



HAS THE CANADIAN PUBLIC DEBT BEEN TOO HIGH? A QUANTITATIVE ASSESSMENT

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Abstract

This paper provides a quantitative analysis on whether the historically sizable public debt that the Canadian governments have accumulated might be close to its welfare maximizing level. As the public provision of liquidity to borrowing constrained individuals coupled with an increased supply of safe assets can be welfare improving, I consider a two-region model with an integrated asset market and incomplete insurance markets. The home country features a rich life-cycle setup, where the income dynamics rely on state of the art estimates obtained from previous studies using income tax returns. The main features are ex-ante labor earnings heterogeneity, both in levels and in growth rates, together with persistent and permanent shocks. When the public expenditure is assumed to be wasteful, I find that the optimal quantity of public debt for Canada is negative, meaning that the government should be a net saver. When the government, with a portion of its expenditure and consistent with the Canadian experience, finances valuable public goods, the long-run public debt is still found to be inefficiently large, but closer to the welfare maximizing level.

Keywords: Public debt, Incomplete markets, Welfare.

JEL Classifications: D52, E21, E62, H63.

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1 Introduction

A rationale for governments to accumulate public debt is the provision of liquidity to borrowing constrained individuals. In an influential paper, [Aiyagari and McGrattan \(1998\)](#) argued that the long-run average of the U.S. public debt is close to the optimal one. They found that the increase in the asset demand brought about by larger values of public debt has important General Equilibrium (GE, hereafter) effects, leading to positive –yet remarkably small– welfare effects. In this paper, I undertake an equilibrium analysis tailored to the Canadian economy. The Canadian experience represents an interesting case, because the long-run average of the public debt has been sizable, and the pre-tax labor income dynamics have been similar to the U.S., as shown by [Baker and Solon \(2003\)](#) and [Brzozowski, Gervais, Klein and Suzuki \(2010\)](#). However, the resulting labor income inequality has been partially offset by a more generous redistribution, and public debt has also been accumulated because of a more extensive provision of public goods. Finally, the size of these two economies is drastically different, with plausibly very different implications for the magnitude of general equilibrium effects in asset markets.

Some of the main findings in [Aiyagari and McGrattan \(1998\)](#) rest on three important assumptions: a closed economy framework, infinitely-lived households, and wasteful public expenditures.

In a closed economy setup, an increase in public debt is associated with an increase in the aggregate demand for assets, leading to a potentially large response of the equilibrium interest rate. In steady-state comparisons, this can be beneficial for social welfare, as saving in a risk-free asset becomes a more effective tool to smooth income fluctuations. At the same time, higher interest rates crowd out investment in physical capital, reducing aggregate output and worsening social welfare, calling for a quantitative analysis to determine the overall effect on welfare. Such effects would not be present in a basic Small Open Economy framework (SOE). With interest rates determined in international financial markets, and domestic public debt held by foreign savers, welfare improvements induced by higher interest rates would be prevented by assumption. Differently, the public provision of liquidity channel, relaxing the households’ borrowing constraint with public transfers partially financed by public debt, would still be operational in a SOE model. The OECD data plotted in [Figure 1](#) show that there is considerable dispersion in the public debt/GDP ratio for both large and small economies with access to international financial markets. These economies also display a large and positive average long-run value for this ratio. These arguments call for an open economy analysis to better measure the importance of the liquidity provision explanation. I consider a model with two regions, with the home economy calibrated to Canadian microeconomic and macroeconomic evidence, and the other economy representing the Rest-of-the-World (RoW). The two-region model does not rule out a priori the possibility of large public debts to be welfare improving via the improved self-insurance attained by saving in a safe asset. One might expect consistently small GE effects on the interest rate, which is what I actually find. However, these could still be important for the welfare effects, especially in an environment where saving has multiple goals, and the demand for redistribution is driven by several concurrent causes (namely, the degree of market incompleteness, and the nature of labor market shocks).

[[Figure 1](#) about here]

A model with infinitely-lived households prevents some key life-cycle dimensions from playing a role in how detrimental incomplete insurance possibilities might be. In particular, young and asset-poor individuals typically prefer more public liquidity, unless this positive effect on their welfare is offset by lower wages (due to a crowding out of investment in physical capital) and more expensive borrowing (due to an increase in the asset demand). Thanks to the availability of both longitudinal surveys and administrative data, the literature on earnings dynamics has proposed several econometric models to capture their evolution over the life-cycle. Even though a consensus has yet to be reached on an accurate –and parsimonious– statistical process, there is extensive evidence supporting labor income growth heterogeneity. [Baker and Solon \(2003\)](#) is one of the few systematic studies on Canadian administrative data, while [Baker \(1997\)](#), [Guvenen \(2009\)](#), [Haider \(2001\)](#) and [Hause \(1980\)](#) are other influential contributions in this field. Recently, also [Peterman and Sager \(2018\)](#) have included life-cycle motives in their analysis of optimal public debt, finding that these have important implications for their welfare measure, the expected utility of the newborns. In particular, and contrary to [Aiyagari and McGrattan \(1998\)](#), they find that large public debts are detrimental for welfare, and the optimal policy is one displaying Public Savings. However, their contribution differs from mine, as I consider two open economies, a richer specification for the income process, a calibration targeting the Canadian economy, and an extension with more general preferences that leads to a different explanation for large public debts. In terms of the income dynamics, I rely on state of the art estimates obtained from income tax returns by [Baker and Solon \(2003\)](#). Their econometric specification nests the standard persistent/permanent decomposition of income shocks into a framework with heterogeneity in the growth rate of labor earnings during the life-cycle. This formulation leads to substantial heterogeneity in income dynamics during the active years in the labor market, together with different pensions upon retirement. To the best of my knowledge, there are no other contributions in the literature of optimal public debt considering such a general specification for the income dynamics. The considerable amount of redistribution present in Canada via the fiscal system might be effective in counteracting the substantial earnings risk that Canadian households have faced. This begs the question of whether the liquidity provision attained with public debt is warranted, as it might inefficiently provide excessive insurance against labor market shocks. A two-region life-cycle model allows to accurately quantify the importance of the GE effects on both wages and the interest rate. This is a critical aspect, as these influence the effectiveness of self-insurance via the accumulation of risk-free assets, versus the lost income and lower pensions arising from a lower level of aggregate capital. In this setup I find that, under several modeling assumptions, the optimal quantity of debt for Canada is large and negative. This means that the federal and provincial governments should be net savers, rather than being net borrowers.¹ To facilitate the comparison with calibration studies done on the U.S., I then conduct some quantitative experiments assuming that the Canadian economy is closed. In these counterfactual scenarios, I do find that sizable public debts can be optimal, at least according to some welfare measures. I also find a generational conflict, as in this case the welfare effects of very young individuals are opposite in sign to

¹I performed an extensive robustness analysis, and a subset of the results are reported in Appendix D. The main outcomes found in the two-region model without valuable public goods apply also when considering Canada as a SOE, and when there is an exogenous 2% differential between borrowing and lending interest rates. An extension to the SOE model allows for financial markets to demand a risk premium for larger public debt/GDP ratios. In this case the results differ considerably, but this version of the model leads to some evident counterfactual implications.

those of older individuals, an outcome that is not observed in the open economy versions of the model.

