach (2009) can use, with some confidence, the Congressional Budget Office (CBO) projections, while the Canadian context presents a number of challenges.[31] First and foremost, in the period that is relevant for the analysis, PBO (2022) documents that there was a growing discontent in the private sector regarding the reliability of the federal government's fiscal forecasts. Cunning political agendas were seen as the reason behind the systematic errors in the projections. Ultimately, the widespread lack of trust led to a profound reform, which culminated in the creation of the Parliamentary Budget Officer. Unfortunately, this institution was established only in 2006, which corresponds with the end of my sample period.[32] Lacking reliable, and plausibly exogenous, public debt projections disseminated by independent institutions, I start by using the current public debt as the explanatory variable of interest in the regressions. I then follow Hamilton (2018), constructing the 5-year-ahead debt projections using a simple linear forecasting model. Although neither variable is an ideal measure of exogenous changes in public debt, using Hamilton's specification is motivated by some of its properties, which make it a desirable –and increasingly popular– filtering device. Rather than using it to extract the short-run fluctuations from variables with a long-run trend, I use it to obtain the predictable component, with four lags used as regressors, and twenty quarters as the time horizon of the prediction.

[30] For more details on the methodology, see Gurkaynak, Sack and Wright (2007) and the appendix in Laubach (2009). I do not use Svensson's extension, as it has two more parameters that introduce numerical instability, but the added flexibility is not needed, as it is meant to improve the fit at very long maturities.

[31] Cohen and Follette (2003) show that the quality of the CBO projections deteriorated after 1998, becoming systematically inaccurate. Perhaps, also the results based on the CBO projections should be interpreted with some caution, as it is not realistic to assume that the private sector would trust projections that are known to be consistently unreliable.

[32] Another complication is that the available fiscal forecasts were produced over short time horizons, as typically only two years were included in the reports. Moreover, the forecasts were annual, which creates a mismatch with my quarterly dataset.

Equity premium: For the computation of the equity premium, I follow Lettau and Ludvigson (2001), who consider the cointegration residuals among consumption (of non-durables and services), labor income, and household wealth. I construct these cointegration residuals because they typically outperform alternative measures of the equity premium in forecasting one-quarter-ahead excess returns of the S&P 500 over T-bills. Like Lettau and Ludvigson (2001), I start by running a Phillips-Ouliaris test, which is a residual-based cointegration test. To obtain the residuals, I use OLS to estimate a regression of log consumption on log labor income, log household wealth and a constant. The hypothesis of no cointegration is rejected at the 5% level as all of the augmented Dickey–Fuller test statistics, from models with up to four lags, are below the relevant critical value (-2.89). I also use Johansen's procedure to test for the number of cointegration relationships. Regarding the lag selection, both the Akaike information criterion and the likelihood ratio test indicate that the appropriate number of lags is 3. Using 3 lags, a constant in the cointegrating equations, and a linear trend in the undifferenced data, Johansen's test provides evidence of a single cointegration relation among log consumption, log labor income, and the log of household wealth. Both the trace and the maximum eigenvalue statistics reject the assumption of no cointegration and fail to reject the assumption of one cointegrating relation. Finally, a vector error-correction model with 3 lags is specified, and the cointegration residuals are computed based on its estimated parameters.[33]

Having explained the preliminary steps required to gather the data and construct the variables needed in the econometric models, I now turn to the regression analysis.

[**Table 2 about here**]

4.2.2 Empirical results

Table 2 reports the estimation results of six econometric models. The first five columns deal with reduced-form specifications estimated with OLS, while the last column is based on a six-equation Vector Error-Correction (VEC) model. The dependent variable of the single-equation models is always an annualized real rate of return, expressed as a percentage. Following Engen and Hubbard (2005), I mainly use the 5-year-ahead 10-year forward rate. In the first column, the dependent variable is the current rate of return of 10-year government bonds

[33] The series for consumption and labor income come from National accounting, and begin in 1961Q1. Unlike for the US, household net worth data are not readily available for Canada. I estimate this variable using the following simple steps. Both the Capital/GDP and the Public Debt/GDP ratios are available for Canada, from which I can back out the value of the stock of assets in 1961. Then, in every quarter I add to this initial stock of wealth the personal saving series, which is also available. The implicit assumption for the procedure to be valid is that in the initial period the whole stock of assets was held by domestic households. This seems a plausible approximation, because I consider a time well before the financial liberalization began. The initial value of the capital stock is likely to be contaminated by measurement error. I then discard the first part of the sample (until 1976Q1), to use the data where the initial capital stock should not play a role, because it depreciated.

(gbr_{10}). In all the other columns, it is replaced by the 5-year-ahead 10-year forward rate ($fwr_{5,10}$), which in the VEC model represents only one of the dependent variables.[34]

Column (1) refers to a basic specification, where the explanatory variable of interest is the current value of the debt/GDP ratio. The associated point estimate is negative, a result found also by other papers in the literature that do not rely on forward rates or debt projections. For example, Ardagna, Caselli and Lane (2007) and the references cited therein obtain a negative sign. They argue that it can be rationalized by portfolio effects, with investors switching into government bonds when they are assessed to be low-risk assets, an element that is not present in the economic model where both capital and public debt are risk-free. Column (2) considers the preferred specification in the literature. The dependent variable is now the 5-year-ahead 10-year forward rate, and the main regressor is the 5-year-ahead projection of the debt/GDP ratio. The associated point estimate is now positive, but the parameter is not statistically different from zero. Column (3) keeps the same dependent variable, but the debt/GDP regressor now consists of its current value. The parameter estimate changes sign, but it is not statistically different from zero. Comparing the estimates in regressions (2) and (3) highlights the effect of using debt projections, while comparing the estimates in regressions (1), (2) and (3) highlights the effect of using forward rates. Using the variables favored in this field has an impact on the sign of the coefficient of interest, but in this application the inference is not affected, and the conjectured portfolio effects might be a statistical artifact.

Columns (4) and (5) present two robustness checks, motivated by the time series properties of the Canadian data.[35] In particular, Dickey-Fuller, DF-GLS, and Perron stationarity tests on both the dependent variable and the regressor of interest display mixed results. These series might feature a stochastic trend.

In an attempt to deal with the possible non-stationarity of the dependent variable, regression (4) extends the basic specification by adding a linear time trend. The estimated coefficient on the debt/GDP ratio stays positive, yet it is not statistically different from zero. The residuals of the regressions considered so far are autocorrelated, which could lead to erroneous inference. For instance, the Durbin-Watson statistics for regressions (1)-(4) are between 1.74 and 2.33. Therefore, column (5) reports the results of a Cochrane-Orcutt regression. The estimated autoregressive coefficient of the error term is negative, being -0.19. Nevertheless, also in this case, the estimated coefficient on public debt is positive and with a large p-value.

Motivated by the evidence indicating the potential presence of unit roots in both the forward rates and the

[34] For brevity's sake, for the economic interpretation of the control variables, I point to Engen and Hubbard (2005) and Laubach (2009). Here it suffices to say that in regressions (1)-(4) they are jointly statistically significant at the 15% level. All of the covariates in the reduced-form regressions are included as endogenous variables in the VEC model. Also, since in my dataset there is a limited number of securities whose maturity is longer than 10 years, as a robustness check I performed the analysis using the 5-year-ahead 5-year forward rate. The results are similar, and are reported in Appendix F.

[35] I also considered some additional cases. In particular: a) I repeated the analysis fitting the yield curves on the monthly interest rate data, and then performing the time aggregation; b) as dependent variable, I also used the 2-year-ahead and 3-year-ahead 5-year forward rates. The main results are similar.

debt/GDP ratio, I perform a Vector Error-Correction (VEC) analysis. Estimating a cointegration relationship is desirable, because it is conceptually closer to the steady-state equilibria considered in the economic model. Since my dataset is quarterly, the short-run dynamics are likely to play a prominent role, making this analysis particularly informative. The estimation of the error correction model finds a cointegration relationship, a result that is established formally using Johansen's methodology. Regarding the lag selection, both the Schwarz's Bayesian and the Hannan and Quinn information criteria indicate that the model should include one lag. Using one lag, a constant in the cointegrating equations, and a linear trend in the undifferenced data, Johansen's test provides evidence of a single cointegration relation. Both the trace and the maximum eigenvalue statistics reject the assumption of no cointegration and fail to reject the assumption of one cointegrating relation. The coefficients of the cointegration equation are listed in column (6). The estimated parameter on the debt/GDP ratio is now 0.036, namely positive and larger than in the previous regressions, and it is also statistically different from zero at the 7% level.

Overall, keeping the data limitations in mind, the empirical analysis for Canada provides some evidence supporting the two-region theoretical model. The VEC results are consistent with positive and statistically significant responses of the interest rate to changes in public debt. This is at odds with the implications of the SOE model, which predicts that the estimated parameters in all of the specifications should not be statistically significant. Finally, it is worth noting that the quantitative response of the two-region model to changes in public debt falls inside the 95% confidence interval built around the VEC estimate.

5 Quantitative analysis for the US economy

In this section, I focus on the US economy, discussing the welfare effects stemming from counterfactual public debt policies, considering both steady state comparisons and transitional dynamics.

5.1 A welfare analysis of public debt policies in the US

Here, I analyze the welfare effects of counterfactual public debt policies for the US, a large open economy. The welfare analysis is based on both steady-state comparisons and transitional dynamics between steady-states.[36] Since the overall procedure is similar to the one presented above, the calibration is reported and discussed in Appendix A.

[**Figure 7 about here**]

[36]The computational burden of the model with transitional dynamics is substantial, as storing the decision rules and distributions requires computers with more than 64Gb of RAM.

5.1.1 Welfare analysis in the long-run

Figure 7 shows the long-run equilibrium outcomes for the US economy. Overall, the results are similar to the ones obtained for the Canadian case, with some noteworthy distinctions. These are due to the different calibrations, with the estimated income processes, the income tax codes, and the size of the home economy representing the most prominent disparities. In the two-region model, the main differences in the results are the magnitude of the welfare effects (that are approximately 50% larger for the US), a stronger response of the interest rate (13 times larger) that causes a sizable drop in production, and responses that display more curvature of both the consumption dispersion and the income tax parameter.[37]

The mechanisms at play are similar, as the related welfare effects decompositions in the bottom part of Figure 5 show. The welfare changes are mostly driven by the level effect, with aggregate consumption falling sharply in d. The right panels in Figure 7 plot the results when the US economy is assumed to be closed. In this case, the long-run welfare effects are clearly hump-shaped and their maximum is reached for public debts of $d = 170\%$ for social welfare and $d = 230\%$ for the newborns' welfare. The results are remarkably different from what Peterman and Sager (2018) found, showing that the nature of labor market risk is an important element of the analysis.

Figure 6 indicates that also in the US calibration of the two-region model there is little dispersion in the welfare effects over the life-cycle. Differently, the closed economy case can display a generational conflict, as younger and older individuals sometimes have welfare effects with opposite signs.

5.1.2 Welfare analysis accounting for transitional dynamics

A welfare analysis focusing on long-run outcomes neglects the short-run costs and benefits of changing the public debt. These can be important, as higher (lower) public debts are achieved with larger (smaller) transitional budget deficits, which increase (reduce) the households' disposable income. In the US calibration of the open economy model, I also study the welfare implications of public debt policies taking the transitional dynamics into account. At time $t = 0$, the world economy is in the initial steady-state, with $d_0 = 0.6$. I assume that public debt starts changing in period $t = 1$, and reaches its new long-run value in $T_d = 15$ periods, while at $T = 150$ the world economy is in the final steady-state. The government credibly commits to a linear evolution of the sequence of debts $\{d_t\}_{t=1}^{t=T}$, and the equilibrium dynamics are caused by this change in public debt policy.

The results are presented in Figure 8. The first panel confirms that incorporating the transitional dynamics

[37] It is worth reporting that the interest rate response to a 1% change in public debt lies in the 95% confidence intervals built around the estimates reported by Engen and Hubbard (2005) in Table 2, Ardagna, Caselli and Lane (2007) in Table 2, and Laubach (2009) in Table 4.

is important, as now increasing public debt considerably improves both welfare measures.[38] Panels 2 and 3 plot the paths of domestic aggregate consumption and the tax schedule variable for two new debt policies: one that eliminates public debt ($d = 0\%$) and one that doubles the public debt/GDP ratio ($d = 120\%$). The world economy goes through a long adjustment phase towards the new steady-state, needed for capital to slowly converge to its new long-run value, which is reached in less than 150 periods. The dynamics of domestic consumption are characterized by a jump when the new policy is implemented at time $t = 1$, followed first by a rapid adjustment while the debt dynamics are underway, and by a more gradual change thereafter. The dynamics of λ_t display "turnpike" behavior. In the case where the final steady-state is $d = 0\%$, disposable income must fall substantially when the new policy is implemented, and stays virtually pinned to this lower value until the debt dynamics are completed. This explains why social welfare is more negatively affected: older generations alive at time $t = 1$ experience a large cut in their disposable income, which immediately translates into lower consumption, leading to a sizable welfare loss as, unlike the newborns, they are not around long enough to eventually enjoy an improvement in their consumption. The dynamics of the specular case that increases d by the same amount, where the final steady-state is $d = 120\%$, are almost symmetric. Panels 4 and 5 show the implications of working with different time horizons (T_d) for the completion of the public debt dynamics, now equal to either 10 or 20 periods. Overall, the welfare effects are not heavily influenced by this margin because there are two opposite effects. For example, in the counterfactual that eliminates public debt, the dynamics of aggregate consumption display a sharper downward adjustment when the debt dynamics are completed in only 10 periods, implying larger short-run costs. At the same time, consumption recovers more quickly, as the economy starts reaping the long-run benefits earlier.

[**Figure 8 about here**]

A mechanical implication of the steady-state analysis is that, for a fixed level of taxes and public transfers, there is a negative long-run relationship between public debt and government expenditure, because the government's intertemporal budget constraint must be satisfied. For constant taxes, lower public debts imply lower interest payments, freeing up resources that can be used to increase government expenditure. However, the OECD data show that at the annual frequency there is no such negative relationship, as these data are contaminated by aggregate shocks and reflect transitional dynamics. In the short-run, the relationship between public debt and government expenditure features a positive correlation.[39] A possible interpretation of this

[38] Social welfare is now based on the aggregation of the value functions of all the agents that are alive when the new policy is implemented at time $t = 1$.

[39] Since public debts are very persistent, I specify a dynamic panel data model with one lag, estimated with the Arellano-Bond

finding is that, in order to obtain changes in the level of public debt, considerable fiscal adjustments need to be undertaken. Governments worried about the size of their public debts typically implement cuts to their public expenditures, entering a period of fiscal rigor. Similarly, public debts increase in periods of fiscal negligence.

The final version of the model with transitional dynamics imposes the estimated short-run relationship between public debt and public expenditure. For an announced sequence of public debts, I can invert the estimated relationship to find the applicable sequence of public expenditures $\{g_t\}_{t=1}^{t=T}$. The counterfactual analysis then mimics the implications of either cuts in public spending to reduce public debt, or increases in public spending that lead to a rise in public debt. In these counterfactuals I limit the lower bound of d to -50%, because in the OECD data the ranges of d and g are between 6% and 238%, and between 17% and 47%, respectively. Panel 6 in Figure 8 displays the welfare effects profiles for this case, which restores the optimality of public savings. In this case, the sizable short-run welfare costs are more than offset by the large welfare gains of reducing wasteful public expenditure.

6 Conclusions

Sizable long-run averages of public debt are prevalent in both small and large economies with access to international asset markets. The long-run results of either version of the two-region model considered in this paper cast doubts on the liquidity provision rationale for public debt, as equilibria with a negative quantity of debt afford large welfare gains. Quantitatively, in a model with two open economies, the muted response of the interest rate dampens some of the positive effects of public debt. Even though the data plotted in Figure 1 show a positive correlation between the size of GDP and the public debt/GDP ratio, in the subsample selecting the smaller economies there is substantial variation in d. Arguably, the results in this paper make the liquidity provision a quantitatively unimportant explanation for public debt in the long-run for several small –and open– countries.

As for Canada, the results of the welfare analysis showed that the long-run value of its public debt has been too high. The generous redistribution already in place, and the minimal equilibrium effects on the interest rate, make public debt an ineffective tool for the provision of resources to liquidity constrained individuals. These findings support the argument that the long-run value of public debt should not be increased, and a period of fiscal adjustments might be warranted. However, because of the large state space, in this version of the model the short-run welfare analysis is not computationally feasible, preventing an accurate assessment of the short-run welfare costs.

estimator. Morever, in order to capture non-linearities, in the set of regressors I include a quartic polynomial in government expenditure. The non-linear terms are important, because they prevent government expenditure from becoming negative unreasonably quickly when extrapolating its value with $d \ll 0$. The regression results and the plot of the estimated non-linear relationship between d and g are reported in Appendix F. In a previous version of the paper, I ran OLS and fixed-effects regressions on the OECD panel data, still obtaining a positive (conditional) correlation.

As for the US, I showed that the short-run costs of increased taxes can offset the long-run benefits, leading to welfare gains stemming from a public debt increase. However, this result rests on the assumption that wasteful public expenditure cannot be reduced. Using OECD panel data, I estimated the short-run correlation between public debt and public expenditure. Imposing this relationship in the model with transitional dynamics reversed the result. Cuts in public expenditure consistent with the empirical evidence can tame the temporary increase in taxes needed to reduce public debt, which restores the optimality of public savings. These considerations are especially important in the current times, as the detrimental effects of the COVID-19 pandemic have forced virtually every government to run extremely large public deficits, in some cases of unprecedented size. When the effects of the pandemic shock will be over, many economies will be left with public debts of extraordinary magnitude. It is reasonable to expect that this trend will be reverted, especially in light of the expected increase in interest rates triggered by a change in monetary policy. A meticulous analysis of the public sector, and the nature of its expenditures, will be key to identifying which government outlays can be classified as wasteful, hence targeted in future cuts, and which ones are productive and highly valuable. In this regard, the Greek public debt crisis that started unfolding in 2009 has shown that drastic and cursory "austerity" policies can lead to civil unrest, with enormous economic and social costs.

The findings also demonstrated that using closed economy models for the quantitative assessment of the welfare effects of fiscal policies can lead to erroneous conclusions, even for large economies, such as the US. Since inaccurate results can be caused by artificially strong GE effects, the models should be validated against the empirical response of prices. That said, in many applications, finding credible sources of exogenous variation is a considerable challenge.

In terms of future research, the model could be extended to take into consideration valuable government services coupled with political economy mechanisms, and the associated distortions. These seem to be potentially important explanations for large public debts, as advocated by Battaglini (2011), Alesina and Passalacqua (2016), and Yared (2019). This identifies a promising avenue for future equilibrium quantitative research, along the lines of Krusell, Quadrini and Rios-Rull (1997), Azzimonti, Battaglini and Coate (2016) and Azzimonti and Yared (2019).

Parameter	*Value*	*Target*
Set Externally - Home Economy (CA):		
Model Period	*Year*	*Frequency of T-4 Tax Return Data*
J *- Maximum Age*	75	*Certain death at age* 101
J_R *- Maximum Working Age*	41	*Retirement at age* 66
π_j^d *- Death probabilities*	-	*Canadian Life Tables, average for 1990-2007, see Appendix F*
γ *- Rate of Technological Change*	0.02	*Average growth rate*, PWT9.0
α *- Capital share*	0.345	*Labor share of output* $= 65.5\%$, PWT9.0
θ *- Risk Aversion*	1.5	*Elasticity of Intertemporal Substitution* $= 0.75$
ρ_ε *- Persistence of the AR(1) income shocks*	0.540	Baker and Solon (2003)
σ_ε *- S.d. of the AR(1) income shocks*	0.358	Baker and Solon (2003)
σ_s *- S.d. of the Random Walk income shocks*	0.0837	Baker and Solon (2003)
σ_{f_α} *- S.d. of the level fixed effect*	0.366	Baker and Solon (2003)
σ_{f_β} *- S.d. of the slope fixed effect*	0.0095	Baker and Solon (2003)
$\sigma_{f_\alpha,f_\beta}$ *- Covariance between fixed effects*	-0.0031	Baker and Solon (2003)
m_{f_α} *- Mean of the level fixed effect*	10.102	*Intercept of a Log-earnings regression on LFS data*
m_{f_β} *- Mean of the slope fixed effect*	0.0131	*Gradient of a Log-earnings regression on LFS data*
g *- Government Consumption share of GDP*	0.214	$G/GDP = 21.4\%$, *average for 1990-2007*
τ_c *- Consumption Tax Rate*	0.124	*Average GST and PST*, Mendoza, Razin and Tesar (1994)
ψ *- Income Tax Progressivity*	0.09	*Estimate from Log-net income regressions on SCF data*
ϕ_R *- Pension Replacement Rate*	0.479	*Average pension/average earnings ratio*, OECD (2015)
d *- Government Long-Run Debt/GDP ratio*	0.685	*Debt/GDP* $= 68.5\%$, *average for 1990-2007*
$\underline{a}$ *- Borrowing limit*	0	*Households cannot borrow*
Set Externally - Rest-of-the-World:		
$\pi_j^{d,RoW}$ *- Death probabilities*	-	*Cross-country average Life Expectancy, see Appendix F*
θ^{RoW} *- Risk Aversion*	1.5	*Elasticity of Intertemporal Substitution* $= 0.75$
ε_j^{RoW} *- Labor efficiency units*	-	*Cross-country average of productivity profiles, see text*
ρ_ε^{RoW} *- Persistence of the AR(1) income shocks*	0.949	*Cross-country average*
σ_ε^{RoW} *- S.d. of the AR(1) income shocks*	0.152	*Cross-country average*
ϕ_R^{RoW} *- Pension Replacement Rate*	0.610	*Cross-country average*, OECD (2015)
α^{RoW} *- Capital share*	0.364	*Cross-country average implied by US calibration*
Calibrated in Equilibrium:		
β *- Discount factor*	1.031	$K/Y = 2.745$, PWT9.0
β^{RoW} *- Discount factor*	1.098	$r = 3.0\%$, *Average rate of return*
δ *- Capital depreciation rate*	0.008	$I/Y = 25.5\%$, PWT9.0
δ^{RoW} *- Capital depreciation rate*	0.005	$I^{RoW}/Y^{RoW} = 26.1\%$, PWT9.0
$\bar{\mu}^{RoW}$ *- Measure of RoW*	30.71	*Home GDP/Global GDP* $= 2.6\%$, PWT9.0

Table 1: Calibration of the Two-Region Model, Canadian case. Parameters Set Externally for Canada and the RoW, and Calibrated in Equilibrium.

	(1)	(2)	(3)	(4)	(5)	(6)
	gbr_{10}	$fwr_{5,10}$	$fwr_{5,10}$	$fwr_{5,10}$	$fwr_{5,10}$	$fwr_{5,10}$
Net debt/GDP	-0.070	0.023	-0.023	0.023	0.027	0.036
	(0.110)	(0.524)	(0.562)	(0.525)	(0.430)	(0.068)
GDP growth	0.493	-0.758	0.600	-0.735	-0.895	-2.118
	(0.500)	(0.235)	(0.408)	(0.233)	(0.111)	(0.000)
Oil price (CAD)	-0.039	0.009	-0.046	0.018	0.012	0.003
	(0.347)	(0.673)	(0.267)	(0.479)	(0.683)	(0.832)
Equity premium	80.31	41.36	98.57	39.03	38.66	55.19
	(0.002)	(0.034)	(0.000)	(0.063)	(0.043)	(0.000)
BoC gov. assets	-0.035	0.064	0.239	0.156	0.143	0.118
	(0.785)	(0.418)	(0.068)	(0.485)	(0.507)	(0.022)
Time trend				-0.059	-0.042	
				(0.667)	(0.728)	
AR(1)					-0.188	
Constant	10.64	-5.02	-8.49	0.17	-1.87	6.75
	(0.079)	(0.352)	(0.129)	(0.990)	(0.875)	-
Debt projection	NO	YES	NO	YES	YES	YES
N	72	49	72	49	48	47
R^2	0.355	0.152	0.284	0.156	0.202	0.309

Table 2: Model validation. *Notes*: The dependent variable, always expressed in percentage points, is the current annualized real rate of return of 10-year government bonds (gbr_{10}), or the real 5-year-ahead 10-year forward rate ($fwr_{5,10}$). The Net debt/GDP regressor is either its current value, or its 5-year-ahead projection computed using a linear model with four autoregressive lags. (1)-(5) list the coefficients from reduced-form equations, while (6) is the estimated cointegrating equation from a VEC model (the coefficient on $fwr_{5,10}$ is normalized to 1). Huber-White standard errors in all regressions, but (5) and (6). In (6), the R^2 is the average across the six equations in the VEC model. p-values in parentheses.

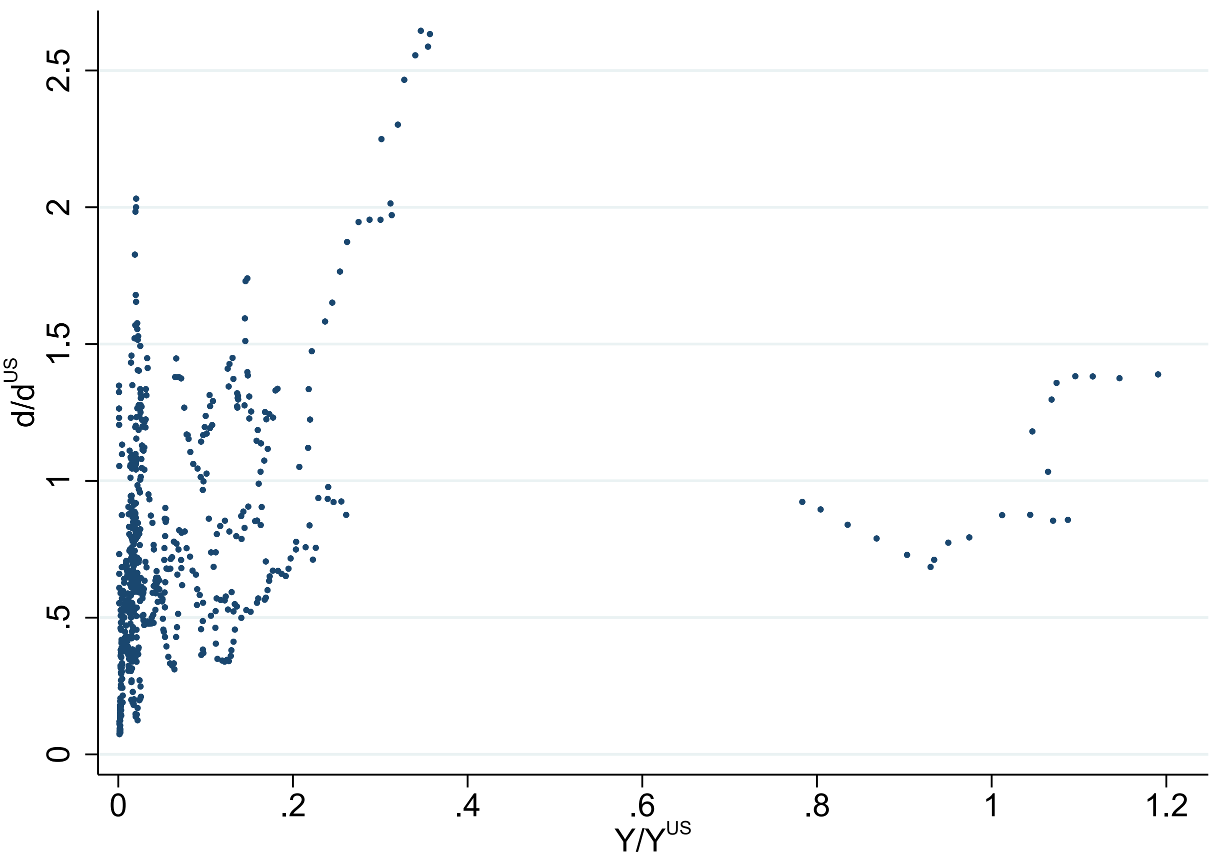

Figure 1: The Liquidity Channel Vs. the OECD Evidence, OECD data 1995-2015. The $x-$axis refers to the GDP of a specific country in a given year (Y) relative to the average GDP for the US over the sample period (Y^{US}). The $y-$axis refers to the public debt/GDP ratio (d) of a specific country in a given year relative to the average public debt/GDP ratio for the US over the sample period (d^{US}).

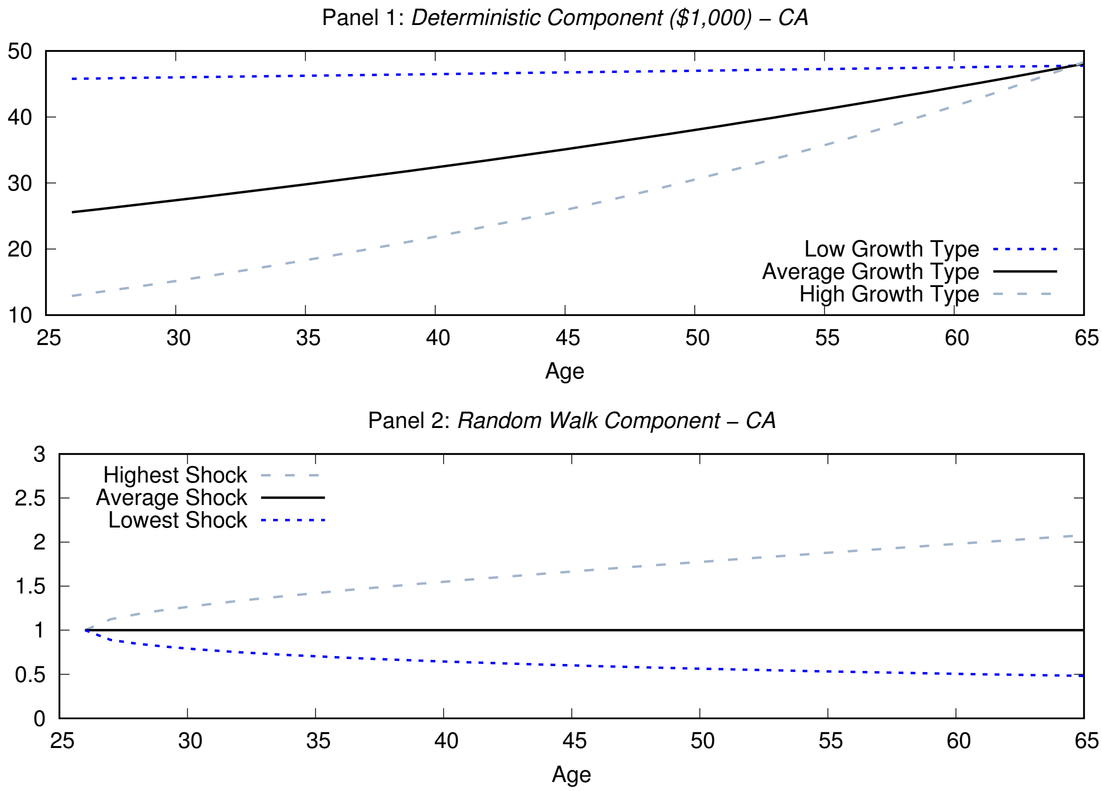

Figure 2: Pre-tax Earnings Dynamics over the Life-Cycle for the Canadian economy. The top panel plots the individual effects component of labor earnings (in thousand of 2010 CAD$), while the bottom panel the stochastic random walk component.

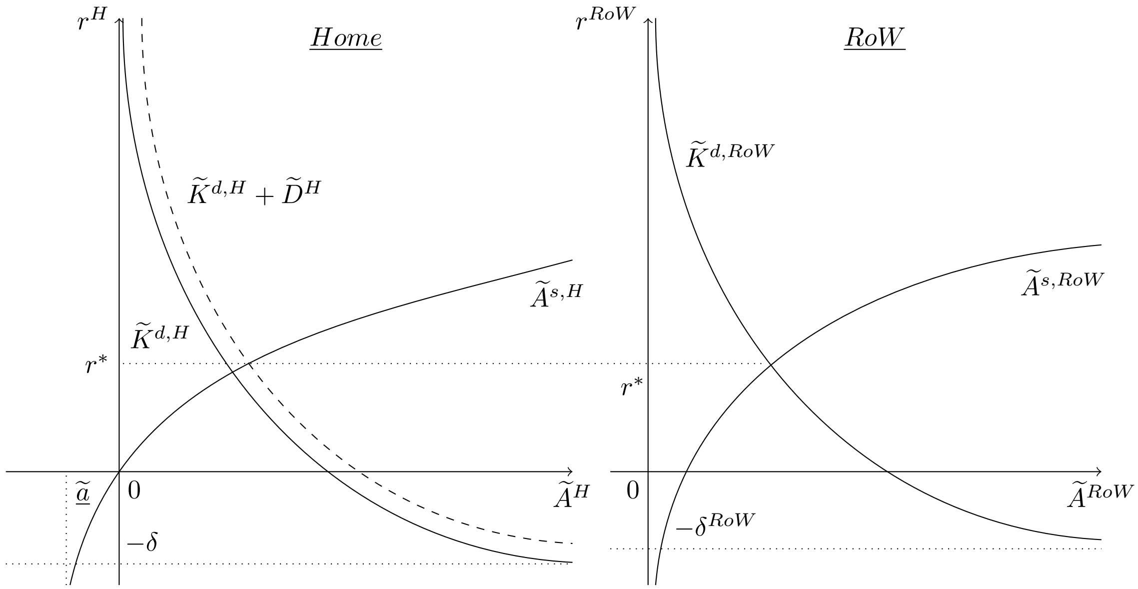

Figure 3: Equilibrium determination in the two-region economy. H stands for the Home economy and RoW for the Rest of the World.