A model with wasteful public expenditure has some appealing features, as positive welfare effects of public debt could then be interpreted as lower bounds. However, in a scenario where public debt is found to be unequivocally detrimental for welfare, the notion of an entirely wasteful public expenditure can be problematic. Governments do provide valuable services, either in terms of public infrastructure, a hypothesis considered in the literature by [Chatterjee, Gibson, and Rioja \(2017\)](#) and [Chatterjee, Gibson, and Rioja \(2018\)](#), or in terms of publicly provided goods that directly affect the welfare of the households. This instance is ruled out by most papers in the literature on public debt with incomplete markets, as typically public spending is interpreted as consumption of the public sector, and it is wasteful by assumption.² The Canadian experience is noteworthy in this dimension, as key goods and services, such as health and education, are publicly provided. To capture this feature, I extend the model by including a public good that affects the utility of the households. Since there is limited direct information on the utility weight that individuals might place on private vs. public goods, and the results can be heavily affected by this element, I rely on a stylized political economy argument to identify this key parameter. The status quo public policy is determined according to a vote-maximizing mechanism. In this light, I then provide an alternative explanation for large public debts. Lowering the public debt can be achieved by partially cutting the public expenditure, a channel that I will estimate empirically. If the wasteful consumption of the public sector cannot easily be targeted as a source of budget savings, the welfare of the households will eventually be negatively affected when the provision of public goods falls below some (endogenously determined) minimum level. The results show that the long-run Canadian public debt has been inefficiently high, and that a positive –but moderate– public debt is optimal.

In the next subsection I briefly present some evidence related to public debt and public expenditure in the Canadian context.

1.1 The Canadian public debt and Public Expenditure in the Long Run

Figure 2 includes three panels plotting the time series of Canadian public debt and public expenditure. The top panel displays the longest available series for the gross Canadian public debt, expressed as a fraction of GDP. Canadian public debt has been quite volatile, with the public debt/GDP ratio being anywhere between 30% and 150%. As observed in several other countries, public debt spiked during WWII, and quickly fell to less extreme values shortly thereafter. In this dataset, the long-run average of the public debt/GDP ratio in the post WWII era (1952-2012) is 68.5%, while the average for the whole sample period is 61.8%. The middle panel relies on Cansim data, which measure the net public debt/GDP ratio.³ These data cover a considerably shorter period, but are quarterly and disaggregated by level of government. It is interesting to notice that in the last decade the Federal public debt as a share of GDP has been declining, but it has been replaced by public debt accumulated by other levels of government. The quantitative model will be constrained by data limitations

²Unlike public transfers, which can be beneficial, as stressed by [Floden \(2001\)](#), and that are included in my framework.

³The Net public debt is obtained subtracting financial assets from the Gross public debt as, in principle, financial assets could be used to pay back part of the debt. In order of importance, the Government of Canada's financial assets are mainly composed of: taxes receivable, investment in enterprise Crown corporations, foreign exchange accounts, and cash and/or cash equivalents.

for the reference time period, especially with respect to the income dynamics. It is worth reporting that the Cansim data in the 1990-2007 period, namely pre-dating the great recession and closer to the income data, show a public debt/GDP ratio of 68.5%, which is identical to the statistic reported above. For these reasons, I am going to consider this figure as the long-run value of the Canadian public debt/GDP ratio.

The bottom panel displays the time series of Canadian public expenditure/GDP ratio. Abstracting from the short-run fluctuations, a noteworthy feature of this series is the emergence of two long-run levels of public expenditure. The first regime covers the years from the early 1960's until the early 1990's, where a public expenditure/GDP ratio around 25% was the norm. Starting in the 2000's, this ratio has settled around a lower value of approximately 20%. This indicates that changes in public debt are positively correlated with changes in public expenditure, a fact that will be more carefully established empirically below, using OECD data. In the 1990-2007 period, the average public expenditure/GDP ratio was 21.4%.

[Figure 2 about here]

1.2 Related Literature

The literature on public debt has been very active of late, due to the large public debts accumulated by several countries in the aftermath of the great recession, and the recent public debt crises that have been observed in Europe. [Alesina and Passalacqua \(2016\)](#) and [Yared \(2018\)](#) are two comprehensive surveys reviewing both the classical and the more recent explanations for public debt. Both contributions point out the need for methodical quantitative work, especially in models with a political economy explanation for public debt.

There is limited work on the role of aggregate shocks and public debt in models with household heterogeneity. [Desbonnet and Kankanamge \(2017\)](#) consider a standard version of the RBC model with incomplete markets, finding that the optimal amount of public debt is negative. In their paper, they also perform a comprehensive decomposition analysis for the steady-state version of the model. However, they do not allow for an open economy analysis or for a life-cycle component, which is found by [Peterman and Sager \(2018\)](#) to be the most important element in their decomposition of the welfare effects. Since the income processes these two contributions are working with are quite parsimonious, it is not straightforward that similar quantitative effects would be obtained in a model with more general –and empirically grounded– income dynamics. [Rohrs and Winter \(2015\)](#) and [Rohrs and Winter \(2017\)](#) emphasize the distinction between the private vs. public provision of liquidity, also accounting for the transitional dynamics. The computational burden of my framework does not make the study of the transition between steady states feasible. However, given the open economy set-up, I speculate that the transition could be completed relatively quickly, making the steady-state comparisons possibly quite accurate. As mentioned above, [Chatterjee, Gibson, and Rioja \(2017\)](#) argue that the role of public investment is quantitatively important in the determination of the optimal debt/GDP ratio. Instead, in my most general setup, I will focus on a different role of the public sector, namely the provision of public goods.

Although understanding how the Canadian public debt has reached what I will find to be an inefficiently high level is beyond the scope of this paper, [Desbonnet and Weitzenblum \(2012\)](#) argue that the short-run gains can create a temptation to deviate toward higher levels of debt. Another possible argument is that short lived

governments have used generous public transfers for electoral purposes, an instance that is discussed extensively in [Yared \(2018\)](#). In this paper I am going to propose a model with public goods provision that is closer in spirit to the latter channel. For tractability, I do not consider a model with endogenous sovereign default, an element included in [D’Erasmus and Mendoza \(2016\)](#).

The rest of the paper is organized as follows: Section 2 presents an open economy model for Canada. Section 3 discusses the model calibration. Section 4 presents the results. Section 5 a model extension with public goods. Section 6 concludes and a set of Appendices present an in-depth discussion of the theoretical model, further information on the data used for the quantitative analysis, and additional results.