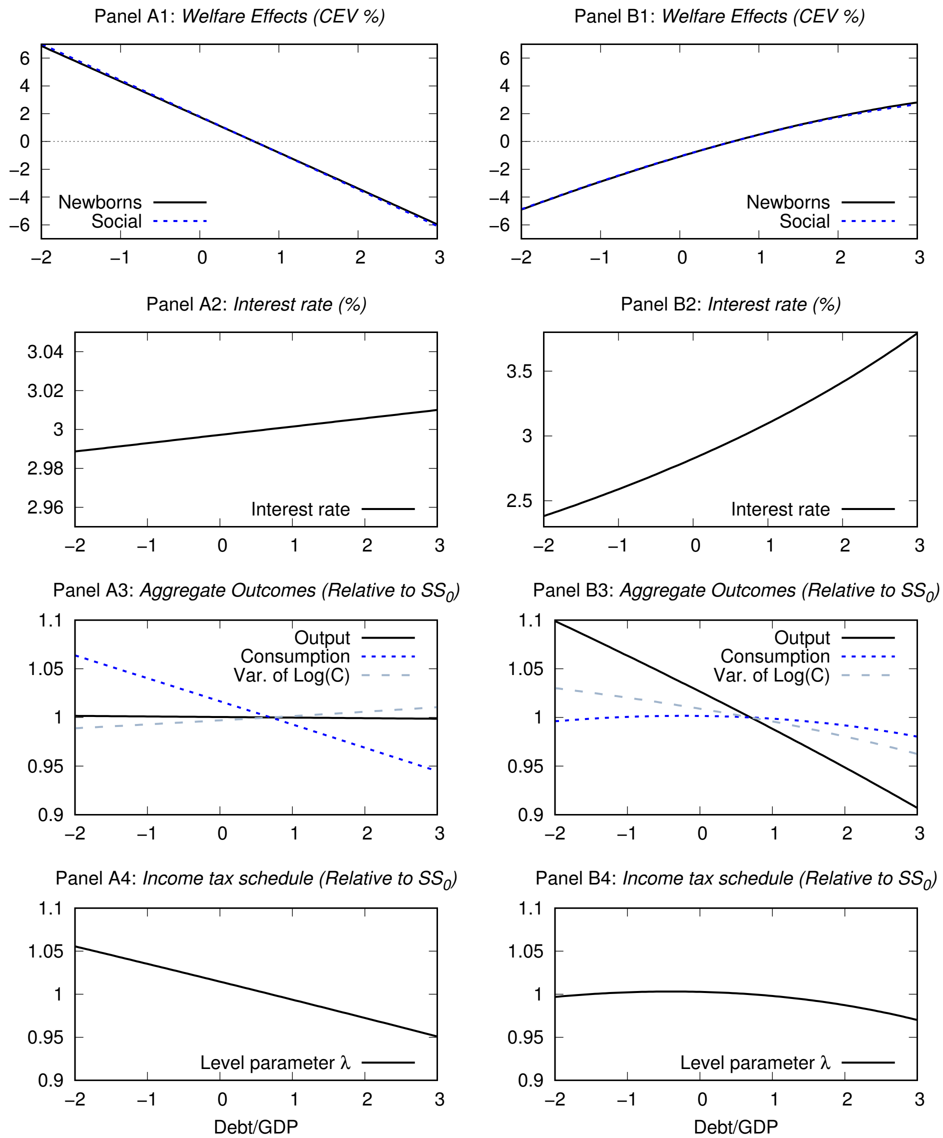

Figure 4: Equilibrium outcomes for the two-region economy (left panels, A) and the closed economy (right panels, B), Canadian case. *Notes*: The welfare effects and the interest rate are displayed as percentages. The aggregate outcomes and the parameter λ of the after-tax income schedule are relative to the initial steady state (SS_0).

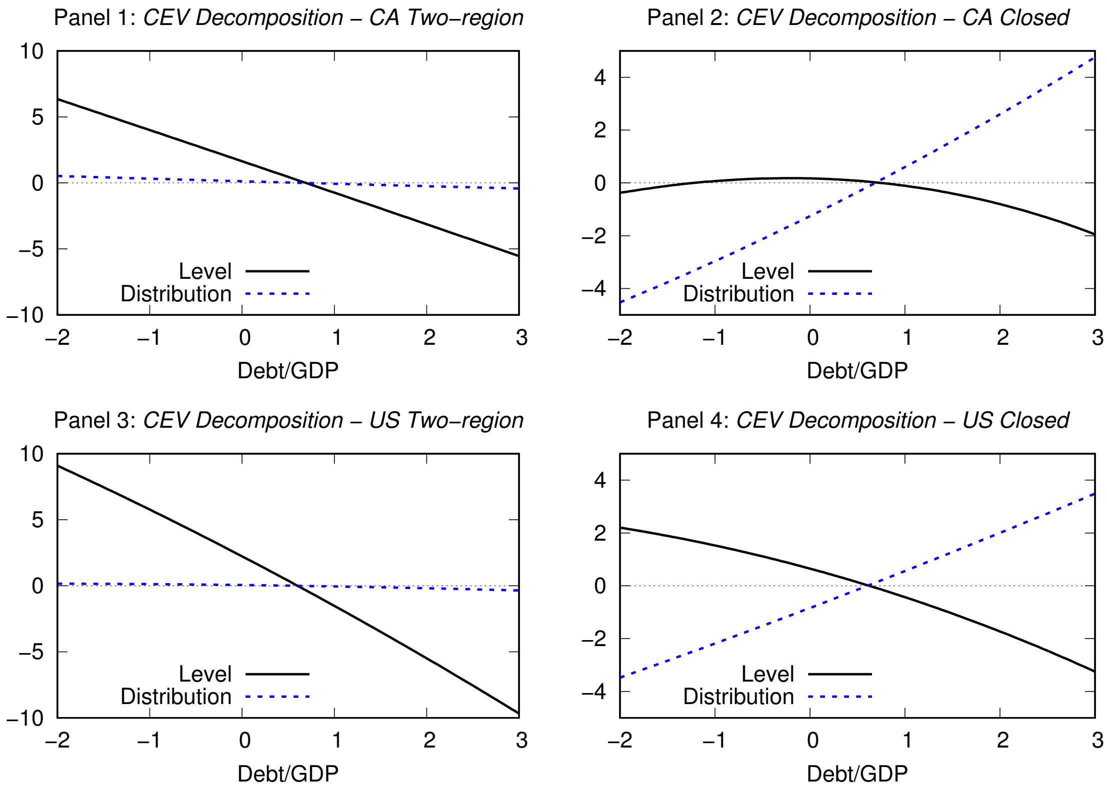

Figure 5: Welfare Effects (CEV) Decomposition for Canada (top panels, labeled CA) and the US (bottom panels). The welfare effects are displayed as percentages.

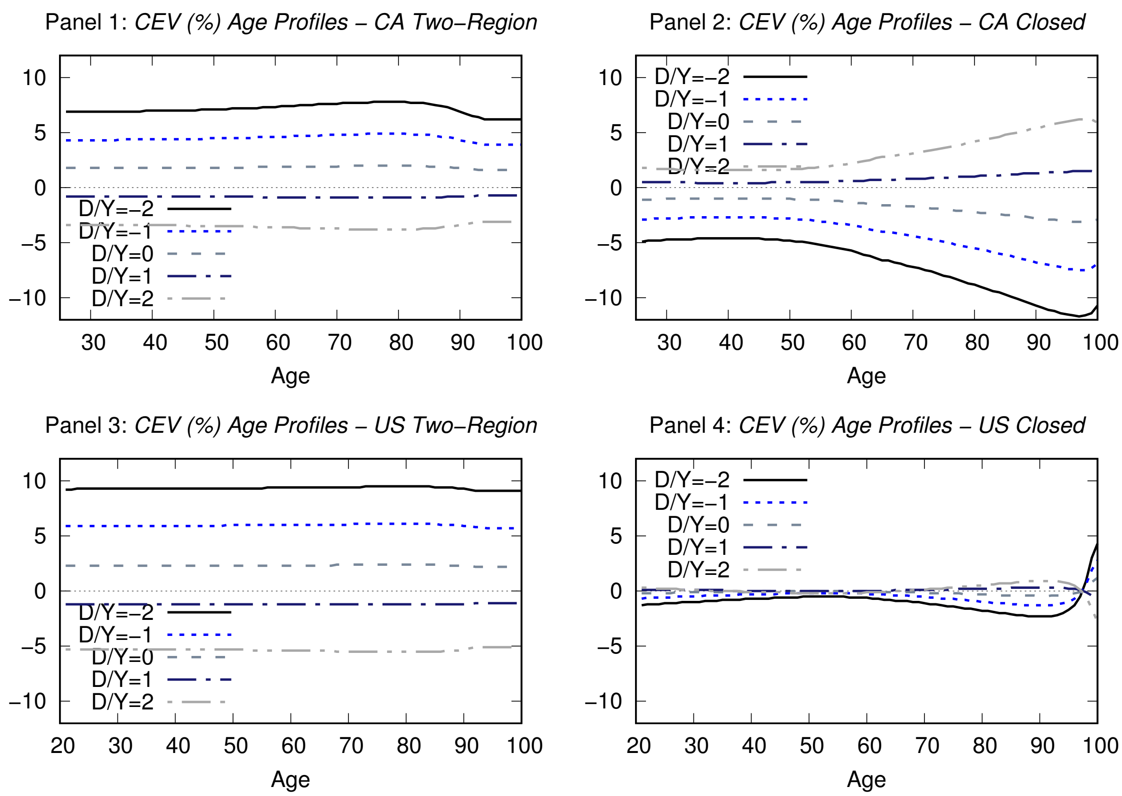

Figure 6: Age-Dependent Welfare Effects. The welfare effects are displayed as percentages.

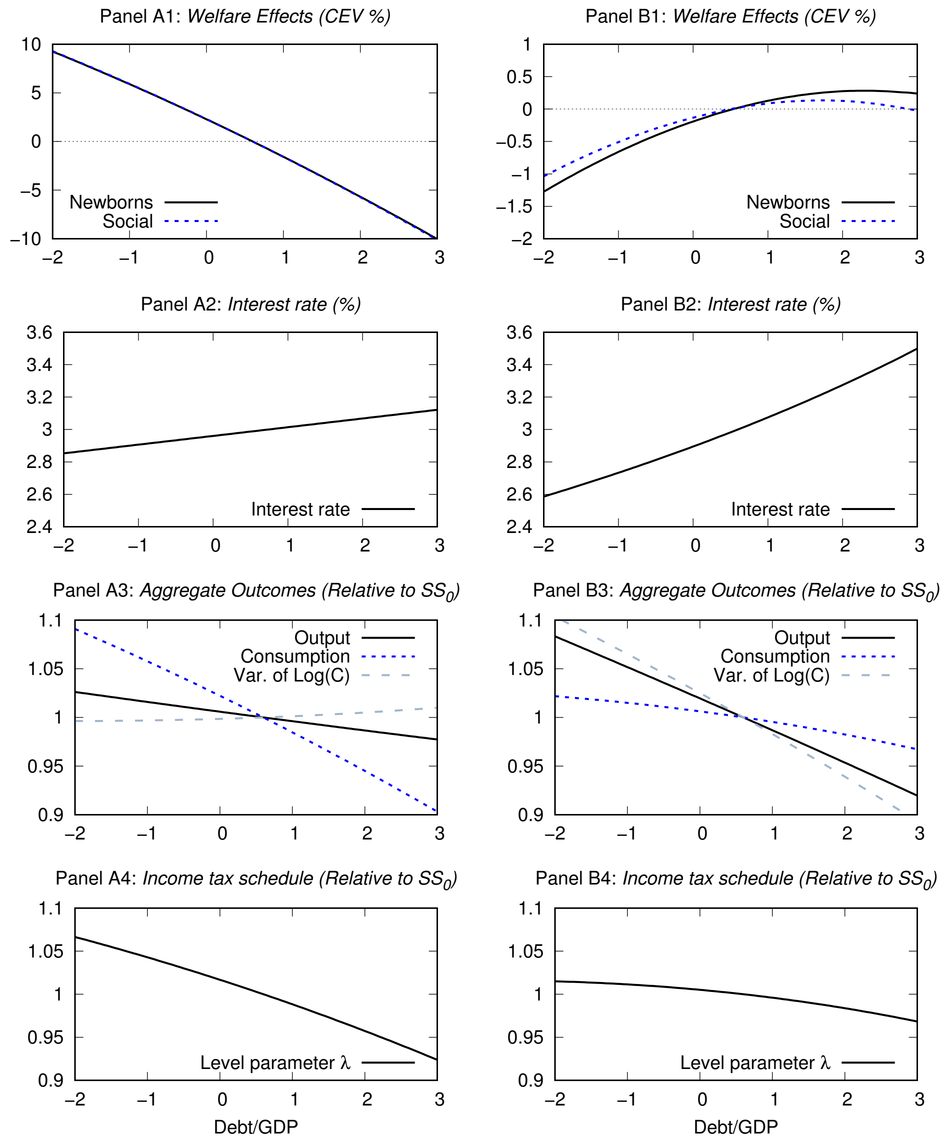

Figure 7: Equilibrium outcomes for the two-region economy (left panels, A) and the closed economy (right panels, B), US case. *Notes*: The welfare effects and the interest rate are displayed as percentages. The aggregate outcomes and the parameter λ of the after-tax income schedule are relative to the initial steady state (SS_0).

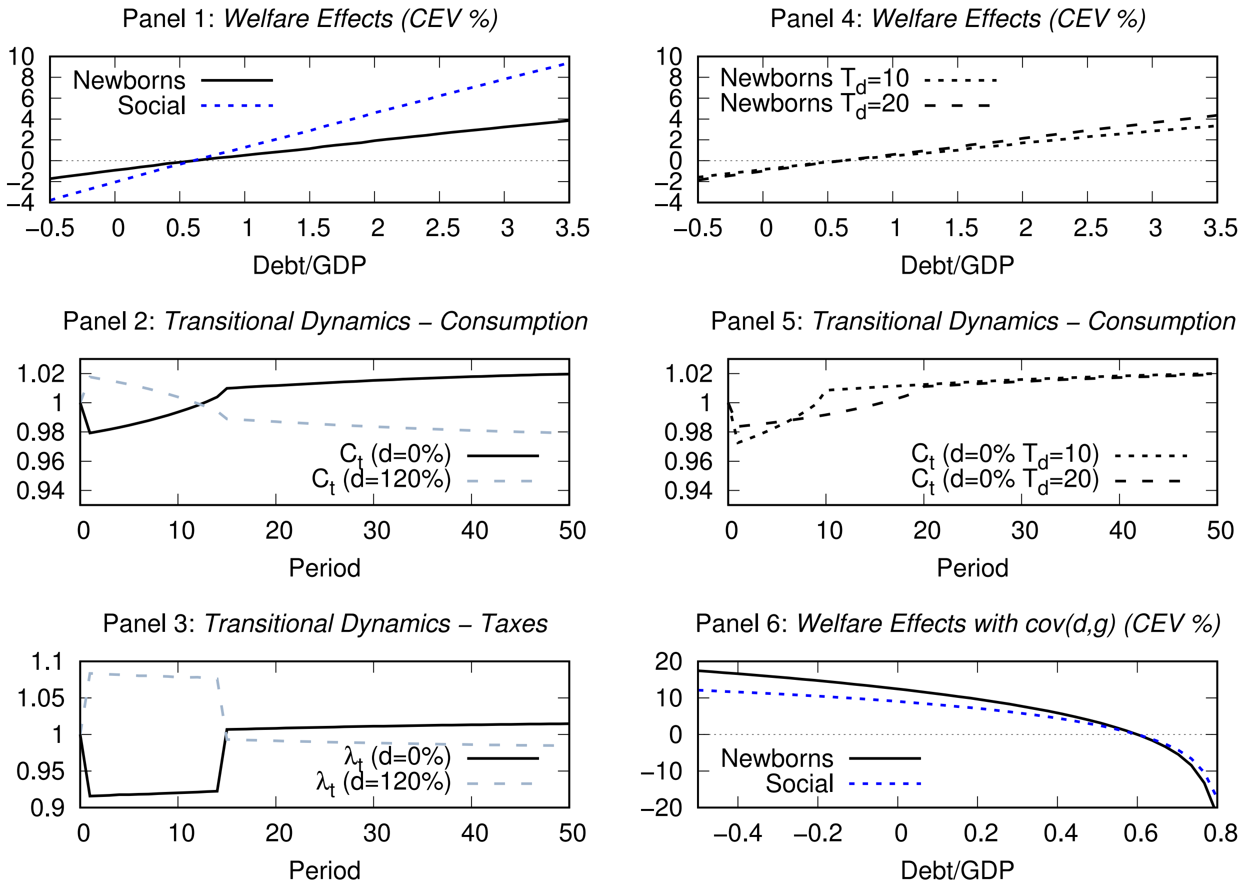

Figure 8: Equilibrium outcomes with transitional dynamics for the two-region economy, US case. *Notes*: The welfare effects and the interest rate are displayed as percentages. The aggregate outcomes and the parameter λ_t of the after-tax income schedule are relative to the initial steady state (SS_0). T_d stands for the number of periods needed for d to reach its new long-run value. Panels 2 and 5 plot the transitional dynamics of two specific cases, both starting from $d_0 = 0.6$, and transitioning to either $d = 0$ or $d = 1.2$.

Appendix A - Calibration for the US case

This Appendix discusses the calibration when the home economy represents the US. I begin by discussing the American parameters, then I move to show the ones for the RoW region.

[Table 3 about here]

A.1 Home economy parameters

Table 3 reports the full list of calibrated parameters with their values and associated empirical targets. The parameters in the top part of the Table are the ones that apply to the US, and that can be set externally, without solving the model.

Demographics: The following parameters are related to the demographic aspects: J_R, J, π_j^d. Agents become economically active at age 21, as the PSID data allow to include younger individuals. They retire at age $J_R = 66$, and they can live up to $J = 101$ years. The survival probabilities π_j^d are obtained from the US Life Tables.[40]

Technology: The US labor share is computed from the PWT9.0, and its long-run average implies a capital share $\alpha = 37.1\%$. The growth rate of technological change γ is chosen to match the average long-run growth of per-capita income for the US economy, leading to $\gamma = 1.7\%$.[41]

Preferences: The relative risk aversion is set to $\theta = 1.50$, which matches an elasticity of intertemporal substitution of 0.75, a common value in the literature.

Income process and pensions: The estimation of the statistical model for the US labor earnings dynamics is based on Panel Study on Income Dynamics (PSID) data.[42] Since the estimates in Guvenen (2009) rely on hourly wages, derived from yearly earnings and hours worked, I implement the same minimum distance estimator to obtain the parameters estimates for yearly earnings. The point estimates are quite similar to the results in Guvenen (2009). $\rho_\varepsilon = 0.743$ and $\sigma_\varepsilon = 0.223$ are my estimates of the autocorrelation and s.d. of the innovations for the persistent (AR1) component. The heterogeneity at labor market entry is captured by the dispersion of the two individual effects: $\sigma_{f_\alpha} = 0.305$ is the s.d. of the level individual effect, while $\sigma_{f_\beta} = 0.023$

[40] The two top panels in Figure 14 in Appendix F plot the survival probabilities over the life-cycle together with the resulting age distribution.

[41] The RoW economy grows at the same rate as the home economy, and the world economy grows along a BGP.

[42] To achieve tractability of the model with transitional dynamics, I need to reduce the number of continuous state variables. The random walk component is not considered in the US calibration, an assumption also made by Guvenen (2009).

is the s.d. of the slope individual effect. Finally, the correlation index between the two individual effects is negative and high, being -0.747.[43] The average of the level fixed effect is $m_{f_\alpha} = 10.26$ and the average of the slope fixed effect is $m_{f_\beta} = 0.0147$. A noteworthy difference in my estimates relates to the magnitude of the negative correlation between the level and slope individual effects. This is larger for yearly earnings, as it is well known that more productive employees work longer hours, and this difference widens during the life-cycle. The American and Canadian income dynamics for the average workers have similar characteristics, while they differ quite substantially for the workers at the top and bottom of the distribution. The US estimates feature a dispersion in growth rates that is considerably larger: the top 10% enjoys annual growth rates above 4%, while the bottom 10% experiences a 1% reduction. These differences partially explain the discrepancy in results for the closed economy versions of the model.

The pension replacement rate is taken from OECD (2015), and its value is $\phi_R = 0.448$.

Taxes and government: The consumption tax rate is $\tau_c = 0.05$, which is the estimate reported by Mendoza, Razin and Tesar (1994). The degree of progressivity parameter is $\psi = 0.18$, which is taken from Heathcote, Storesletten and Violante (2017).[44]

The wasteful government consumption and the public debt/GDP ratio are borrowed from Aiyagari and McGrattan (1998), their values being $g = 0.2$ and $d = 0.6$.

Asset markets: The borrowing limit is set to $\underline{a} = 0$, such that households cannot borrow.

A.2 Rest of the world parameters

The middle portion of Table 3 reports the full list of the calibrated RoW parameters with their values and associated empirical targets.

Demographics: Just like for the Canadian calibration, the survival rates are constructed using OECD data. The resulting profiles match the 1960-2007 cross-country average in the life expectancy at birth (72 years) and at age 65 (14 years).[45]

Preferences: Since there is no evidence that US and foreign households differ systematically in their attitudes towards risk, I set the relative risk aversion parameter equal in the two economies, $\theta = \theta^{RoW} = 1.50$.

[43] Figure 11 in Appendix F plots the joint bi-normal density implied by these estimates.

[44] This estimate might be noisy, because of the imputations required to obtain a measure of after-tax income. Since this value implies that the US fiscal system is more progressive than the Canadian one, I also calibrated the model using an ad-hoc value of $\psi = 0.07$. The related results are very similar, and are plotted in Figure 16 in Appendix F.

[45] See Figure 15 in Appendix F.

Income process and labor input: The life-cycle profile of the deterministic component of labor productivity is obtained by computing the cross-country average of the profiles reported in Lagakos et al. (2018), excluding the US and including Canada. The persistence and the standard deviation of the autocorrelated income shocks are obtained averaging the available estimates in the literature on income dynamics. This procedure gives $\rho_{\varepsilon}^{RoW} = 0.941$ and $\sigma_{\varepsilon}^{RoW} = 0.158$. By construction, in the RoW each country's labor input averages to 1.[46]

Asset markets: The borrowing limit is set to $\underline{a}^{RoW} = 0$, such that foreign households cannot borrow.

A.3 Parameters set in equilibrium

Six parameters do not have a direct empirical counterpart, and need to be set in equilibrium. They are reported in the bottom part of Table 3, and they are selected to match the statistics that follow.

From the PWT9.0, I compute the long-run average of the capital/GDP ratio for the US, which is $K^{US}/Y^{US} = 2.84$. Also from the PWT9.0, I compute the ratio between the US GDP and the World GDP, which is equal to $Y^{US}/(Y^{US} + Y^{RoW}) = 31\%$. For comparability with the Canadian calibration, I target an equilibrium interest rate $r^* = 3\%$ and a current account that is approximately zero. The full calibration, matching the targets above in the initial steady state, leads to: $\beta = 1.038$, $\beta^{RoW} = 1.092$, $\alpha^{RoW} = 0.357$, and $\mu_{RoW} = 2.77$.

The last two parameters, δ and δ^{RoW}, are chosen to match the investment share of GDP in the two regions. Solving the relevant formulas for the calibration targets $I^{US}/Y^{US} = 22.5\%$ and $I^{RoW}/Y^{RoW} = 24.8\%$ gets: $\delta = [r^*(I^{US}/Y^{US}) - \alpha\gamma]/[\alpha - (I^{US}/Y^{US})] = 0.003$ and $\delta^{RoW} = [r^*(I^{RoW}/Y^{RoW}) - \alpha^{RoW}\gamma]/[\alpha^{RoW} - (I^{RoW}/Y^{RoW})] = 0.012$.

[46] See Figure 15 in Appendix F.

Parameter	*Value*	*Target*
Set Externally - Home Economy (US):		
Model Period	*Year*	*Frequency of PSID Data*
J *- Maximum Age*	81	*Certain death at age* 101
J_R *- Maximum Working Age*	46	*Retirement at age* 66
π_j^d *- Death probabilities*	-	*US Life Tables*
γ *- Rate of Technological Change*	0.017	*Average growth rate*, PWT9.0
α *- Capital share*	0.371	*Labor share of output* $= 62.9\%$, PWT9.0
θ *- Risk Aversion*	1.5	*Elasticity of Intertemporal Substitution* $= 0.75$
ρ_ε *- Persistence of the AR(1) income shocks*	0.743	*Minimum distance estimate on PSID data*
σ_ε *- S.d. of the AR(1) income shocks*	0.223	*Minimum distance estimate on PSID data*
σ_{f_α} *- S.d. of the level fixed effect*	0.305	*Minimum distance estimate on PSID data*
σ_{f_β} *- S.d. of the slope fixed effect*	0.023	*Minimum distance estimate on PSID data*
$\sigma_{f_\alpha,f_\beta}$ *- Correlation between fixed effects*	-0.747	*Minimum distance estimate on PSID data*
m_{f_α} *- Mean of the level fixed effect*	10.26	*Intercept of a Log-earnings regression on PSID data*
m_{f_β} *- Mean of the slope fixed effect*	0.0147	*Gradient of a Log-earnings regression on PSID data*
g *- Government Consumption share of GDP*	0.2	$G/GDP = 20\%$, Aiyagari and McGrattan (1998)
τ_c *- Consumption Tax Rate*	0.05	Mendoza, Razin and Tesar (1994)
ψ *- Income Tax Progressivity*	0.18	Heathcote, Storesletten and Violante (2017)
ϕ_R *- Pension Replacement Rate*	0.448	*Average pension/average earnings ratio*, OECD (2015)
d *- Government Long-Run Debt/GDP ratio*	0.60	$Debt/GDP = 60\%$, Aiyagari and McGrattan (1998)
$\underline{a}$ *- Borrowing limit*	0	*Households cannot borrow*
Set Externally - Rest-of-the-World:		
$\pi_j^{d,RoW}$ *- Death probabilities*	-	*Cross-country average Life Expectancy, see Appendix F*
θ^{RoW} *- Risk Aversion*	1.5	*Elasticity of Intertemporal Substitution* $= 0.75$
ε_j^{RoW} *- Labor efficiency units*	-	*Cross-country average of productivity profiles, see text*
ρ_ε^{RoW} *- Persistence of the AR(1) income shocks*	0.941	*Cross-country average*
σ_ε^{RoW} *- S.d. of the AR(1) income shocks*	0.158	*Cross-country average*
ϕ_R^{RoW} *- Pension Replacement Rate*	0.657	*Cross-country average*, OECD (2015)
Calibrated in Equilibrium:		
β *- Discount factor*	1.038	$K^{US}/Y^{US} = 2.840$, PWT9.0
β^{RoW} *- Discount factor*	1.092	$r = 3.0\%$, *Average rate of return*
α^{RoW} *- Capital share*	0.357	*Current Account* ≈ 0
δ *- Capital depreciation rate*	0.003	$I^{US}/Y^{US} = 22.5\%$, PWT9.0
δ^{RoW} *- Capital depreciation rate*	0.012	$I^{RoW}/Y^{RoW} = 24.8\%$, PWT9.0
$\bar{\mu}^{RoW}$ *- Measure of RoW*	2.77	*Home GDP/Global GDP*$= 31\%$, PWT9.0

Table 3: Calibration of the Two-Region Model, US case. Parameters Set Externally for the US and the RoW, and Calibrated in Equilibrium.

References

Aiyagari, R., and McGrattan, E. (1998). "The optimum quantity of debt," *Journal of Monetary Economics*, Vol. 42, 447-469.

Alesina, A., and Passalacqua, A. (2016). "The Political Economy of Government Debt," *Handbook of Macroeconomics*, Vol. 2, 2599-2651.

Ardagna, S., Caselli, F., and Lane, T. (2007). "Fiscal Discipline and the Cost of Public Debt Service: Some Estimates for OECD Countries," *The B.E. Journal of Macroeconomics (Topics)*, Vol. 7, Article 28.

Azzimonti, M., Battaglini, M., and Coate, S. (2016). "The costs and benefits of balanced budget rules: Lessons from a political economy model of fiscal policy," *Journal of Public Economics*, Vol. 136, 45-61.

Azzimonti, M., and Yared, P. (2019). "The Optimal Public and Private Provision of Safe Assets," *Journal of Monetary Economics*, forthcoming.

Baker, M. (1997). "Growth-Rate Heterogeneity and the Covariance Structure of Life-Cycle Earnings," *Journal of Labor Economics*, Vol. 15, 338-375.

Baker, M., and Solon, G. (2003). "Earnings Dynamics and Inequality among Canadian Men, 1976–1992: Evidence from Longitudinal Income Tax Records," *Journal of Labor Economics*, Vol. 21, 289-321.

Battaglini, M. (2011). "The Political Economy of public debt," *Annual Review of Economics*, Vol. 3, 161-189.

Brzozowski, M., Gervais, M., Klein, P., and Suzuki, M. (2010). "Consumption, income, and wealth inequality in Canada," *Review of Economic Dynamics*, Vol. 13, 52-75.

Chatterjee, S., Gibson, J., and Rioja, F. (2017). "Optimal public debt redux," *Journal of Economic Dynamics and Control*, Vol. 83, 162-167.

Chatterjee, S., Gibson, J., and Rioja, F. (2018). "Public investment, debt, and welfare: A quantitative analysis," *Journal of Macroeconomics*, Vol. 56, 204-217.

Cohen, D., and Follette, G. (2003). "Forecasting Exogenous Fiscal Variables in the United States," *FEDS Working Papers*, 2003-59.

D'Erasmo, P., and Mendoza, E. (2016). "Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default," *Journal of the European Economics Association*, Vol. 14, 7-44.

Desbonnet, A., and Weitzenblum, C. (2012). “Why do governments end up with debt? short-run effects matter,” *Economic Inquiry*, Vol. 50, 905-919.

Desbonnet, A., and Kankanamge, C. (2015). “Public Debt and Aggregate Risk,” *Macroeconomic Dynamics*, Vol. 21, 1996-2032.

Domeij, D., and Floden, M. (2010). “Inequality trends in Sweden 1978-2004,” *Review of Economic Dynamics*, Vol. 13, 179-208.

Engen, E., and Hubbard, G. (2005). “Federal Government Debt and Interest Rates,” *NBER Macroeconomics Annual*, Vol. 19, 83-138.

Floden, M. (2001). “The Effectiveness of Government Debt and Transfers as Insurance,” *Journal of Monetary Economics*, Vol. 48, 81-108.

Gorodnichenko, Y., Peter, K. S., and Stolyarov, D. (2010). “Inequality and volatility moderation in Russia: Evidence from micro-level panel data on consumption and income,” *Review of Economic Dynamics*, Vol. 13, 209-237.

Gurkaynak, R., Sack, B., and Wright, J. (2007). “The US Treasury yield curve: 1961 to the present,” *Journal of Monetary Economics*, Vol. 54, 2291-2304.

Guvenen, M.F. (2007). “Learning Your Earning: Are Labor Income Shocks Really Very Persistent?,” *American Economic Review*, Vol. 97, 687-712.

Guvenen, M.F. (2009). “An Empirical Investigation of Labor Income Processes,” *Review of Economic Dynamics*, Vol. 12, 58-79.

Haider, S. (2001). “Earnings Instability and Earnings Inequality of Males in the United States: 1967–1991,” *Journal of Labor Economics*, Vol. 19, 799-836.

Hamilton, J. (2018). “Why You Should Never Use the Hodrick-Prescott Filter,” *Review of Economics and Statistics*, Vol. 100, 831-843.

Hause, J. (1980). “The Fine Structure of Earnings and the On-the-Job Training Hypothesis,” *Econometrica*, Vol. 48, 1013-1030.

Heathcote, J., Storesletten, K., and Violante, G. (2017). “Optimal Tax Progressivity: An Analytical Framework,” *Quarterly Journal of Economics*, Vol. 132, 1-62.

Huggett, M. (1996). "Wealth distribution in life-cycle economies," *Journal of Monetary Economics*, Vol. 38, 469-494.

Krusell, P., Quadrini, V., and Rios-Rull, J.V. (1997). "Politico-economic equilibrium and economic growth," *Journal of Economic Dynamics and Control*, Vol. 21, 243-272.

Lagakos, D., Moll, B., Porzio, T., Qian, N., and Schoellman, T. (2018). "Life Cycle Wage Growth across Countries," *Journal of Political Economy*, Vol. 126, 797-849.

Laubach, T. (2009). "New Evidence on the Interest Rate Effects of Budget Deficits and Debt," *Journal of the European Economic Association*, Vol. 7, 858-885.

Le Blanc, J., and Georgarakos, D. (2013). "How Risky is their Income? Labor Income Processes in Europe," *Mimeo.*

Lettau, M., and Ludvigson, S. (2001). "Consumption, Aggregate Wealth, and Expected Stock Returns," *Journal of Finance*, Vol. 56, 815-849.

Lise, J., Sudo, N., Suzuki, M., Yamada, K., and Yamada, T. (2014). "Wage, income and consumption inequality in Japan, 1981–2008: From boom to lost decades," *Review of Economic Dynamics*, Vol. 17, 582-612.

Mendoza, E., Razin, A., and Tesar, L. (1994). "Effective tax rates in macroeconomics Cross-country estimates of tax rates on factor incomes and consumption," *Journal of Monetary Economics*, Vol. 34, 297-323.

Nelson, C., and Siegel, A. (1987). "Parsimonious Modeling of Yield Curves," *Journal of Business*, Vol. 60, 473–489.

OECD. (2015). *Pensions at a Glance 2015: OECD and G20 indicators,* OECD Publishing, Paris.

Parliamentary Budget Officer. (2022). *History* (last accessed 2 January 2022):
`https://www.pbo-dpb.gc.ca/en/history--historique`

Peterman, W., and Sager, E. (2018). "Optimal public debt with Life Cycle Motives," *FEDS Working Paper* 2018-028.

Rios-Rull, J.V. (1996). "Life-Cycle Economies and Aggregate Fluctuations," *Review of Economic Studies*, Vol. 63, pp. 465-489.

Rios-Rull, J.V. (1994). "On the Quantitative Importance of Market Completeness," *Journal of Monetary Economics*, Vol. 34, pp. 463-496.

Rohrs, S., and Winter, C. (2015). “Public versus private provision of liquidity: Is there a trade-off?,” *Journal of Economic Dynamics and Control*, Vol. 53, 314-339.

Rohrs, S., and Winter, C. (2017). “Reducing Government Debt in the Presence of Inequality,” *Journal of Economic Dynamics and Control*, Vol. 82, 1-20.

Yared, P. (2019). “Rising Government Debt: Causes and Solutions for a Decades-Old Trend,” *Journal of Economic Perspectives*, Vol. 33, 115-140.

Yetman, J. (2017). “The evolution of inflation expectations in Canada and the US,” *Canadian Journal of Economics*, Vol. 50, 711-737.

Appendix B - Public Debts and Public Expenditures in the Long Run for Canada and the US (Not for publication)

Figure 2 includes three panels plotting the time series of the Canadian and American public debts and public expenditures. The top panel displays the longest available series for the gross public debts, expressed as a fraction of GDP. Both public debts have been quite volatile, with the public debt/GDP ratio being anywhere between 20% and 120%. As observed in several other countries, public debt spiked during WWII, and quickly fell to less extreme values shortly thereafter. In this dataset, the long-run average of the public debt/GDP ratio in the post WWII era (1952-2012) is 68.5% for Canada and 55.9% for the US. The middle panel relies on Canadian (Cansim) data, which measure the net public debt/GDP ratio.[47] These data cover a considerably shorter period, but are quarterly and disaggregated by level of government. In Canada, during the last decade, the federal public debt as a share of GDP has been declining, but it has been replaced by public debt accumulated by other levels of government. As for the reference time period, the quantitative model is constrained by data limitations, especially with respect to the income dynamics. It is worth reporting that the Cansim data in the 1990-2007 period, namely pre-dating the great recession and closer to the income data, show a public debt/GDP ratio of 68.5%, which is identical to the statistic reported above. For these reasons, I am going to consider this figure as the long-run value of the Canadian public debt/GDP ratio.

The bottom panel displays the time series of the public expenditure/GDP ratio. Abstracting from the short-run fluctuations, a noteworthy feature of this series is the emergence of two long-run levels of public expenditure. The first regime covers the years from the early 1960's until the early 1980's, where a public expenditure/GDP ratio around 25% (22%) was the norm for Canada (US). Starting in the 2000's, this ratio has settled around a lower value of approximately 20% (19%) for Canada (US). This indicates that changes in public debt are positively correlated with changes in public expenditure, a fact that is established empirically, using OECD data. In the 1990-2007 period, the average public expenditure/GDP ratio was 21.4% and 19.1%.