2 A Two-Region Model

I start by considering a model with two asymmetric economies that share an integrated asset market. There are no restrictions to capital movements, but both economies have incomplete markets, whereby the workers cannot buy insurance for the labor market risks they are facing. For tractability, there is no international mobility of workers, also because migration flows do not seem to be driven by changes in public debt policies. Since the home economy is representing Canada, its size is considerably smaller compared to the RoW economy.⁴

One might wonder why the two-region framework is to be favored compared to the simpler Small Open Economy (SOE) one. One reason is that Canada’s shares in global GDP and in capital markets are not trivially small, being 2.16% and 2.26%, respectively. Although it is plausible to expect a limited interest rate response arising from different public debt policies, under incomplete markets and with rich income dynamics, this could still give rise to positive welfare effects of public debt. The SOE framework eliminates the possible positive role of public debt through improved self insurance by assumption. In a SOE, an increase in public debt is absorbed by the world financial market without any effect on the foreign and domestic interest rates. On the one hand, this precludes the crowding out of investment, which would lead to a lower domestic capital stock, output and wages. At the same time, in the SOE scenario, public debt is partially wasteful as the additional asset income is enjoyed by foreigners, leaving the domestic residents to pay higher taxes that translate into a worsening of the current account. Solving the model with the SOE setup, I found that the optimal steady-state public debt is unambiguously negative and large, because public wealth would provide income subsidies, rather than taxes, to the domestic households via the asset income accrued on the accumulated public wealth. This outcome is distant from the available empirical evidence on public debt, questioning the suitability of the SOE assumption.⁵

⁴Because of the NAFTA agreement, considering the U.S. and Mexico as the relevant group of foreign economies would seem appealing. However, the zero long-run average of the Canadian net lending position is inconsistent with the sizable net borrowing position of the U.S., invalidating the two-region set-up.

⁵I also solved the SOE version of the model closing the model assuming an ad-hoc domestic interest rate determination rule, adapting the ideas discussed by [Schmitt-Grohe and Uribe \(2003\)](#). I estimated the interest rate elasticity to public debt on a sample of OECD countries, such that the interest rate is increasing in the home country’s public debt. Also in this case the optimal public debt/GDP ratio was found to be negative and large, in excess of 200%, albeit with a different and dubious mechanism. For more details see Appendix D.

The two-region model is an extension of the [Huggett \(1996\)](#) economy, appropriately modified to allow for several sources of heterogeneity in labor income, public debt, and two open economies producing the same final good.⁶ The home economy has an Overlapping Generations (OLG) structure. One of the reasons for working with an OLG model is to accommodate the Canadian empirical evidence on the labor income dynamics, that emphasize the importance of the life-cycle elements.

The economy is a production economy with an endogenous asset distribution, where a government collects taxes to finance an exogenously given stream of public expenditure, public transfers, a pension scheme, and the interest rate costs on the accumulated public debt. Besides the workers, there is a representative firm that produces the final output relying on a Cobb-Douglas production function on aggregate labor and aggregate capital.

Demographics: Time is discrete. The economy is populated by finitely lived agents facing an age-dependent death probability π_j^d . Age is denoted with j and there are J overlapping generations, each consisting of a continuum of agents. At age J_R all agents that are still alive become retirees. Regarding the life expectancy, in the benchmark case there is a perfectly competitive annuity market, where the agents share their mortality risk. It follows that in this version of the model there are no accidental bequests, as the average asset holdings of the individuals that die are zero.⁷

Preferences: Agents' preferences are assumed to be time-separable and represented by the utility function $U(\cdot)$. Agents' utility is defined over stochastic consumption $\{c_j\}_{j=1}^J$ sequences: their aim is to choose how much to consume (c_j) and how much to save in an interest bearing asset (a_{j+1}) in each period of their lives, in order to maximize their objective function. The agents' problem can be defined as:

$$\max_{\{c_j, a_{j+1}\}_{j=1}^J} \mathbb{E}_1 U(c_1, \dots, c_J) = \max_{\{c_j, a_{j+1}\}_{j=1}^J} \mathbb{E}_1 \sum_{j=1}^J \beta^{j-1} \left[\prod_{s=1}^j (1 - \pi_s^d) \right] u(c_j)$$

where \mathbb{E}_1 represents the expectation operator at labor market entry over the idiosyncratic sequences of shocks, and $\beta > 0$ is the subjective discount factor. In the benchmark formulation, I assume that $u(c_j) = \frac{c_j^{1-\theta}}{1-\theta}$, that is the per-period utility function is strictly increasing in consumption, strictly concave, satisfies the Inada conditions, and has a constant relative risk aversion. The labor supply is fixed and equal to the time endowment multiplied by the value of the efficiency units.⁸

⁶For more details on the model, see Appendix A, [Rios-Rull \(1994\)](#), [Rios-Rull \(1996\)](#), [Huggett \(1996\)](#), and [Guisarri \(2007\)](#), among many others.

⁷This formulation has the advantage of leaving the newborns with just their labor income, the government transfers and no additional resources, as they are born with no assets and receive no accidental bequests. I also tried a more conventional formulation with annuities that affect the ex-post interest rate of the surviving agents on an age-to-age basis. Since agents can borrow, this formulation had the drawback that the surviving agents had to take care of the outstanding debts of the agents dying with negative asset holdings. Finally, in an alternative case, there are accidental bequests denoted by b , which are distributed uniformly to all the surviving agents, and the newborns.

⁸Since the T-4 tax returns that form the basis for the estimation of the stochastic income process do not include the number of hours worked, it is not possible to estimate a process for wages or allow for a flexible labor supply margin.

Labor Income Dynamics: Newborn agents are ex-ante identical, but the idiosyncratic realizations of a series of shocks induce substantial ex-post heterogeneity. In particular, upon entering the labor market, the workers observe two characteristics that affect their earnings potential: the immutable individual effects for both the entry level (f_α) and the growth rate of labor income (f_β).

Given the rich dynamics displayed by the Canadian labor income data, the model allows for rich labor earnings dynamics. This feature allows to capture the latent demand for redistribution and liquidity provision, and the need for self insurance. There are five channels that contribute to the determination of the total efficiency units ϵ that the workers supply in the labor market. First, there is an age component j , which interacts with one of the fixed effects, namely the growth rate of individual productivity f_β . Second, there is the level fixed effect f_α , which is correlated with f_β . The two fixed effects are jointly normally distributed and are observed also by the employers at labor market entry. Finally, there are two time-varying shocks, one being a stochastic component ε that follows a stationary AR(1) process with normally distributed shocks ν^ε , and the other one being a stochastic component s that follows a random walk process with normally distributed shocks ν^s . The income dynamics over the life-cycle for a generic individual i are driven by an exogenous process, specified as follows:

$$\left\{ \begin{array}{l} \epsilon_{i,j} = f_{\alpha,i} + f_{\beta,i} * (j - 1) + \varepsilon_{i,j} + s_{i,j} \\ \varepsilon_{i,j} = \rho_\varepsilon \varepsilon_{i,j-1} + \nu_{i,j}^\varepsilon \\ s_{i,j} = s_{i,j-1} + \nu_{i,j}^s \\ (f_{\alpha,i}, f_{\beta,i}) \stackrel{iid}{\sim} N(m_{f_\alpha}, m_{f_\beta}; \sigma_{f_\alpha}^2, \sigma_{f_\beta}^2, \sigma_{f_\alpha, f_\beta}), \nu_{i,j}^\varepsilon \stackrel{iid}{\sim} N(0, \sigma_\varepsilon^2), \nu_{i,j}^s \stackrel{iid}{\sim} N(0, \sigma_s^2) \end{array} \right.$$

The estimated process implies that individuals with large labor earnings at labor market entry tend to have low wage growth over the life-cycle, while workers with low entry earnings enjoy a faster income growth. The top panel in Figure 3 shows these trajectories for three worker types.