[47] The net public debt is obtained subtracting financial assets from the gross public debt as, in principle, financial assets could be used to pay back part of the debt. In order of importance, the Government of Canada's financial assets are mainly composed of: taxes receivable, investment in enterprise Crown corporations, foreign exchange accounts, and cash and/or cash equivalents.

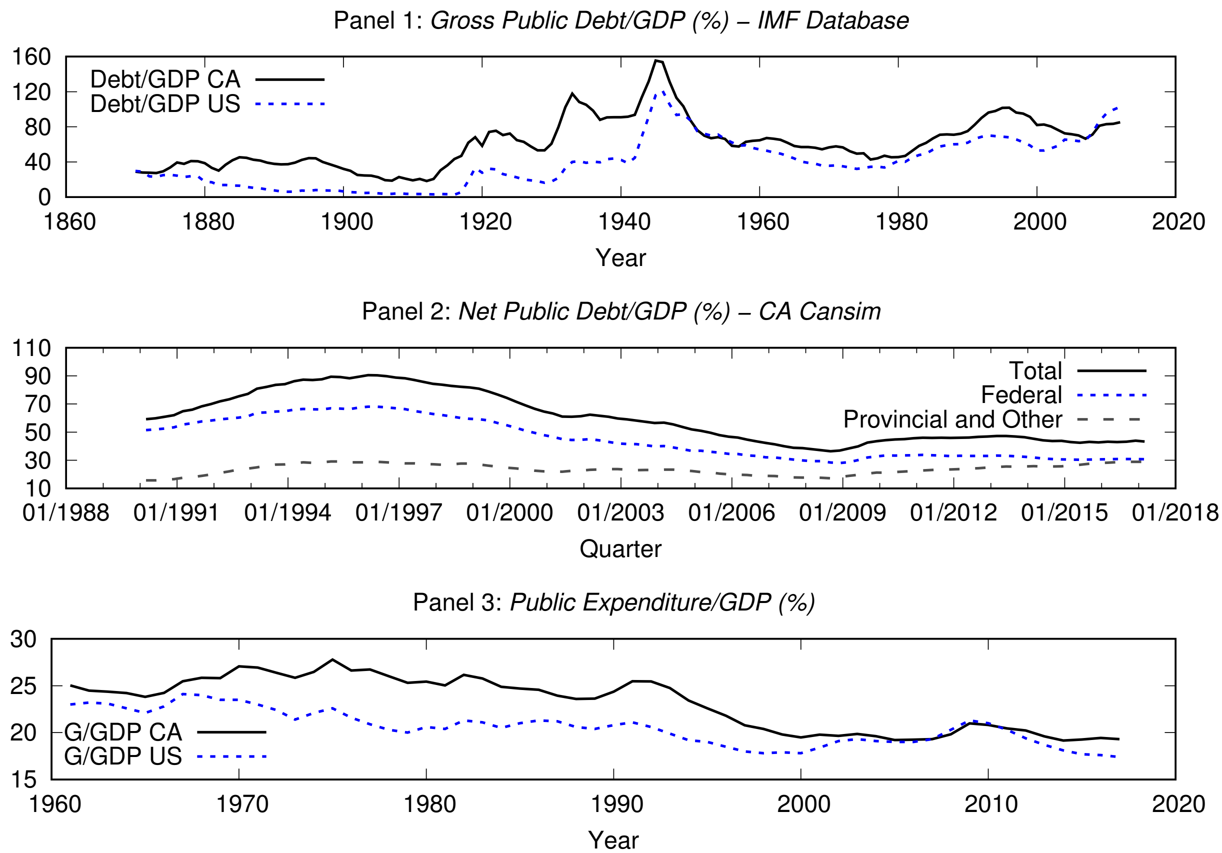

Figure 9: Time series evolution of Gross public debt/GDP in Canada and the US (top panel), Net public debt/GDP in Canada (middle panel), and public expenditure/GDP in Canada and the US (bottom panel). The Gross public debt/GDP series were compiled by Abbas, Belhocine, El-Ganainy and Horton (2011), and are included in a database maintained by the IMF. The Net public debt/GDP series is from Cansim data, Table 0378-0125. The Public Expenditure/GDP is computed from FRED Economic Data.

References

Abbas, A., Belhocine, N., El-Ganainy, A., and Horton, M. (2011). "Historical Patterns and Dynamics of public debt–Evidence From a New Database," *IMF Economic Review*, Vol. 59, 717-742.

Appendix C - The OLG Model and its Recursive Representation (Not for publication)

Stationary Equilibrium

The economy is a production economy with endogenous asset distributions, where a government collects taxes to finance an exogenously given stream of public expenditures, interest payments on the accumulated public debt, a pension scheme, and redistribution across the households. The model is an extension of the Huggett (1996) economy, appropriately modified to allow for several sources of heterogeneity in labor income. Beside the workers, in each of the two regions, there is a representative firm that produces the final output relying on a Cobb-Douglas production function on aggregate labor and aggregate capital.

Time is discrete. The economy is populated by finitely lived agents facing age-dependent death probabilities, π_j^d. Age is denoted with j and there are J overlapping generations, each consisting of a continuum of agents. At age J_R all agents that are still alive become retirees. Regarding the mortality risk, I consider two different environments. In the benchmark case, there are accidental bequests denoted by $\widetilde{b}$, which are distributed uniformly to all the surviving agents (and the newborns). In an alternative case, there is a perfectly competitive annuity market, where the agents share their mortality risk. It follows that in the second formulation there are no accidental bequests, as the average asset holdings of the individuals that die are zero.

Preferences: Agents' preferences are assumed to be time-separable and represented by the expected utility function $\mathbb{E}U(.)$. Agents' utility is defined over stochastic consumption $\{\widetilde{c}_j\}_{j=1}^J$ sequences: their aim is to choose how much to consume $(\widetilde{c}_j)$ and how much to save in an interest bearing asset $(\widetilde{a}_{j+1})$ in each period of their lives, in order to maximize their objective function. The agents' problem can be defined as:

$$\max_{\{\widetilde{c}_j,\widetilde{a}_{j+1}\}_{j=1}^J} \mathbb{E}_1 U(\widetilde{c}_1,...,\widetilde{c}_J) = \max_{\{\widetilde{c}_j,\widetilde{a}_{j+1}\}_{j=1}^J} \mathbb{E}_1 \sum_{j=1}^{J} \beta^{j-1}(1+\gamma)^{(1-\theta)t} \left[\prod_{h=1}^{j} \left(1-\pi_h^d\right)\right] \frac{\widetilde{c}_j^{1-\theta}}{1-\theta}$$

where $\mathbb{E}_1$ represents the expectation operator over the idiosyncratic sequences of shocks, and $\beta > 0$ is the subjective discount factor. The labor supply is fixed and equal to the time endowment multiplied by the value of the efficiency units. Notice that the economy is supposed to be on a BGP already at time $-J$, and $t = j-1$.

Endowments: Agents differ in their labor endowments ϵ. In the home economy, there are five channels that contribute to the determination of the total efficiency units that the workers supply in the labor market, $\epsilon_{j,s,\varepsilon,f_\alpha,f_\beta}$. First, there is an age component j, which interacts with one of the fixed effects, namely the growth rate of individual productivity. Second, there are two correlated fixed effects, f_α and f_β, that are jointly normally distributed and are observed at labor market entry. Finally, there are two time-varying shocks, one being a stochastic component ε that follows a stationary $AR(1)$ process with normally distributed shocks ν^{ε}, and the other one being a stochastic component s that follows a random walk process RW with normally distributed shocks ν^{s}. The income dynamics of the life-cycle for a generic individual i are represented as follows:

$$\begin{cases} \epsilon_{i,j} = f_{\alpha,i} + f_{\beta,i} * (j-1) + \varepsilon_{i,j} + s_{i,j} \\ \varepsilon_{i,j} = \rho_{\varepsilon}\varepsilon_{i,j-1} + \nu^{\varepsilon}_{i,j} \\ s_{i,j} = s_{i,j-1} + \nu^{s}_{i,j} \\ (f_{\alpha,i}, f_{\beta,i}) \overset{iid}{\sim} N(m_{f_\alpha}, m_{f_\beta}; \sigma^2_{f_\alpha}, \sigma^2_{f_\beta}, \sigma_{f_\alpha,f_\beta}), \nu^{\varepsilon}_{i,j} \overset{iid}{\sim} N(0,\sigma^2_{\varepsilon}), \nu^{s}_{i,j} \overset{iid}{\sim} N(0,\sigma^2_{s}) \end{cases}$$

where $\varepsilon_{i,0} = s_{i,0} = 0$, $\forall i$. Since the regression analysis is performed on labor earnings in logs, the total efficiency units a worker is endowed with is the exponential transformation of the product of all the components described above. It follows that labor earnings are $y^w_j = \widetilde{w} \exp\left(\epsilon_{j,s,\varepsilon,f_\alpha,f_\beta}\right)$. After the common retirement age J_R, the labor endowment drops to zero, and the agents receive a pension $\widetilde{\overline{y}}_R\left(f_\alpha, f_\beta\right)$ paid for with the contributions of the agents that are active in the labor market. The pension is a fixed replacement rate ϕ_R of the type-dependent average labor earnings, and agents pay proportional taxes (τ_R) to contribute to the balanced-budget pension scheme. Income taxes are levied to finance interest payments on public debt $r\widetilde{D}$, public expenditure $\widetilde{G}$, and redistribution. The fiscal system is progressive and characterized by two parameters: λ, which mainly captures a proportional effect, and ψ, which mainly captures the progressivity of the system. In the benchmark version of the model, agents cannot insure against their mortality risk. On average, agents die with positive wealth, which is distributed evenly to all the surviving agents. Newborns enter the economy with a zero asset endowment and with the average realization of the stochastic component of labor earnings, which is normalized to 1.

Now I first define the problem of the agents in its recursive representation, then I provide a formal definition of the equilibrium concept used in this model, the recursive competitive equilibrium. The individual state variables are: age $j \in \mathcal{J} = \{1, ..., J\}$, the level individual effect $f_\alpha \in \mathcal{F}_\alpha = \{f_{\alpha,\min}, ..., f_{\alpha,\max}\}$, the slope individual effect $f_\beta \in \mathcal{F}_\beta = \{f_{\beta,\min}, ..., f_{\beta,\max}\}$, the random walk component of the stochastic labor productivity $s \in \mathcal{S} = \{s_{\min}, ..., \overline{s}, ..., s_{\max}\}$, the auto-regressive component of the stochastic labor productivity $\varepsilon \in \mathcal{E} = \{\varepsilon_{\min}, ..., \overline{\varepsilon}, ..., \varepsilon_{\max}\}$ and asset holdings $\widetilde{a} \in \mathcal{A} = \left[\underline{\widetilde{a}}, \widetilde{\overline{a}}\right]$. Notice that $\widetilde{a}$ is discretized using an exponentially growing grid with 201 points. ε is discretized with the Rouwenhorst method, using a 11-state Markov chain. s is discretized using a 5-state grid whose elements are functions of the standard deviation, and are changing over the working age, as the variance of the random walk increases linearly. This method is an extension for a life-cycle set-up of the popular Tauchen/Hussey procedure. The transition function of the auto-regressive component of the labor endowment shock is represented by the matrix $\Pi(\varepsilon', \varepsilon) = [\pi(v, z)]$, where each element $\pi(v, z)$ is defined as $\pi(v, z) = \Pr\{\varepsilon_{j+1} = z | \varepsilon_j = v\}$, $v, z \in \mathcal{E}$. In every period the exogenous labor endowments are given by $\epsilon_{j,s,\varepsilon,f_\alpha,f_\beta} = f_\alpha \times (f_\beta * (j-1)) \times \varepsilon \times s$. The stationary distribution of working-age agents is denoted by $\mu_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta)$ while that of retirees by $\mu_j^R(\widetilde{a}, f_\alpha, f_\beta)$. Φ_j denotes the share of each cohort j in the total population. These satisfy the recursion $\Phi_{j+1} = \left(\frac{1-\pi_j^d}{1+g_n}\right)\Phi_j$, and are normalized to add up to 1.

In the foreign economy, the individual state variables are: age $j \in \mathcal{J} = \{1, ..., J\}$, the auto-regressive component of the stochastic labor productivity $\varepsilon^{RoW} \in \mathcal{E}^{RoW} = \{\varepsilon_{\min}^{RoW}, ..., \overline{\varepsilon}^{RoW}, ..., \varepsilon_{\max}^{RoW}\}$ and asset holdings $\widetilde{a}^{RoW} \in \mathcal{A}^{RoW} = \left[\underline{\widetilde{a}}^{RoW}, \overline{\widetilde{a}}^{RoW}\right]$.

C.1 - Problem of the Agents

The model is solved backwards, starting from the terminal age J and with the assumption that the terminal utility value is zero, i.e. $V_{J+1} = 0$.

Problem of the retirees. The value function of a retired agent in the home economy whose current age is j and whose current de-trended asset holdings are equal to $\widetilde{a}$ is denoted with $V_j^R(\widetilde{a}, f_\alpha, f_\beta)$. The problem of these agents can be represented as follows:

$$V_j^R(\widetilde{a}, f_\alpha, f_\beta) = \max_{\widetilde{c},\widetilde{a}'}\left\{\frac{\widetilde{c}^{1-\theta}}{1-\theta} + \beta(1+\gamma)^{(1-\theta)}\left(1-\pi_j^d\right)V_{j+1}^R(\widetilde{a}', f_\alpha, f_\beta)\right\} \tag{1}$$

$$\begin{aligned} &s.t.\\ &(1+\tau_c)\widetilde{c} + (1+\gamma)\widetilde{a}' = \lambda\left[r\widetilde{a} + \widetilde{\overline{y}}_R(f_\alpha, f_\beta)\right]^{1-\psi} + \widetilde{a} + \widetilde{b}\\ &\widetilde{c} \geq 0, \quad \widetilde{a}' \geq 0 \end{aligned}$$

The value functions are indexed also by the two individual effects, as the pension benefits are a fixed replacement rate of the average labor earnings. Because of the related computational burden, the transitory shocks do not enter in the computation of the pension, as they would require keeping track of another continuous state variable. Instead, for the pension computation I consider the average labor earnings arising from the permanent components of the labor efficiency units and the unconditional means of the other stochastic elements. Since the AR1 component is mean reverting, and the dispersion of the RW component is not too wide, this simplification will not affect heavily the results. In the budget constraint notice the presence of the after-tax income, $\lambda\left[r\widetilde{a} + \widetilde{\overline{y}}_R(f_\alpha, f_\beta)\right]^{1-\psi}$, where taxable income includes capital income $r\widetilde{a}$ and the pension payment

$\widetilde{\widetilde{y}}(.)$. Wealth and (possibly) the accidental bequest $\widetilde{b}$ are not taxed. In the model with annuities, there are no accidental bequest, but the gross interest rate (net of taxes) is adjusted by the average survival probability $(1-\pi^d)$.

Problem of the workers. The value function of a working-age agent whose current age is j, whose current asset holdings are equal to $\widetilde{a}$, whose current efficiency units shock is ε and whose individual effects are f_α and f_β is denoted with $V_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta)$. The problem of these agents can be represented as follows:

$$V_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta) = \max_{\widetilde{c}, \widetilde{a}'} \left\{ \frac{\widetilde{c}^{1-\theta}}{1-\theta} + \beta(1+\gamma)^{(1-\theta)}\left(1-\pi_j^d\right) \sum_{\varepsilon', s'} \pi(\varepsilon', \varepsilon)\, \pi(s', s)\, V_{j+1}(\widetilde{a}', s', \varepsilon', f_\alpha, f_\beta) \right\} \tag{2}$$

$$s.t.$$

$$(1+\tau_c)\widetilde{c} + (1+\gamma)\widetilde{a}' = \lambda\left[r\widetilde{a} + (1-\tau_R)\widetilde{w}\epsilon_{j,s,\varepsilon,f_\alpha,f_\beta}\right]^{1-\psi} + \widetilde{a} + \widetilde{b}$$

$$\widetilde{a}_0 = 0, \quad \widetilde{c} \geq 0, \quad \widetilde{a}' \geq \underline{\widetilde{a}}$$

Non-retired agents have to set optimally their consumption/savings plans. They enjoy utility from consumption, and face some uncertain events in the future. In the next period they can still be alive, and with joint probability $\pi(\varepsilon', \varepsilon)\, \pi(s', s)$ they transit from their current efficiency units shocks (ε, s) to a new pair (ε', s'). Working-age agents pay a proportional tax τ_R on their labor earnings to finance the pension scheme. They also pay income taxes, and their after-tax income is $\lambda\left[r\widetilde{a} + (1-\tau_R)\widetilde{w}\epsilon_{j,s,\varepsilon,f_\alpha,f_\beta}\right]^{1-\psi}$. Finally, they are born with the average shock $(\overline{\varepsilon}, s=0)$, with no wealth and are subject to an exogenous borrowing constraint, $\underline{\widetilde{a}} = 0$.

Problem of the foreign retirees. The value function of a retired agent in the RoW whose current age is j and whose current asset holdings are equal to $\widetilde{a}$ is denoted with $V_j^{R,RoW}(\widetilde{a})$. The problem of these agents can be represented as follows:

$$V_j^{R,RoW}\left(\widetilde{a}\right)=\max_{\widetilde{c},\widetilde{a}'}\left\{\frac{\widetilde{c}^{1-\theta}}{1-\theta}+\beta^{RoW}(1+\gamma)^{(1-\theta)}\left(1-\pi_j^{d,RoW}\right)V_{j+1}^{R,RoW}\left(\widetilde{a}'\right)\right\} \tag{3}$$

$$\begin{aligned}
&s.t.\\
&\widetilde{c}+(1+\gamma)\widetilde{a}'=(1+r)\widetilde{a}+\widetilde{\widetilde{y}}_R^{RoW}+\widetilde{b}^{RoW}\\
&\widetilde{c}\geq 0,\quad \widetilde{a}'\geq 0
\end{aligned}$$

Problem of the foreign workers. The value function of a foreign agent whose whose current age is j, whose current asset holdings are equal to $\widetilde{a}$, whose current efficiency units shock is ε is denoted with $V_j^{RoW}\left(\widetilde{a},\varepsilon\right)$. The problem of these agents can be represented as follows:

$$V_j^{RoW}\left(\widetilde{a},\varepsilon\right)=\max_{\widetilde{c},\widetilde{a}'}\left\{\frac{\widetilde{c}^{1-\theta}}{1-\theta}+\beta^{RoW}(1+\gamma)^{(1-\theta)}\left(1-\pi_j^{d,RoW}\right)\sum_{\varepsilon'}\pi^{RoW}\left(\varepsilon',\varepsilon\right)V_{j+1}^{RoW}\left(\widetilde{a}',\varepsilon'\right)\right\} \tag{4}$$

$$\begin{aligned}
&s.t.\\
&\widetilde{c}+(1+\gamma)\widetilde{a}'=(1+r)\,\widetilde{a}+(1-\tau_R^{RoW})\widetilde{w}^{RoW}\epsilon_{j,\varepsilon}^{RoW}+\widetilde{b}^{RoW}\\
&\widetilde{c}\geq 0,\quad \widetilde{a}'\geq \underline{a}^{RoW}=0
\end{aligned}$$

Foreign agents have to set optimally their consumption/savings plans. They enjoy utility from consumption, and face some uncertain events in the future. In the next period, with probability $\pi^{RoW}\left(\varepsilon',\varepsilon\right)$ they transit from their current income shock ε to the value ε'. Working-age agents pay a proportional tax τ_R^{RoW} on their labor earnings to finance the pension scheme.

C.2 - Recursive Stationary Equilibrium

Definition 1 *For given public policies* $\left\{\lambda,\psi,\tau_c,\widetilde{\phi_R},\widetilde{\phi_R}^{RoW},\widetilde{G}/\widetilde{Y},\widetilde{D}/\widetilde{Y}\right\}$ *a recursive stationary equilibrium is a set of home decision rules,* $\left\{c_j\left(\widetilde{a},s,\varepsilon,f_\alpha,f_\beta\right),a_j'\left(\widetilde{a},s,\varepsilon,f_\alpha,f_\beta\right)\right\}_{j=1}^{J_R-1}$ *and* $\left\{c_j^R\left(\widetilde{a},f_\alpha,f_\beta\right),a_j^{R\prime}\left(\widetilde{a},f_\alpha,f_\beta\right)\right\}_{j=J_R}^{J}$, *foreign decision rules,* $\left\{c_j^{RoW}\left(\widetilde{a},\varepsilon\right),a_j^{RoW\prime}\left(\widetilde{a},\varepsilon\right)\right\}_{j=1}^{J_R-1}$ *and* $\left\{c_j^{R,RoW}\left(\widetilde{a}\right),a_j^{R,RoW\prime}\left(\widetilde{a}\right)\right\}_{j=J_R}^{J}$, *home value functions,* $\left\{V_j\left(\widetilde{a},s,\varepsilon,f_\alpha,f_\beta\right)\right\}_{j=1}^{J_R-1}$ *and* $\left\{V_j^R\left(\widetilde{a},f_\alpha,f_\beta\right)\right\}_{j=J_R}^{J}$, *foreign value functions,* $\left\{V_j^{RoW}\left(\widetilde{a}\right)\right\}_{j=1}^{J_R-1}$, *and* $\left\{V_j^{R,RoW}\left(\widetilde{a}\right)\right\}_{j=J_R}^{J}$, *prices* $\left\{r,\widetilde{w},\widetilde{w}_{RoW}\right\}$, *a set of home stationary distributions,* $\left\{\mu_j\left(\widetilde{a},s,\varepsilon,f_\alpha,f_\beta\right)\right\}_{j=1}^{J_R-1}$ *and* $\left\{\mu_j^R\left(\widetilde{a},f_\alpha,f_\beta\right)\right\}_{j=J_R}^{J}$, *and a set of foreign stationary distributions,* $\left\{\mu_j^{RoW}\left(\widetilde{a}\right)\right\}_{j=1}^{J_R-1}$, *and* $\left\{\mu_j^{R,RoW}\left(\widetilde{a}\right)\right\}_{j=J_R}^{J}$, *such that:*

- Given relative prices $\{r, \widetilde{w}\}$, taxes and pension benefits $\widetilde{\widetilde{y}}_R(f_\alpha, f_\beta)$, the individual home policy functions $\left\{c_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta), a'_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta)\right\}_{j=1}^{J_R-1}, \left\{c_j^R(\widetilde{a}, f_\alpha, f_\beta), a_j^{R\prime}(\widetilde{a}, f_\alpha, f_\beta)\right\}_{j=J_R}^{J}$ solve the household problems (1) and (2), and $\{V_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta)\}_{j=1}^{J_R-1}, \left\{V_j^R(\widetilde{a}, f_\alpha, f_\beta)\right\}_{j=J_R}^{J}$ are the associated value functions.

- Given relative prices $\{r, \widetilde{w}_{RoW}\}$, pension taxes and benefits $\widetilde{\widetilde{y}}_R^{RoW}$, the individual foreign policy functions $\left\{c_j^{RoW}(\widetilde{a}, \varepsilon), a_j^{RoW\prime}(\widetilde{a}, \varepsilon)\right\}_{j=1}^{J_R-1}$ and $\left\{c_j^{R,RoW}(\widetilde{a}), a_j^{R,RoW\prime}(\widetilde{a})\right\}_{j=J_R}^{J}$, solve the household problems (3) and (4), and $\left\{V_j^{RoW}(\widetilde{a})\right\}_{j=1}^{J_R-1}, \left\{V_j^{R,RoW}(\widetilde{a})\right\}_{j=J_R}^{J}$, are the associated value functions.

- Given relative prices $\{r, \widetilde{w}\}$, $\widetilde{K}/L$ solves the domestic firm's problem.

- Given relative prices $\{r, \widetilde{w}_{RoW}\}$, $\widetilde{K}_{RoW}/L_{RoW}$ solves the foreign firm's problem.

- The labor markets are in equilibrium, and the labor inputs L and L_{RoW} correspond to the total supply of labor efficiency units in both regions

$$L = \sum_{j=1}^{J_R-1} \Phi_j \int_{\mathcal{A}\times\mathcal{S}\times\mathcal{E}\times\mathcal{F}_\alpha\times\mathcal{F}_\beta} \epsilon_{j,s,\varepsilon,f_\alpha,f_\beta} \, d\mu_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta)$$

$$L^{RoW} = \int_{\mathcal{Q}} \sum_{j=1}^{J_R-1} \Phi_j^{RoW} \int_{\mathcal{A}\times\mathcal{E}} \epsilon_{j,\varepsilon}^{RoW} \, d\mu_j^{RoW}(\widetilde{a}, \varepsilon) \, dq$$

- The global asset market clears

$$\begin{aligned}\widetilde{K} + \widetilde{K}^{RoW} + \widetilde{D} = &\sum_{j=1}^{J_R-1} \Phi_j \int_{\mathcal{A}\times\mathcal{S}\times\mathcal{E}\times\mathcal{F}_\alpha\times\mathcal{F}_\beta} a'_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta) \, d\mu_j(.) + \sum_{j=J_R}^{J} \Phi_j \int_{\mathcal{A}\times\mathcal{F}_\alpha\times\mathcal{F}_\beta} a_j^{R\prime}(\widetilde{a}, f_\alpha, f_\beta) \, d\mu_j^R(.) + \\ &\int_{\mathcal{Q}} \sum_{j=1}^{J_R-1} \Phi_j^{RoW} \int_{\mathcal{A}\times\mathcal{E}} a_j^{RoW\prime}(\widetilde{a}, \varepsilon) \, d\mu_j^{RoW}(.) \, dq + \int_{\mathcal{Q}} \sum_{j=J_R}^{J} \Phi_j^{RoW} \int_{\mathcal{A}} a_j^{R,RoW\prime}(\widetilde{a}) \, d\mu_j^{R,RoW}(.) \, dq\end{aligned}$$

- The two pension systems are budget-balanced, and the payroll taxes $\left(\tau_R, \tau_R^{RoW}\right)$ are set to satisfy this condition.

- The government's intertemporal budget constraint is satisfied, that is total tax revenues $\widetilde{TX}$ (obtained from taxing labor income, asset income, pensions, and consumption) minus total government expenditure (the sum of government purchases $\widetilde{G}$ and public transfers $\widetilde{TR}$) are equal to the growth-adjusted interest costs of public debt, namely public debt is constant over time

$$(r-\gamma)\frac{\widetilde{D}}{\widetilde{Y}} = \frac{\tau_c \widetilde{C} + \int \widetilde{y}_i di - \lambda \int \widetilde{y}_i^{1-\psi} di - \widetilde{G}}{\widetilde{Y}}$$

- The stationary distributions $\left\{\mu_j\left(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta\right), \mu_j^R\left(\widetilde{a}, f_\alpha, f_\beta\right), \mu_j^{RoW}\left(\widetilde{a}, \varepsilon\right), \mu_j^{R,RoW}\left(\widetilde{a}\right)\right\}$ satisfy

$$\mu_{j+1}\left(\widetilde{a}', s', \varepsilon', f_\alpha, f_\beta\right) = \int \nu\left(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta, j, \widetilde{a}', s', \varepsilon'\right) d\mu_j\left(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta\right) \tag{5}$$

$$\mu_{j+1}^R\left(\widetilde{a}', f_\alpha, f_\beta\right) = \int \nu^R\left(\widetilde{a}, f_\alpha, f_\beta, j, \widetilde{a}'\right) d\mu_j^R\left(\widetilde{a}, f_\alpha, f_\beta\right) \tag{6}$$

$$\mu_{j+1}^{RoW}\left(\widetilde{a}', \varepsilon'\right) = \int \nu^{RoW}\left(\widetilde{a}, \varepsilon, \widetilde{a}', \varepsilon'\right) d\mu_j^{RoW}\left(\widetilde{a}, \varepsilon\right) \tag{7}$$

$$\mu_{j+1}^{R,RoW}\left(\widetilde{a}'\right) = \int \nu^{R,RoW}\left(\widetilde{a}, \widetilde{a}'\right) d\mu_j^{R,RoW}\left(\widetilde{a}\right) \tag{8}$$

In equilibrium the measure of agents in each state is time invariant and consistent with individual decisions, as given by the above four equations (5), (6), (7), and (8), where $\nu\left(.\right)$, $\nu^R\left(.\right)$, $\nu^{RoW}\left(.\right)$ and $\nu^{R,RoW}\left(.\right)$ are the transition functions.

C.3 - Welfare Measures

- The ex-ante welfare measure W^N is the expected utility of a new-born in the steady-state, namely the value function of the age 1 individuals, evaluated at the initial conditions and integrated with respect to the individual effects:

$$W^N = \int_{\mathcal{F}_\alpha \times \mathcal{F}_\beta} V_1 \left(\widetilde{a} = 0, s = \overline{s}, \varepsilon = \overline{\varepsilon}, f_\alpha, f_\beta\right) d\mu_1 \left(\widetilde{a} = 0, s = \overline{s}, \varepsilon = \overline{\varepsilon}, f_\alpha, f_\beta\right) \tag{9}$$

- The social welfare measure W^S is utilitarian, i.e. it weights the agents' lifetime utilities by their mass in the steady-state

$$W^S = \sum_{j=1}^{J_R - 1} \Phi_j \int_{\mathcal{A} \times \mathcal{S} \times \mathcal{E} \times \mathcal{F}_\alpha \times \mathcal{F}_\beta} V_j \left(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta\right) d\mu_j \left(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta\right) + \sum_{j=J_R}^{J} \Phi_j \int_{\mathcal{A} \times \mathcal{F}_\alpha \times \mathcal{F}_\beta} V_j^R \left(\widetilde{a}, f_\alpha, f_\beta\right) d\mu_j^R \left(\widetilde{a}, f_\alpha, f_\beta\right) \tag{10}$$

- The consumption based welfare measure ϖ is the percentage increase in consumption in all states of the world that makes welfare in the counterfactual economy $W_1^i(\varpi)$ equal to welfare in the baseline one W_0^i, $i \in \{N, S\}$:

$$\begin{aligned} W_0^i &= W_1^i(\varpi) \\ \varpi &= \left(\frac{W_1^i}{W_0^i}\right)^{\frac{1}{1-\theta}} - 1 \end{aligned} \tag{11}$$

Appendix D - Data, Estimation and Empirical Targets (Not for publication)

- The estimation was performed with STATA SE 17.1.

- The public version of the Canadian Labour Force Survey, Survey of Consumer Finances, and Survey of Labour and Income Dynamics data was downloaded from the following website:

  `http://dli-idd-nesstar.statcan.gc.ca/webview/`

- The Canadian Life Tables can be downloaded from the following website:

  `https://www150.statcan.gc.ca/n1/pub/84-537-x/84-537-x2018002-eng.htm`

- The public debt and GDP data were obtained from the OECD and the PWT9.0 databases.

- The CPI and the net public debt data for Canada were obtained from the CANSIM database.

- The Canadian public expenditure/GDP was computed dividing the Government Final Consumption Expenditure series by the GDP series. Both series were retrieved using the FRED website: the id of the former is NAEXKP03CAA189S and for the latter is NAEXKP01CAA189S. The US public expenditure/GDP series is computed by the BEA, was retrieved on FRED, and the series id is: A822RE1A156NBEA.

  `https://fred.stlouisfed.org/`

- The Canadian national net lending or net borrowing was obtained from the CANSIM database, series v62305900, and was adjusted for inflation using the GDP deflator, series v1997756.

- The real interest rate on Treasury Bills for Canada used for the model calibration was computed subtracting the inflation rates (obtained from the CPI) from the nominal interest rates. Both series were retrieved using the FRED website: the id of the interest rates series is INTGSTCAM193N and the id for the CPI is CANCPIALLMINMEI. Both series are monthly, but the first series reports an annual rate. This was transformed to its monthly counterpart, and the resulting average real monthly interest rate was then converted to its annual counterpart.

- The degree of progressivity parameter ψ is estimated using the 1972, 1986, and 1998 waves of the Survey of Consumer Finances (SCF), and the 2007 wave of the Survey of Labour and Income Dynamics (SLID). The SLID superseded the SCF, introducing a panel dimension, while still collecting the same information on income. In the SCF, I use the variable *majinc* to construct a model-consistent income variable (I keep in the sample respondents in categories 2, 5, 6, and 7). In the three SCF waves, the pre-tax (post-tax) income variable that I use is labeled totinc, TOTINC, and HHTOTINC (incftax, INCAFTTX, HHINCAFT). In the SLID wave, the pre-tax (post-tax) income variable that I use is labeled ttinc27 (atinc27). The four point estimates for $1 - \psi$ are $0.925, 0.9062, 0.887$, and 0.920.

- The monthly interest rates series used for the model validation were obtained from the CANSIM database, series v122529, v122531, v122532, v122533, v122538, v122539, v122540, v122542, v122543 and v122544. They are also available for download at two Bank of Canada websites:
  `https://www.bankofcanada.ca/rates/interest-rates/lookup-bond-yields/`
  `https://www.bankofcanada.ca/rates/interest-rates/t-bill-yields/selected-treasury-bill-yields-10-year`

- The real GDP growth was computed from the quarterly real GDP. The latter series was retrieved on FRED, and the series id is NAEXKP01CAQ657S.

- The oil price (Global price of WTI Crude) and the Canada/US Foreign Exchange Rate series were retrieved on FRED. The series id's are: POILWTIUSDM and EXCAUS.

- The monthly assets and liabilities for the Bank of Canada were retrieved at:

  https://www.bankofcanada.ca/rates/banking-and-financial-statistics/bank-of-canada-assets-and-liabilities-month-end-formerly-b1/

- The series used to compute the equity premium were mostly obtained from the CANSIM database. All series are seasonally adjusted at annual rates. The Personal expenditure on non-durable goods is series v498571. The Personal expenditure on services is series v498572. Wages, salaries and supplementary labour income are series v498166. Personal savings are series v498164. The Population is series V1. The CPI index is series CPALCY01CAQ661N in FRED, and its base year is 2015. The estimated cointegration equation is $log(consumption) = 1.0 * log(labor\, income) + 0.2 * log(wealth) + 8.8$, and it satisfies desirable properties, such as very low p-values associated with the estimated coefficients (they are virtually zero), and mean reversion (i.e., a trend is absent).

- As for the yield curves, I implemented a non-linear optimization method, because of the well-known collinearity issues in fitting the linearized version of the model. I used a simplex method, and the procedure was restarted 100 times, with the vector of optimizers randomly perturbed by a factor between $0 - 10\%$ to reduce the likelihood of selecting local minima.