[Figure 3 about here]

The average growth type included in the Figure represents 25.4% of the working age population, while both the low and high growth types each represent 11.4% of the working age population.⁹ Given the strong negative correlation between the two individual effects, the high growth types tend to start their career with a low level of earnings and vice versa. An interesting feature of the earnings dynamics attributable to the fixed effects is that, absent the other shocks, close to the retirement age these three worker types would enjoy virtually the

⁹I discretize the fixed effects with six types in each dimension, for a total of thirty six possible pairs of fixed effects values (i.e., worker types at labor market entry). Notice that, not to clutter the figure, the average type plotted in the graph corresponds to the four middle types in the quantitative model. In order to maximize the accuracy of the numerical computations, I undertook a form of pruning of the discrete points in the support of the bi-normal density (taking into account that the two random variables are negatively correlated), and the symmetry of the shares at the extreme points of the grid is obtained by construction. By way of comparison, the other two extreme categories, high level-high growth and low level-low growth types, represent less than 0.001% of the labor force.

same level of earnings, of about \$57,500.¹⁰ There is a type of labor income convergence taking place, as upon retirement (abstracting from other shocks) these worker types command virtually the same labor earnings. A possible interpretation of these dynamics sees individuals with high educational achievements on one side of the spectrum, who often begin their careers with low paying internships and co-op experiences. On the other side of the spectrum there could be athletes hired by professional teams, young entertainers, and other individuals involved in the show-business or other artistic careers, whose pay can be relatively large from early on, and possibly decline over time. In terms of the consumption smoothing behavior and the latent demand for public liquidity, the low growth types are in a favorable position. Their high entry earnings allow them to start saving early on, providing them with a buffer stock of wealth that can be used against negative income shocks. For these agents, self insurance is extremely effective and not very costly, as they are also saving for retirement and to smooth their initial high labor earnings over their lifetime. Differently, both the average and high growth types are constrained for two reasons: they would like to borrow against their substantial future incomes and also to partially offset negative income shocks.

The bottom panel of Figure 3 plots the earnings dynamics over the life-cycle attributable to the Random Walk component. Two patterns are worth pointing out. First, since the variance of the shocks increases linearly, the dispersion of these permanent shocks grows over the life-cycle. Second, since these shocks in logs are symmetric around zero, their average is zero, while the average of the RW component in levels is greater than one, because of Jensen’s inequality.¹¹

Pensions: There is a Pay-as-you-go pension system, financed by the contributions of the employed workers. There is dispersion in the pensions, as the pension benefits $\bar{y}_R(f_\alpha, f_\beta)$ depend on the individual fixed effects.¹² The pension benefits are financed with a proportional tax τ_R on the current labor earnings, and the equilibrium value of this tax guarantees a balanced budget.

Taxes, Government Outlays and public debt: There is a flat consumption tax rate τ_c and the income taxes are proportional. The income tax rate τ is set to satisfy the intertemporal budget constraint of the government. Every individual is entitled to an unconditional lump-sum transfer tr_G . The transfer combined with the flat income tax is a parsimonious specification to obtain a progressive fiscal system. The government finances wasteful and exogenous public expenditure G . Since the government’s intertemporal budget constraint has to be satisfied, total tax revenues T (obtained from taxing labor income, asset income, pensions, and consumption) minus total government outlays (the sum of government purchases G and total public transfers

¹⁰In terms of dollar values, the deterministic component of earnings might appear low (the base year is 2010). However, one has to take into consideration that total labor earnings are on average higher compared to the values depicted in this plot, because the average of the other stochastic components of the income process is different from one.

¹¹The theoretical unconditional average of the stationary AR(1) component in levels is one. By construction, the Rouwenhorst method used to discretize it ensures that this property is preserved. See Figure 11 in Appendix D.

¹²For computational reasons, I cannot keep track of the actual individual contributions to the system, as it would require an additional continuous state variable. The simpler approach used here assigns pensions on the basis of the average labor market histories, namely it considers the role of both the level and the growth fixed effects, while it collapses the effect of the other shocks to their unconditional averages.

TR).

The evolution of public debt D over time is obtained by considering the intertemporal budget constraint of the government, where D' stands for the next period value:

$$D' = (1 + r)D + G + TR - T$$

In a steady state, public debt is constant over time, and it is such that the related interest costs are equal to the excess of taxes beyond the total government outlays:

$$rD = T - G - TR$$

In a steady state aggregate output Y is also constant, and the previous relationship can be considered relative to GDP, where $d \equiv D/Y$, $t \equiv T/Y$, $g \equiv G/Y$ and $\xi \equiv TR/Y$:

$$r \frac{D}{Y} = \frac{(T - G - TR)}{Y} \rightarrow d = \frac{(t - g - \xi)}{r} \rightarrow t = rd + g + \xi$$

The last version of the formula represents the equilibrium condition that, for given public debt, government expenditure and public transfers policies, can be used to obtain the value of the tax rate τ .

Technology: Production in both the home economy and in the foreign one relies on labor and capital to produce a homogeneous good that can be sold internationally. The production functions are Cobb-Douglas, with region-specific inputs and parameters. The interpretation of the RoW region is that there is a measure $\bar{\mu}_{RoW}$ of foreign economies, each sharing the same preferences and the same technology, a simple Cobb-Douglas production function with aggregate labor (L_{RoW}) and capital (K_{RoW}) as inputs, and with a Total Factor Productivity parameter (Z_{RoW}).¹³ Also in these countries there are incomplete insurance markets and the workers are facing labor income risk. For tractability, I assume that the RoW region is populated by infinitely lived agents, that are subject to an exogenous borrowing constraint, and that their income dynamics are represented by a simple AR(1) stochastic process. It follows that in every foreign economy the aggregate labor input is $L_{RoW} = 1$. Given the absence of aggregate shocks, at the aggregate level each foreign country $q \in \mathcal{Q}$ displays the same outcomes and they can be conveniently considered as a unitary region.

$$Y = K^\alpha L^{1-\alpha}$$

$$Y_{RoW,q} = Z_{RoW} (K_{RoW,q})^{\alpha_{RoW}} (L_{RoW,q})^{1-\alpha_{RoW}}$$

The input markets are perfectly competitive, and the firms maximization problem leads to the usual equilibrium conditions equating marginal products to the related input price.

¹³I chose this formulation, instead of an alternative one with the RoW using an aggregate production function with RoW labor and capital as inputs, because it is possible to obtain all the relevant parameters by computing cross-country averages.

$$\begin{aligned}
r &= \alpha K^{\alpha-1} L^{1-\alpha} - \delta = \alpha_{RoW} Z_{RoW} (K_{RoW,q})^{(\alpha_{RoW}-1)} (L_{RoW,q})^{1-\alpha_{RoW}} - \delta_{RoW}, \forall q \in \mathcal{Q} \\
w &= (1 - \alpha) K^\alpha L^{-\alpha} \\
w_{RoW} &= (1 - \alpha_{RoW}) Z_{RoW} (K_{RoW,q})^{\alpha_{RoW}} (L_{RoW,q})^{\alpha_{RoW}}, \forall q \in \mathcal{Q}
\end{aligned}$$

Since in the global asset market there is only one interest rate r , the symmetry between countries can be imposed, such that $K_{RoW,q}/L_{RoW,q} \equiv k_q = k, \forall q$. It follows that total output in the foreign countries $Y_{RoW} \equiv \int_{\mathcal{Q}} Y_{RoW,q} dq$ can be computed as:

$$Y_{RoW} = \int_0^{\bar{\mu}_{RoW}} Z_{RoW} (K_{RoW,q}/L_{RoW,q})^{\alpha_{RoW}} (L_{RoW,q})^{1-\alpha_{RoW}} dq = Z_{RoW} k^{\alpha_{RoW}} \int_0^{\bar{\mu}_{RoW}} 1 dq = Z_{RoW} k^{\alpha_{RoW}} \bar{\mu}_{RoW}$$

Since I focus on the welfare effects for the home economy, stemming from policies implemented by the domestic government, I assume that the RoW region has an inactive public sector.

Figure 4 depicts the determination of the equilibrium in the two-region economy. In the plot, the label *CA* stands for Canada (the Home economy), and *RoW* for the Rest of the World. As in the textbook two-large-economies model, the equilibrium interest rate is found when the current accounts of the two regions are consistent with each other. Given that the Canadian data on the long-run average of its net financial position show a value that is virtually zero, in the initial steady-state (namely, under the status quo Canadian public debt), the current accounts of both regions are forced by the calibration to be equal to zero. Notice that the home economy has two variables determining the total demand for assets, which are the demand for capital by firms ($K^{d,CA}$) coupled with the demand for savings to hold the public debt (D^{CA}). In both economies there is a supply of assets coming from the aggregation of the saving decisions of the households in Canada ($A^{s,CA}$) and in the other region ($A^{s,RoW}$). Any increase in the domestic public debt/GDP ratio brings about an outward shift in the Canadian asset demand. This causes an imbalance in the current accounts, which triggers an interest rate adjustment, until the general equilibrium is restored.¹⁴

[Figure 4 about here]

3 Calibration

In order to assign values to the parameters of the Two-Region model, I rely on a mix of (reduced-form) estimation and calibration (in equilibrium) methods. The initial steady-state is calibrated to reproduce selected long-run features of both the Canadian economy and the RoW. I begin by presenting the calibration for the home economy, then I move to present the one for the RoW region.

¹⁴It is worth mentioning that the welfare effects that will be reported do not stem from the OLG economy being dynamically inefficient, as in all equilibria (status-quo and counterfactuals) the interest rate (net of depreciation) is always positive. This guarantees that the economy is never in the dynamically inefficient region.

3.1 Home Economy Parameters

Table 1 reports the list of the calibrated parameters with their values and associated empirical targets. These parameters are the ones that apply to Canada, and that can be set externally, without solving the model.

[Table 1 about here]

Demographics: The following parameters are related to the demographic aspects: J_R, J, π_j^d . Agents become economically active at age 26, a choice dictated by the lowest age group included in the labor earnings sample. They retire at age $J_R = 66$, and they can live up to $J = 101$ years. These are conventional and fairly innocuous assumptions, also made in other studies calibrating OLG models. The survival probabilities π_j^d are obtained from the Canadian Life Tables. In particular, I averaged the values of each age-dependent survival rate reported for every year of the 1990-2007 period.¹⁵

Technology: The labor share is computed from the Penn World Tables 9.0 (PWT9.0), and its long-run average implies a capital share $\alpha = 34.5\%$. Also the capital depreciation rate is obtained from the PWT9.0, and its long-run average is equal to $\delta = 0.0422$.

Preferences: The relative risk aversion is set to $\theta = 1.50$, which matches an elasticity of intertemporal substitution of 0.75, a common value in the literature.

Income process and pensions: The baseline parameterization of the exogenous stochastic income process mainly relies on the values reported by Baker and Solon (2003). Their estimates, based on the T-4 tax return data, are $\sigma_s = 0.0837$ for the standard deviation (s.d., hereafter) of the permanent –Random Walk– income shocks (RW, hereafter), while $\rho_\varepsilon = 0.54$ and $\sigma_\varepsilon = 0.3579$ are the estimates of the autocorrelation and s.d. of the innovations for the persistent (AR1) component. The heterogeneity at labor market entry is captured by the dispersion of the two individual effects: $\sigma_{f_\alpha} = 0.058$ is the s.d. of the level fixed effect, while $\sigma_{f_\beta} = 0.0095$ is the s.d. of the slope fixed effect. Finally, the covariance between the level and slope fixed effects is $\sigma_{f_\alpha, f_\beta} = -0.0031$. Notice that the resulting correlation index between the two fixed effects is negative and very high, being -0.89 .¹⁶

Baker and Solon (2003) do not provide estimates for two parameters that are needed in the quantitative model for the computation of the labor efficiency profiles: the average growth rate of labor earnings during the life-cycle and the average intercept. Since the tax return data are confidential, I computed these two parameters estimating with OLS a simple log-wage regression on Labour Force Survey (LFS) data. The LFS is a representative sample of the Canadian workforce, and the estimates obtained in the two samples should

¹⁵The two top panels in Figure 11 in Appendix D plot the survival probabilities over the life-cycle together with the resulting age distribution.

¹⁶Figure 10 in Appendix D plots the joint bi-normal density implied by these estimates.

be similar. The average of the level fixed effect is $m_{f_\alpha} = 10.10$ and the average of the slope fixed effect is $m_{f_\beta} = 0.013$.¹⁷

The pension replacement rate is taken from [OECD \(2015\)](#), and its value is $\phi_R = 0.479$.

Taxes and Government: The consumption tax rate is $\tau_c = 0.124$, which is the average of the estimates reported by [Mendoza, Razin and Tesar \(1994\)](#), which include both the PST and the GST.

The wasteful government consumption is set to match the long-run average over the 1990-2007 period of the public expenditure/GDP ratio, which is $g = 0.214$. Over the same time horizon, and as described in Section 1.1, the public debt/GDP ratio is set at $d = 0.685$.

Asset Markets: The borrowing limit is set to $\underline{a} = -0.31$, such that households can borrow up to 20% of the average labor earnings.¹⁸

3.2 Rest of the World Parameters

A number of parameters for the RoW economy can be directly estimated using the PWT9.0. Table 2 reports the full list of the calibrated RoW parameters with their values and associated empirical targets.

[Table 2 about here]

Demographics: The agents in the RoW region are assumed to be infinitely lived. This assumption circumvents the challenging step of obtaining life-cycle profiles for productivity and survival rates.

Technology: The labor share and the capital depreciation rate are computed from the PWT9.0. Since the data quality for some developing countries can be problematic, I exclude the effect of potential outliers by considering the median in each annual cross section. I then take the time average of this statistic's time series. Computed this way, the RoW labor share is 55.9%, which implies $\alpha_{RoW} = 44.1\%$. A similar procedure gives a capital depreciation rate equal to $\delta_{RoW} = 0.0432$. In order to obtain the GDP for the RoW, the number of employed workers is needed. I compute this figure from the PWT9.0.

¹⁷I relied on the monthly LFS from January 1997 to December 2007. Notice that before 1997 wage data were not included in the LFS, so those waves cannot be used. Because of measurement error concerns, in each wave of the dataset and for every age group, I trimmed both the top and bottom 5% of the hourly wages. I also performed the estimation on the whole sample, without trimming, and the values were quite similar. The wage data are adjusted for inflation by using the monthly CPI index. The model is yearly and the [Baker and Solon \(2003\)](#) T-4 data refer to yearly earnings, hence to make the datasets consistent with each other I multiplied the hourly wages in the LFS by the usual number of weekly hours worked. I then multiplied the weekly earnings by an estimate of the number of weeks worked in a year consistent with the data on aggregate number of hours worked available for the Canadian Economy. In the log-wage regression, I included a full set of time dummies, and I added to the intercept the time average of their estimated coefficients. More details are reported in Appendix B.