Appendix E - The Solution Algorithm (Not for publication)

This algorithm represents the computational procedure used to solve the two-region model:

1. Set the home public debt/GDP ratio to its long-run value.
2. Generate a discrete grid over the asset space for the home economy $[\underline{\widetilde{a}}, ..., \widetilde{a}_{\max}]$ and for the RoW $\left[\underline{\widetilde{a}}^{RoW}, ..., \widetilde{a}^{RoW}_{\max}\right]$.
3. Generate a joint discrete grid over the two individual effects $[f_{\alpha,\min}, ..., f_{\alpha,\max}] \times [f_{\beta,\min}, ..., f_{\beta,\max}]$.
4. Generate a discrete grid over the AR(1) income shocks with the Rouwenhorst method for the home economy $[\varepsilon_{\min}, ..., \varepsilon_{\max}]$ and for the RoW $\left[\varepsilon^{RoW}_{\min}, ..., \varepsilon^{RoW}_{\max}\right]$.
5. Generate a discrete grid over the random walk income shocks with a life-cycle adaptation of Tauchen/Hussey's method $[s_{\min}, ..., s_{\max}]$.
6. Guess the interest rate r_0.
7. Guess the home income tax function parameter λ_0.
8. Guess the home accidental bequest $\widetilde{b}_0$ and the RoW accidental bequest $\widetilde{b}^{RoW}_0$.
9. Get the home pension benefits $\widetilde{\widetilde{y}}_{R,0}(f_\alpha, f_\beta)$ and the RoW pension benefits $\widetilde{\widetilde{y}}^{RoW}_{R,0}$.
10. Get the home capital demand $\widetilde{K}_0$ and wages $\widetilde{w}_0$.
11. Get the home saving functions $a'_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta), a^{R\prime}_j(\widetilde{a}, f_\alpha, f_\beta)$ and the home value functions $V_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta)$, $V^R_j(\widetilde{a}, f_\alpha, f_\beta)$.
12. Get the home stationary distributions $\mu_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta), \mu^R_j(\widetilde{a}, f_\alpha, f_\beta)$.
13. Get the RoW capital demand $\widetilde{K}^{RoW}_0$ and wages $\widetilde{w}^{RoW}_0$.
14. Get the RoW saving functions $a^{RoW\prime}_j(\widetilde{a}, \varepsilon), a^{R,RoW\prime}_j(\widetilde{a})$.
15. Get the RoW stationary distributions $\mu^{RoW}_j(\widetilde{a}, \varepsilon), \mu^{R,RoW\prime}_j(\widetilde{a})$.
16. Get the aggregate world capital supply and check the world asset market clearing; Get r_1.
17. Update $r'_0, \lambda'_0, \widetilde{b}'_0$ and $\widetilde{b}^{RoW\prime}_0$ (with a relaxation method).
18. Iterate until the global asset market clearing and aggregate consistency of the pensions are satisfied.
19. Compute the ex-ante welfare W^N of a newborn and the social welfare W^S for the home economy.

20. Get the home consumption functions $c_j(\widetilde{a}, s, \varepsilon, f_\alpha, f_\beta), c_j^R(\widetilde{a}, f_\alpha, f_\beta)$, the RoW consumption functions $c_j^{RoW}(\widetilde{a}, \varepsilon), c_j^{R,RoW}(\widetilde{a})$ and check the final good market clearing.

21. Adjust the parameters $\beta, \beta^{RoW}, \mu^{RoW}, \alpha^{RoW}$ until the empirical targets are matched.

22. Set the public debt/GDP ratio to a counterfactual value on an appropriate grid and repeat steps 2-20 for all the points in this grid.

This algorithm represents the additional steps used to solve the two-region model with transitional dynamics:

1. Assume that at time $t = 0$ the economy is in the initial steady-state with $d = 0.6$, and compute it using the algorithm above.

2. Set the length of time needed to complete the whole transition to the final steady-state to $T = 150$.

3. Assume that at time $t = T$ the economy is in the final steady-state, with $d = d^*$, and compute it using the algorithm above.

4. Set the length of time needed to complete the public debt dynamics, with $T_d = 15$ for the benchmark experiments.

5. Assume that the public debt dynamics evolve linearly, which allows to postulate the whole sequence for the public debt/GDP ratio $\{d_t\}_{t=1}^{t=T}$.

6. Guess sequences for all the transitional equilibrium objects $\left\{r_t, \lambda_t, \widetilde{b}_t, \widetilde{b}_t^{RoW}\right\}_{t=1}^{t=T}$

7. Solve the household problems backward, relying on the fact that at time $t = T$ the decision rules are the ones for the final steady-state.

8. Solve the transitional distributions forward, relying on the fact that at time $t = 0$ the distributions are the ones for the initial steady-state.

9. Aggregate the decision rules and compute the new guesses for the transitional equilibrium objects.

10. Iterate until the asset market clears in every period of the transition, and the sequences $\left\{\lambda_t, \widetilde{b}_t, \widetilde{b}_t^{RoW}\right\}_{t=1}^{t=T}$ converge.

11. Compute the welfare of a newborn and social welfare for the home economy.

12. Set the final public debt/GDP ratio to another counterfactual value on an appropriate grid and repeat steps 3-11 for all the points in this grid.

Appendix F - Additional Results and Plots (Not for publication)

	(1)	(2)	(3)	(4)	(5)	(6)
	gbr_5	$fwr_{5,5}$	$fwr_{5,5}$	$fwr_{5,5}$	$fwr_{5,5}$	$fwr_{5,5}$
Net debt/GDP	-0.0731	0.0202	-0.0304	0.0198	0.0243	0.0341
	(0.109)	(0.580)	(0.452)	(0.578)	(0.480)	(0.109)
GDP growth	0.554	-0.804	0.551	-0.767	-0.932	-2.515
	(0.454)	(0.207)	(0.445)	(0.208)	(0.097)	(0.000)
Oil price (CAD)	-0.0307	0.00813	-0.0478	0.0225	0.0164	0.0019
	(0.467)	(0.708)	(0.250)	(0.379)	(0.578)	(0.899)
Equity premium	65.53	41.18	99.10	37.53	37.25	53.40
	(0.011)	(0.037)	(0.000)	(0.076)	(0.051)	(0.000)
BoC gov. assets	-0.0894	0.0520	0.216	0.196	0.187	0.0925
	(0.490)	(0.511)	(0.096)	(0.377)	(0.385)	(0.092)
Time trend				-0.0927	-0.0781	
				(0.495)	(0.521)	
AR(1)					-0.1888	
Constant	12.59	-3.688	-6.332	4.421	2.560	4.521
	(0.047)	(0.498)	(0.255)	(0.753)	(0.829)	-
Debt projection	NO	YES	NO	YES	YES	YES
N	72	49	72	49	48	47
R^2	0.320	0.162	0.284	0.171	0.220	0.303

Table 4: Model validation. *Notes*: The dependent variable, always expressed in percentage points, is the current annualized real rate of return of 5-year government bonds (gbr_5), or the real 5-year-ahead 5-year forward rate ($fwr_{5,5}$). The Net debt/GDP regressor is either its current value, or its 5-year-ahead projection computed using a linear model with four autoregressive lags. (1)-(5) list the coefficients from reduced-form equations, while (6) is the estimated cointegrating equation from a VEC model (the coefficient on $fwr_{5,5}$ is normalized to 1). Huber-White standard errors in all regressions, but (5) and (6). In (6), the R^2 is the average across the six equations in the VEC model. p-values in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)
	gbn_{10}	$fwn_{5,10}$	$fwn_{5,10}$	$fwn_{5,10}$	$fwn_{5,10}$	$fwn_{5,10}$
Net debt/GDP	-0.00109	-0.00235	-0.0108	-0.00421	0.00109	0.0466
	(0.962)	(0.632)	(0.097)	(0.398)	(0.799)	(0.049)
GDP growth	-0.307	-0.00920	-0.158	-0.00665	0.00550	-5.811
	(0.500)	(0.235)	(0.408)	(0.233)	(0.111)	(0.000)
Oil price (CAD)	0.0133	-0.0106	-0.00881	-0.00198	-0.00200	0.0152
	(0.187)	(0.005)	(0.043)	(0.672)	(0.585)	(0.495)
Equity premium	-9.892	16.33	14.19	13.57	4.547	87.48
	(0.202)	(0.000)	(0.000)	(0.000)	(0.145)	(0.000)
BoC gov. assets	-0.217	-0.00738	-0.0148	0.0826	0.0262	0.054
	(0.000)	(0.445)	(0.179)	(0.003)	(0.134)	(0.516)
Expected inflation	1.215	6.460	0.155	5.593	0.0567	57.25
	(0.001)	(0.001)	(0.030)	(0.003)	(0.954)	(0.000)
Time trend				-0.0576	-0.0182	
				(0.004)	(0.383)	
AR(1)					0.9272	
Constant	10.64	-5.018	-8.495	0.169	-1.868	20.29
	(0.079)	(0.352)	(0.129)	(0.990)	(0.875)	-
Debt projection	NO	YES	NO	YES	YES	YES
N	72	49	72	49	48	47
R^2	0.355	0.152	0.284	0.156	0.202	0.250

Table 5: Model validation. *Notes*: The dependent variable, always expressed in percentage points, is the current annualized nominal rate of return of 10-year government bonds (gbn_{10}), or the nominal 5-year-ahead 10-year forward rate ($fwn_{5,10}$). The Net debt/GDP regressor is either its current value, or its 5-year-ahead projection computed using a linear model with four autoregressive lags. (1)-(5) list the coefficients from reduced-form equations, while (6) is the estimated cointegrating equation from a VEC model (the coefficient on $fwn_{5,10}$ is normalized to 1). Huber-White standard errors in all regressions, but (5) and (6). In (6), the R^2 is the average across the six equations in the VEC model. p-values in parentheses.

	debtgdp
debtgdplag	0.838
	(0.0290)
gtotgdp	91.13
	(29.75)
gtotgdp^2	-4.193
	(1.360)
gtotgdp^3	0.0854
	(0.0272)
gtotgdp^4	-0.000644
	(0.000201)
ssgdp	1.159
	(0.342)
taxgdp	-0.536
	(0.202)
shortintrate	-0.608
	(0.300)
longintrate	1.407
	(0.146)
grgdp	10.93
	(20.09)
Time Dummies	YES
N	452

Table 6: The (D/Y, G/Y) Correlation in the OECD Data. The dependent variable is the public debt/GDP ratio (debtgdp). The explanatory variables are the lagged dependent variable (debtgdplag), a quartic polynomial in the Total Public Outlays/GDP ratio (gtotgdp), the Social Security Benefits/GDP (ssgdp), the Total Tax Revenues/GDP (taxgdp), the growth rate of real GDP (grgdp), and two interest rate measures for the short-run (shortintrate) and for the long run (longintrate). The parameters are estimated using the Arellano-Bond estimator. Standard errors in parentheses.

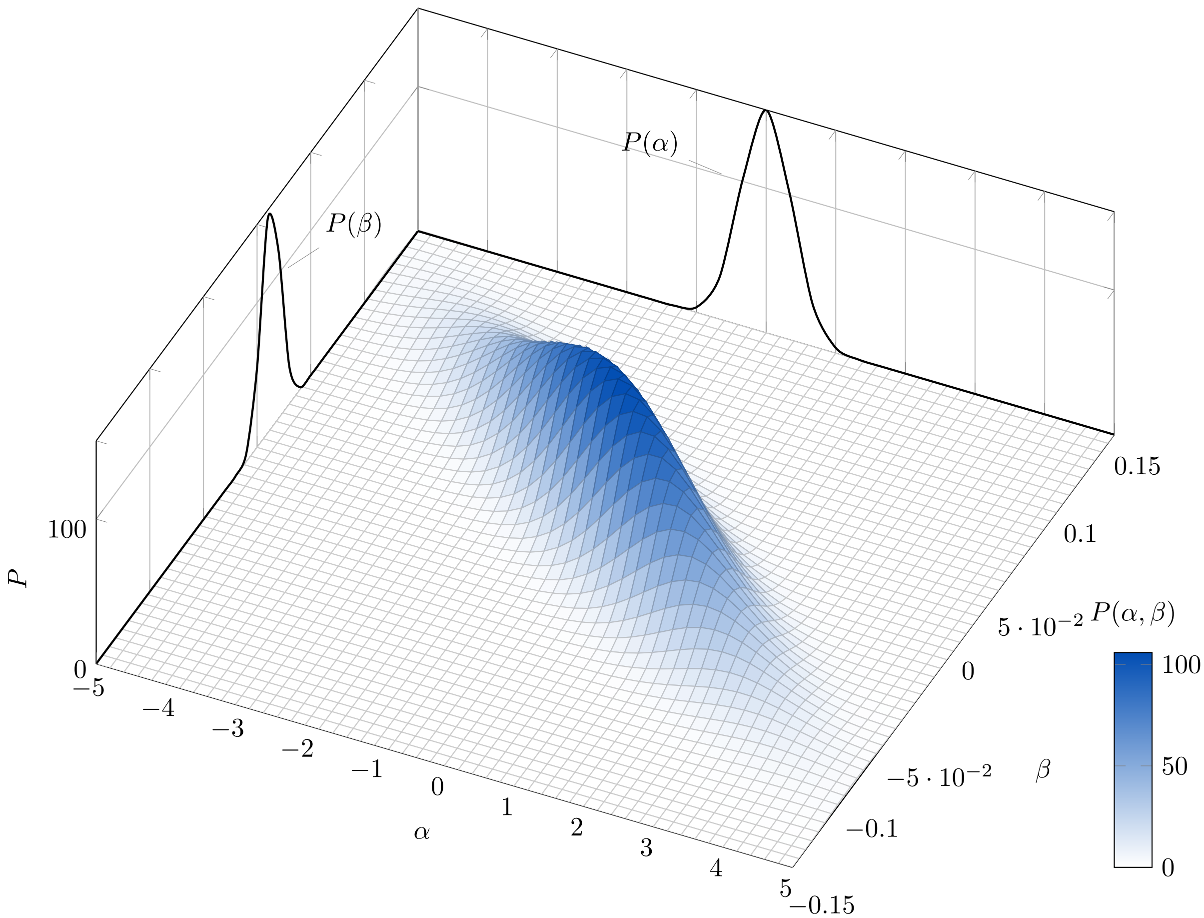

Figure 10: Joint bivariate normal density of the correlated level (α) and slope (β) individual effects for the Canadian economy. For ease of comparison with the joint density $P(\alpha, \beta)$, the marginal densities $P(\alpha)$ and $P(\beta)$ are multiplied by an appropriate normalizing factor.

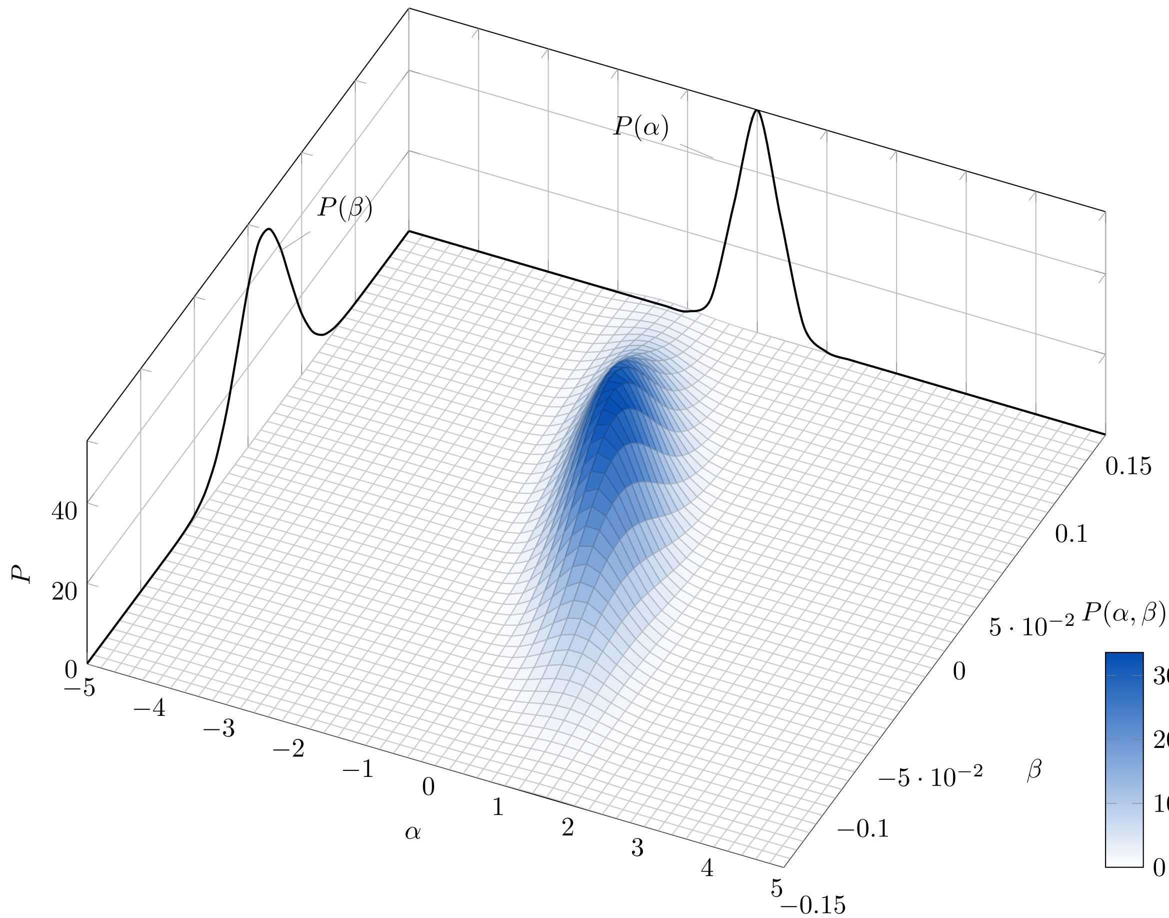

Figure 11: Joint bivariate normal density of the correlated level (α) and slope (β) individual effects for the US economy. For ease of comparison with the joint density $P(\alpha, \beta)$, the marginal densities $P(\alpha)$ and $P(\beta)$ are multiplied by an appropriate normalizing factor.

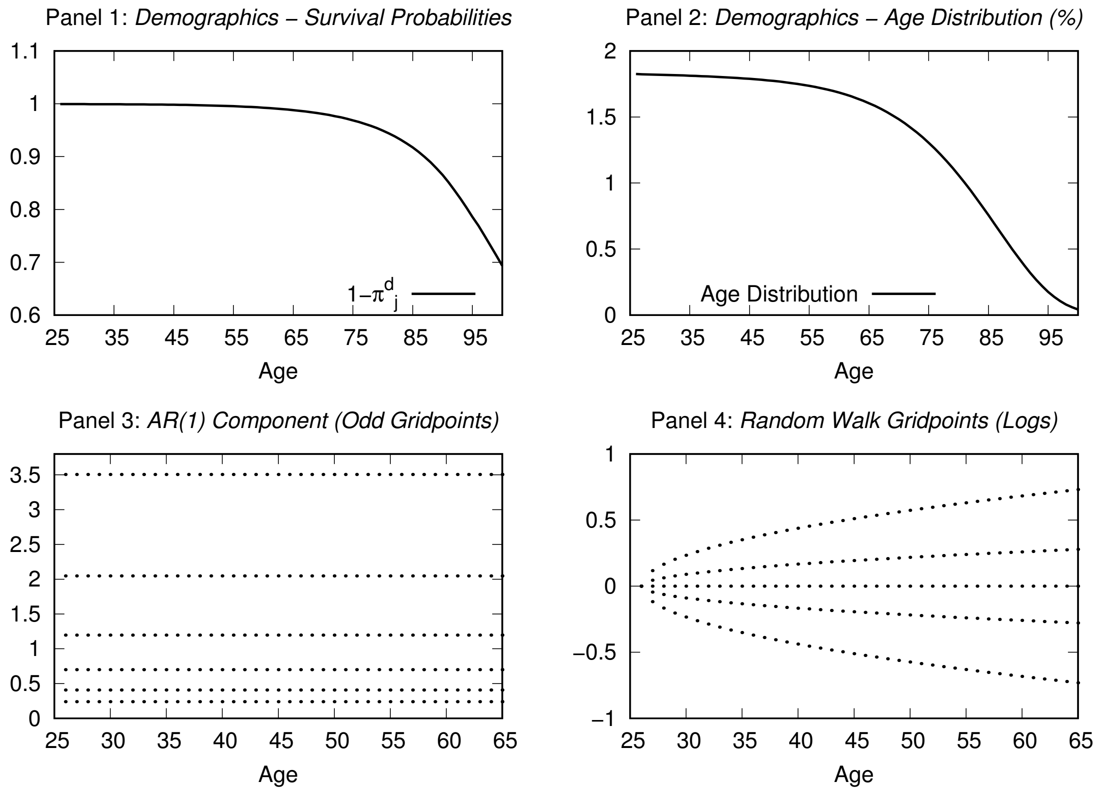

Figure 12: Canadian demographics and discretization of the stochastic component of the Canadian income process.

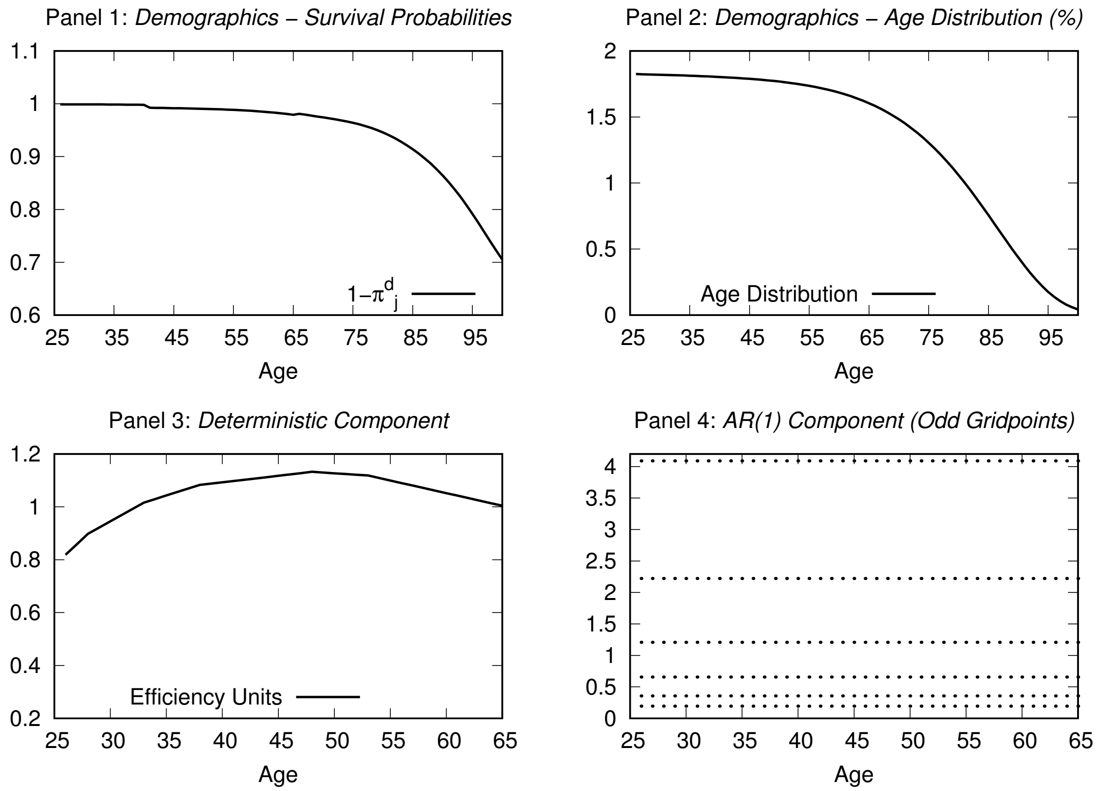

Figure 13: RoW demographics and discretization of the stochastic component of the RoW income process (Canadian calibration).

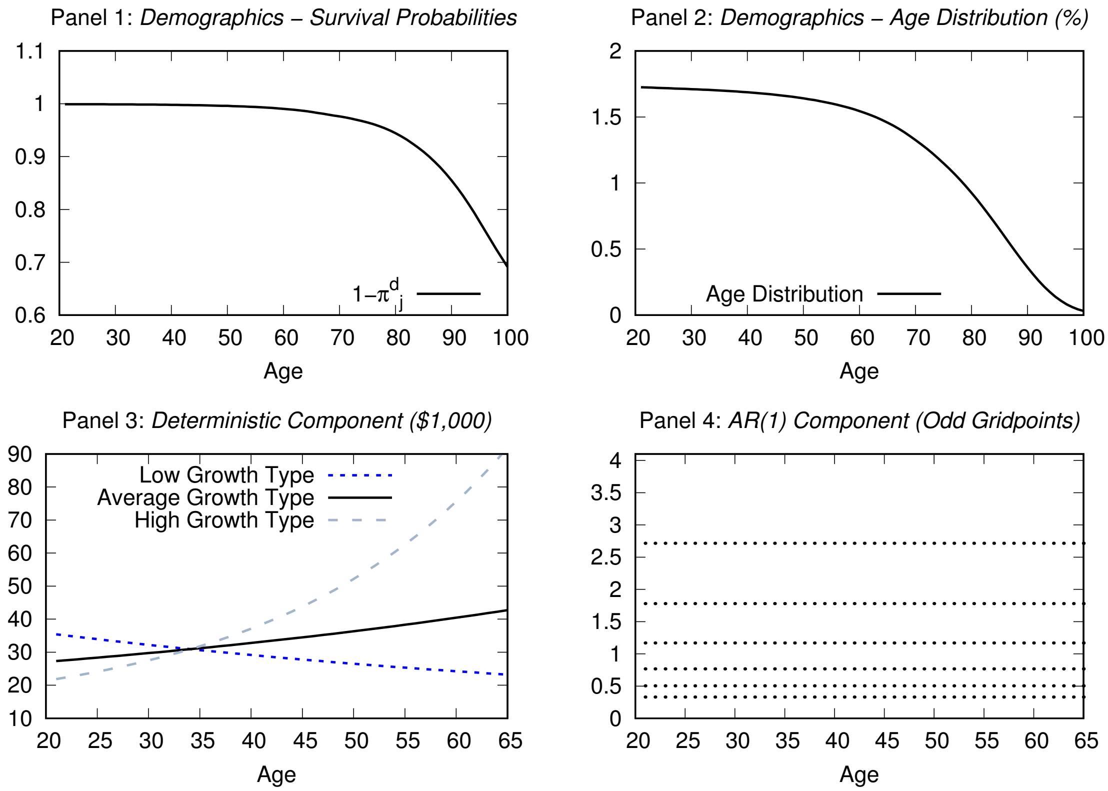

Figure 14: US demographics (top panels), individual effects component of labor earnings, and discretization of the stochastic component of the US income process.

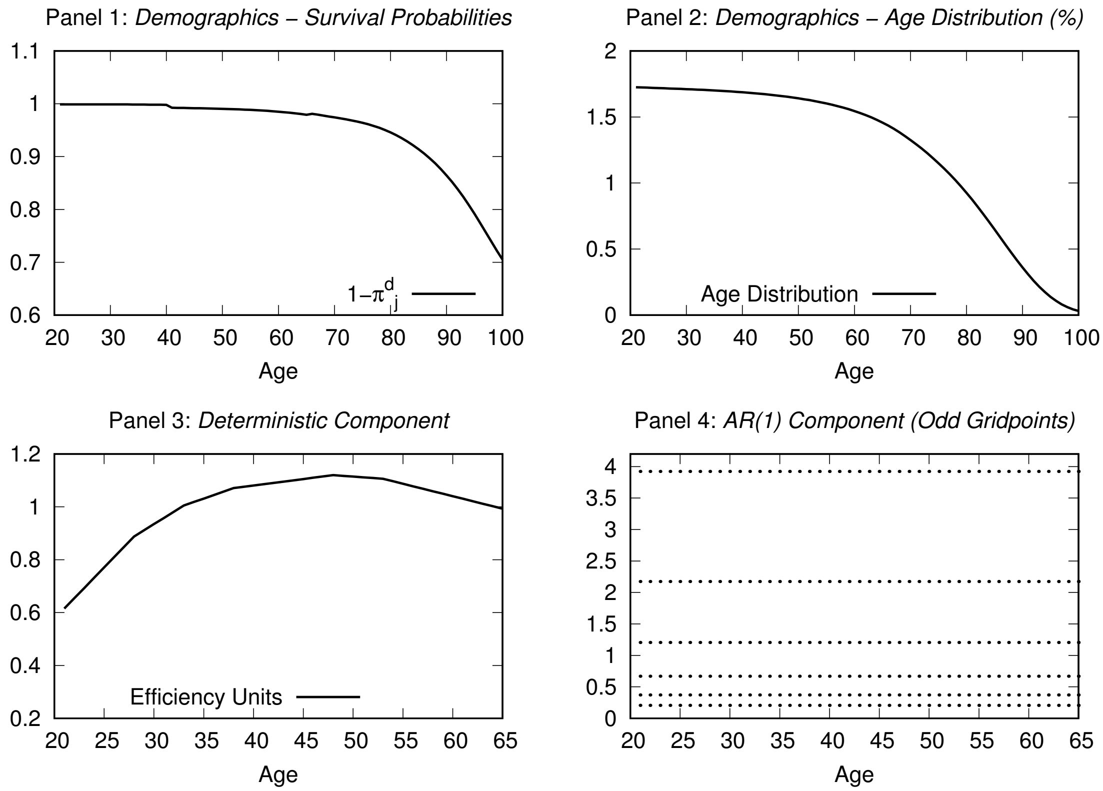

Figure 15: RoW demographics and discretization of the stochastic component of the RoW income process (US calibration).

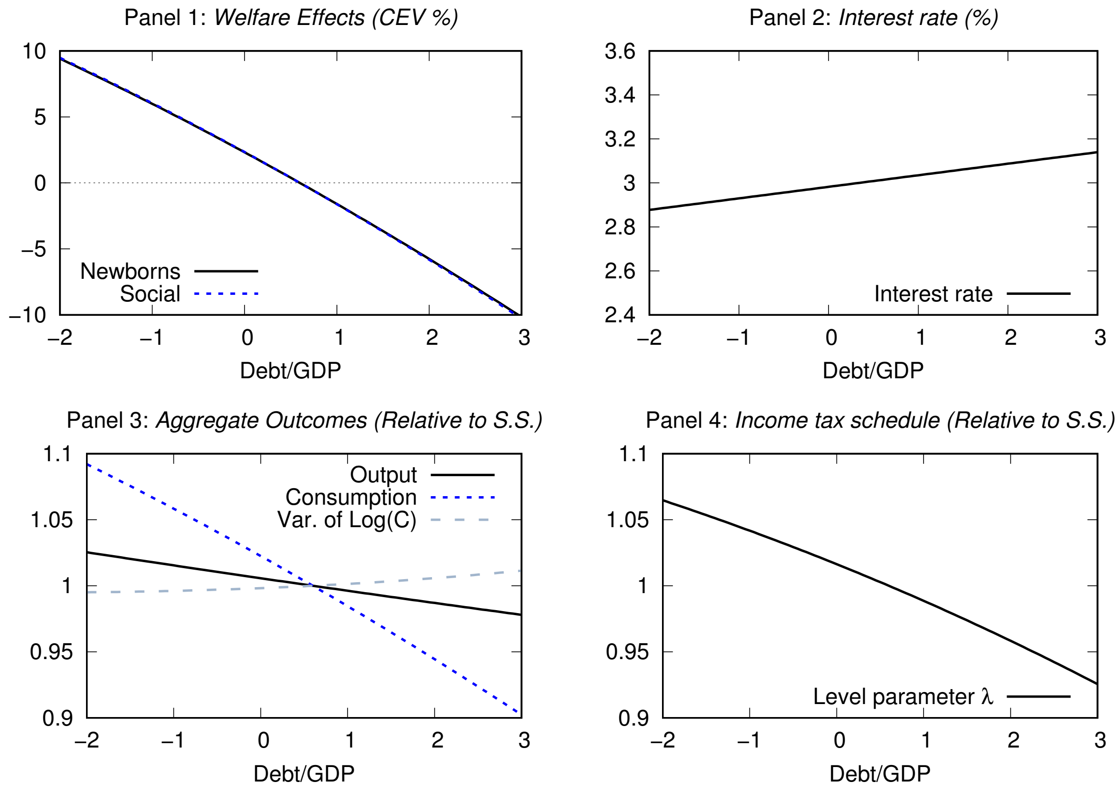

Figure 16: Equilibrium outcomes for the two-region economy, US case with ad-hoc progressivity parameter $\psi = 0.07$.

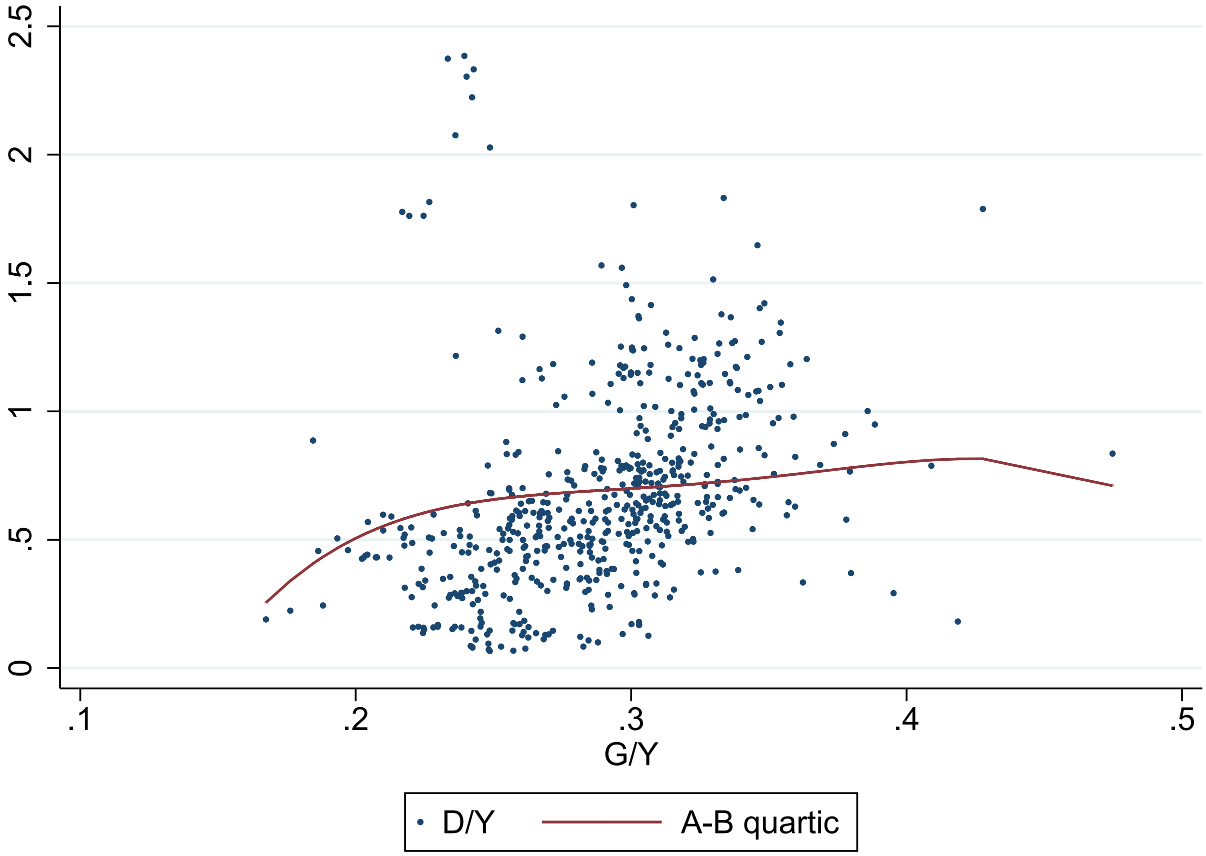

Figure 17: The estimated $D/Y, G/Y$ relationship in the OECD sample.