¹⁸I conducted some robustness checks, setting $\underline{a} = 0$ and the results are not heavily affected by this parameter.

Preferences: Since there is no evidence that Canadian and foreign households differ systematically in their attitudes towards risk, I set the relative risk aversion parameter equal in the two economies $\theta = \theta_{RoW} = 1.50$.

Income process and Labor input: Since many Countries are lacking good-quality panel data with household-level information on labor income, for the RoW I postulate a simple AR(1) process. The persistence and the s.d. of the autocorrelated income shocks are obtained averaging the available estimates in the literature on income dynamics.¹⁹ This procedure gives $\rho_{\varepsilon, RoW} = 0.949$ and $\sigma_{\varepsilon, RoW} = 0.276$. Each country’s labor input averages to 1. The measure of the countries is estimated dividing the total number of workers in the PWT9.0 by the average number of workers, which gives $\mu_{RoW} = 126.8$.

Asset Markets: The borrowing limit is set to $\underline{a}_{RoW} = 0$, such that foreign households cannot borrow.

3.3 Parameters Set in Equilibrium

Four parameters do not have a direct empirical counterpart, and need to be set in equilibrium. These parameters are reported in the middle section of Table 2, and they are selected to match the following statistics.

The Canadian data on the national net lending/net borrowing position (Cansim series v62305900) show that the long-run average is virtually zero. This figure implies that the long-run average of the Canadian current account is approximately equal to zero. Moreover, the average real interest rate for the period 1990-2007 paid by the Canadian Treasury bills was $r^* = 2.79\%$.²⁰ Given that this paper focuses on the welfare effects of Canadian public debt, I chose to target the rate of return of this asset. Since capital and public debt are both risk-free, arbitrage forces their rate of return to be equal. In a perfectly integrated financial market the interest rate for the RoW and home economies must be the same. I then calibrate the Canadian and RoW discount factors to match the two targets above in the initial steady state, obtaining $\beta = 1.0123$ and $\beta_{RoW} = 0.958$.²¹ The TFP for the home economy is normalized to 1, while I set the TFP for the RoW region to match the long-run average of Canada’s share in global asset markets. More precisely, I match the ratio between Canada’s and the RoW’s capital stocks, both taken from the PWT9.0. This figure is equal to $K/K_{RoW} = 2.26\%$, and $Z_{RoW} = 0.31$ matches this target.

The transfers of the Canadian government ξ are also calibrated in equilibrium. This variable affects the degree of inequality in the economy, which is reduced by these unconditional (lump-sum) transfers. $\xi = 0.19$ matches an income inequality pre-post government ratio of $Var[\log(y^{Post})]/Var[\log(y^{Pre})] = 46\%$, which is the average value of this statistic reported by Brzozowski, Gervais, Klein and Suzuki (2010).²²

¹⁹In particular, I consider the estimates for the U.S. reported by Guvenen (2009), for Sweden reported by Domeij and Floden (2010), for Russia reported by Gorodnichenko, Peter and Stolyarov (2010), for France, Germany, Italy, Spain, and the U.K. reported by Le Blanc and Georgarakos (2013), and for Japan reported by Lise et al. (2014).

²⁰I obtained this statistic by converting the nominal monthly series, included in the International Financial Statistics Yearbooks published by the IMF, into a real annual one. More details are reported in Appendix B.

²¹Also in the closed economy version of the model, that eventually will be considered, the home discount factor β is chosen to match an equilibrium interest rate of 2.79%.

²²The parameters reported in the bottom section of Table 2 refer to two parameters that are included in the robustness checks. I consider two extensions to the basic SOE model. In the first one, there is a costly financial intermediation leading to a differential

4 Results

In this section, I discuss the quantitative results related to the two-region model. The analysis is based on implementing a series of counterfactual public debt policies, ranging from $d = -2$ to $d = 2$. I then report the steady state implications for welfare (expressed as % changes in consumption, for both the newborns and society as a whole, with a utilitarian social welfare function), prices, aggregate variables and taxes.

[Figure 5 about here]

Figure 5 presents the equilibrium outcomes for the home economy. The first panel conveys a strong message. According to this model, the Canadian long-run public debt has been inefficiently large. The welfare of both the newborns and the households taken as a whole could be drastically improved by moving from a policy of public debt to a policy of Public Savings. The behavior of both welfare measures is very much alike and they also are quantitatively similar. A striking result is the size of the welfare gains of moving from the long-run public debt/GDP ratio of 68.5% to Public Savings. For example, a public debt/GDP ratio of -200% would entail welfare gains in excess of 20%.

The second panel displays the response of the interest rate. With an overall change of four times the size of Canadian GDP, the interest rate moves from 2.77% to 2.81%. As expected, this is quantitatively small, yet different from a completely flat behavior.

The third panel is devoted to representing the response of GDP and consumption. Given the marginal change in the interest rate, aggregate output only declines mildly, showing a negligible effect of crowding out of investment in physical capital. Differently, the change in aggregate consumption is drastic. This is due to the rising costs of servicing the public debt. This moves from an additional source of income on top of the public transfers, to a disbursement paid mostly to foreigners.²³

This behavior is mirrored by the increase in the proportional tax rate on income, presented in the fourth panel. This variable changes from 25% to 42%, the status quo being characterized by a tax rate of about 35%.

[Figures 6 and 7 about here]

By way of comparison, I conduct some quantitative experiments assuming that the Canadian economy is closed. Figure 6 plots the related results. In this counterfactual scenario, I do find that public debt could be optimal, at least according to some welfare measures, reconciling my results with those of [Aiyagari and McGrattan \(1998\)](#). The first panel in the figure shows that social welfare (arguably, the better measure to base the comparison on, as they have an infinitely-lived household setup) peaks at a positive and substantial value of public debt of 105%. Just like in [Aiyagari and McGrattan \(1998\)](#), this welfare measure is exceptionally flat between borrowing and lending interest rates. The second extension postulates an interest rate determination rule that is a function of the public debt/GDP ratio. For more details see Appendix D.

²³It is worth mentioning that in the status-quo the share of public debt held domestically is indeterminate, because both assets are risk free and pay the same rate of return. However, in some extreme counterfactuals the size of the public debt can get larger than the domestic savings.

for a wide range of public debt policies. Interestingly, the newborns would prefer to have considerable Public Savings of 135%, a result that mirrors the findings in [Peterman and Sager \(2018\)](#), albeit in quite a different environment. This is due to the compounded effect of larger public transfers (partially financed by the public asset income), and larger wages per efficiency units, that ensure better consumption smoothing in the early stages of the life-cycle. The related lower interest rates allow for more capital used in production, leading to higher output and aggregate consumption. Differently from the open economy case, the response of the tax rate is non-monotonic. Compared to the newborns' welfare, social welfare responds differently, as most individuals in the economy hold some wealth and are partially damaged by lower equilibrium interest rates, especially the retirees.

The discrepancy in the welfare measures is made clear by the age profiles of the welfare effects depicted in [Figure 7](#). Unlike in the two-region economy, where the welfare effects have the same sign for virtually all ages, in a closed economy there is a generational conflict. The younger individuals have welfare effects that are opposite in sign compared to most of the older ones. Moreover, this conflict can be acute as the heterogeneity in the welfare effects is sometimes extreme: in the $d = -2$ case, abstracting from the very few individuals older than 90, the range is $[-12\%, +1\%]$.

4.1 Discussion

The quantitative results are in line with the intuition. A small economy with access to integrated financial markets can only have a second order effect in the determination of the interest rates. As a consequence, the liquidity provision effect of public debt is diluted, limiting its effectiveness in improving the self-insurance of borrowing constrained individuals. The results show that the liquidity effect channel is unlikely to be an important explanation for why small economies have also accumulated large public debts. Although the data plotted in [Figure 1](#) show a positive correlation between the size of GDP and the public debt/GDP ratio, in the subsample including the smaller economies there is substantial variation in the latter variable. The results in this section plausibly make the liquidity provision rationale for public debt a quantitatively irrelevant explanation for several small and open Countries. In the next section I extend the model to allow for valuable public services, and to quantify whether a stylized political channel can help explain the large Canadian public debt.

5 A Model with Public Goods

In this extension, the per-period utility function is defined over consumption of a privately provided good c and a pure public good c_G . I assume that there is a Cobb-Douglas aggregator between the private and public good, with η denoting the weight of private consumption in the utility: $u(c_j, c_G) = \frac{(c_j^\eta c_G^{1-\eta})^{1-\theta} - 1}{1-\theta}$.

Given the previous quantitative findings, in this section I am going to consider Canada as a SOE. The government is still in charge of distributing public transfers TR , and collecting taxes T . However, now a given fraction γ_g of total government expenditure G takes the form of a pure public good $c_G = \gamma_g G$.²⁴ In my definition

²⁴ $(1-\gamma_g)G$, the difference between publicly provided goods and total government expenditure, can be interpreted as the inefficiency of the public sector, namely a rents extraction mechanism, which is however assumed to be exogenous and not affected by different

of publicly provided services I include several categories. The first one is the fraction of health expenditure that is covered by the public sector, which for Canada is approximately 70%.²⁵ The other items are public spending on education, parks and defense. According to this notion of public services, their total value accounts for 12.8% of GDP, namely $\gamma_g = 0.128$. In per-capita terms, this corresponds to 2010 CAD\$5,919.²⁶ Conceivably, some of these items may not qualify as pure public goods. However, since the demographic structure is exogenous, the model would imply the same results if I were to model them as publicly provided private goods.²⁷ Incidentally, this formulation captures the fact that fiscal reforms cutting public expenditures might be only imperfectly substituted with their private goods counterpart.

5.1 An Exploratory Fiscal Rigor Mechanism

A plausible explanation for large public debts is that government spending also provides services that are valuable in terms of people’s well-being. “Austerity” policies have typically to be implemented to reduce public debt, with cuts in public expenditure generating the resources for the desired adjustment. Symmetrically, large public debts can be incurred when public expenditure rises quickly.

[Figure 8 about here]

Figure 8 displays the mechanism at play in this extension and hints at a critical feature of this approach. The first two panels show the welfare effects of changing the value of public debt, which partially maps into a decrease of the quantity of public goods provided (on the basis of a correlation that will be estimated below). The first panel focuses on the welfare of the newborns, while the second one on social welfare. Different curves refer to the welfare effects for different values of the weight η . The plots show that both welfare effects behave similarly, qualitatively and quantitatively. It goes without saying that if the weight were to be equal to one, and if we were to consider a negative correlation between public debt policy and public goods provision, we would go back to the results of the previous section. The solid lines refer to a case where the individuals are characterized by a high weight on private consumption, $\eta = 0.95$. As in the previous section, the welfare effects respond almost linearly to changes in public debt. It is apparent that smaller utility weights on the private good bring about a non-linearity, as the gains in terms of lower taxes are offset by a decrease in the provision of public goods. Not only for a low enough η the curves are no longer monotonic, but also the slope can now become positive for low values of public debt, eventually turning into a negative slope for higher values of public debt. In other words, the welfare effects can display an inverse-U shape. This argument should make clear that, absent a reliable estimate for η , almost anything goes. Different weights of private consumption in the utility function imply different preferences for the quantity of public goods that should be provided. As the plot shows, with low values of η , individuals prefer larger public debts that are partially used to finance public goods.

public debt policies.

²⁵In the last two decades, this share has been remarkably stable. The remaining 30% is split between private insurance (12%), out of pocket (15%), and other sources (3%), as reported in [Canadian Institute for Health Information \(2018\)](#).

²⁶Notice that I do not include police expenditures, as they are typically financed with property taxes set at the municipal level.

²⁷Since I assume that there is no congestion, the analysis is providing an upper bound.

The welfare analysis depends crucially on the rate of substitution between private and public goods. A reliable estimate of the curvature parameter in the utility function is needed. Figure 8 shows how the two considered welfare measures are affected by different values of the weight η . The lower the weight of private consumption in the utility function, the more concave the welfare measures become, making a positive optimal public debt more likely. I then use a revealed political preference approach: I calibrate this parameter in order to maximize the political support of the steady-state public policy.²⁸

From the third panel in the plot it is immediate to see what is the source of identification for the parameter η . Different values of η imply different votes in favor of policies distinct from the status quo.²⁹ In particular, I consider two small perturbations around the long-run value of public debt, and I select the value of η that minimizes the (average) probability for the long-run public debt policy to be defeated in a proportional election.³⁰

Many model parameters are calibrated following the same steps used in the previous Section. However, some of them need to be calibrated in equilibrium. Table 3 reports the list of the calibrated parameters with their values and associated empirical targets.

[Tables 3 and 4 about here]

The interest rate is set again to $r = 2.79\%$, but now it is exogenous and fixed. The discount factor is then chosen to set the long-run current account to zero, where $\beta = 1.0138$ matches this target. The value of the public transfer as a share of the GDP is similar to the one used for the benchmark model, with $\xi = 0.19$.

Another less trivial aspect is the relationship between public debt and government expenditure. A mechanical implication of steady state analysis is that, for a fixed level of taxes and public transfers, there is a long-run negative relationship between public debt and government expenditure. The reason is straightforward, because the government's intertemporal budget constraint must be satisfied. For constant taxes, lower public debts imply lower interest payments, freeing up resources that can be used to increase government expenditure. However, empirically there is no such negative relationship. In fact, I ran two regressions on OECD data that show a positive correlation, and are reported in Table 4.³¹ An interpretation of this finding is that, in order to obtain

²⁸Notice that I cannot rely on the median voter theorem, as the policy is multidimensional. Furthermore, individuals differ in several dimensions: keeping track of the indirect utility of all agent types is computationally intractable, as for any possible public policy there is a continuous asset distribution for $(6 \times 6 \times 5 \times 5 \times 41) + (6 \times 6 \times 35) = 38,160$ types. Although there is no guarantee that the indirect utilities are single-peaked, I find that the number of votes are well-behaved.

²⁹This can be interpreted as a model with office motivated politicians, that select the level of public debt in order to maximize the political support for this policy. For a more sophisticated treatment of the political-economy equilibrium in models with incomplete markets see Corbae, D'Erasmus and Kuruscu (2009) and Azzimonti and Yared (2019). Early computational models of voting, for example Krusell, Quadrini and Rios-Rull (1997), circumvented some of the numerical challenges by limiting the amount of heterogeneity.

³⁰In the quantitative implementation, the proportional votes respond smoothly in the parameter and there is no evidence of multiple political equilibria (e.g., if the curves were to display multiple maxima in the neighborhood of the steady-state value). Finally, I also impose the constraint that no alternative policy achieves a majority of votes.

³¹It is worth stressing that these regressions are not designed to identify a causal effect. The goal is to obtain a reliable estimate of the partial correlation between public debt and public expenditure. The dependent variable in the regression is the public debt/GDP ratio, the explanatory variables are the Total Public Outlays/GDP ratio (gtotalgdp), the Social Security Benefits/GDP (ssgdp), the Total Tax Revenues/GDP (taxgdp), and two interest rate measures for the short-run (shortintrate) and for the long run (longintrate).

changes in the level of public debt, considerable fiscal adjustments need to be undertaken. Governments worried about the size of their public debts typically implement cuts to their public expenditures, entering a period of fiscal rigor. Similarly, public debts increase in periods of fiscal negligence. Imposing this correlation in the counterfactual analysis mimics the implications of either cuts in public spending to reduce the public debt, or increases in public spending that imply a rising public debt. Unfortunately, the size of the estimate varies considerably depending on the method used. I used both a Fixed Effects (FE) estimator on the OECD panel data, and a simple OLS estimator on the cross sectional data obtained from computing each country's time averages. In the former case I obtained $\hat{\beta}_{g,d}^{FE} = 1.02$, while in the latter $\hat{\beta}_{g,d}^{OLS} = 8.56$. Since both estimators possess advantages and drawbacks, I consider the quantitative implications of both sets of estimates.³²

For either estimate, the utility weight of consumption is set to the value that maximizes the political support for the observed long-run public debt. In one case, with the OLS estimate of the correlation between G/GDP and D/GDP , this procedure gives a parameter value of $\eta_{OLS} = 0.674$. In the other case, with the FE estimate of the correlation between G/GDP and D/GDP , it gives a parameter value of $\eta_{FE} = 0.704$. For a given utility weight of consumption, the curvature of the utility function needs to be adjusted to obtain the target relative risk aversion of 1.5. In the benchmark case, $\theta_{OLS} = 1.742$ matches this value, while in the other case $\theta_{FE} = 1.710$ does.

[Figure 9 about here]

Figure 9 displays the welfare effects together with the responses of both aggregate consumption and public goods. The two top panels plot the results based on the partial correlation between public debts and public expenditure estimated with OLS on the time-averages by country. The bottom panels refer to the FE estimates on the panel data. Compared to the wasteful public expenditure case considered above, the results are strikingly different. On the one hand, and unlike in the closed version of the model, the behavior of the two welfare measures is remarkably similar. However, the welfare response is no longer monotonic. Moreover, the calibrated weight implies that welfare peaks at positive values of the public debt ratio, which are not too far from the status quo. Because of the sharp negative fall in the public good, the FE case shows that the status quo, supported by a majority of the voters, is a relatively robust equilibrium. This is because the change in welfare afforded by moving to the optimum is relatively small. For a debt ratio below 60% the welfare effects become negative, meaning that a reduction in public debt coupled with a reduction in public goods (and taxes) can be detrimental. For the OLS case, the results are qualitatively similar, but they are less drastic. Not only can public debt reductions determine a welfare loss, but also these losses can be extremely large. In the OLS (FE) case the social welfare maximizing debt ratios of 27% (63%) would improve social welfare by 1.97% (1.44%), while

³²One could interpret this argument as an indication that the welfare analysis should consider transitional dynamics. However, the steady-state model is already computationally demanding (the calibration takes more than 150 hours), making the computation of the transition not feasible. Moreover, a transitional analysis is not necessarily desirable as it would require arbitrary assumptions on the length of the adjustment period and the timing of tax changes. Since there is virtually no guidance in the choice of these variables, there is too large a number of potential combinations that prevent it from making it operational. Finally, from simple Debt dynamics simulations assuming reasonable values for the adjustment period, the interest rate, and the tax changes, I obtained a correlation between D/Y and G/Y that is similar in size to the OLS estimate.

the newborns' maximizing debt ratios of 29% (64%) would improve the newborns' welfare by 1.59% (1.13%). In both cases, the optimal public debt is lower than in the status quo. The results for the two correlations differ markedly with respect to the most extreme adjustment in the public debt/GDP ratio that would make the households considered as a whole indifferent between the status quo and a lower public debt/GDP ratio. In the OLS case, this corresponds to a substantial adjustment leading to a public debt/GDP ratio of -11% , while in the FE case this would be accomplished by a public debt/GDP ratio of 60%, namely an adjustment of only 8 percentage points.

These results have a clear interpretation: Canada should not increase its long-run public debt, even if this finances a proportional increase of publicly provided goods and services. The long-run public debt is already inefficiently high, but because of the asymmetry of the welfare effects response, public debt could be reduced with an associated welfare gain. However, if the cuts in publicly provided goods were to be too extreme, this adjustment could lead to large welfare losses. This outcome would easily materialize if the FE case were to be the relevant one. Moving to a public debt/GDP ratio of 50% would entail welfare losses in the order of 20%. This number is staggering, and rationalizes a cautionary attitude when contemplating the implementation of the so-called "austerity" policies.

6 Conclusions

Sizable public debts are prevalent in small economies with access to international financial markets, and the results of the benchmark two-region model cast doubts on the liquidity provision rationale for public debt. Quantitatively, in a model with two open economies, the negligible response of the interest rate dampens one of the positive effects of public debt. Political economy mechanisms and the associated distortions, advocated for example by [Battaglini \(2011\)](#), [Alesina and Passalacqua \(2016\)](#) and [Yared \(2018\)](#), seem to be quantitatively relevant, a more plausible explanation and a promising avenue for future research.

To conclude, the results show that the long-run value of Canadian public debt has been too high. In a two-region model with incomplete markets and wasteful government consumption there is a stark message: the Canadian government should accumulate public wealth. The generous redistribution already in place and the minimal equilibrium effects on the interest rate make public debt an ineffective tool for the provision of resources to liquidity constrained individuals. Taking into consideration valuable government services, together with public debt changes positively correlated with changes in public expenditure, I still find that the Canadian public debt has been excessive. However, quantitatively the welfare gains of moving to the optimal quantity of debt are substantially smaller, and imply a positive and sizable stock of public debt. For an empirically guided parameterization of the model, the political channel implies an over-accumulation of public debt, which can be reduced at the expenses of cutting public expenditure. The results show that the welfare gains of reducing the long-run level of public debt are fairly small. However, the welfare losses of increasing it can be substantial. The quantitative framework shows social welfare losses in the 1.5 – 15% range when moving to a 80% public debt/GDP ratio, and in the 2.8 – 36% range when moving to a 90% ratio. These figures support the conclusion that the long-run value of public debt should not be increased.

Given the stylized political economy framework assumed in this paper, future work could consider alternative –and perhaps more realistic– voting protocols, as done in [Corbae, D’Erasmus and Kuruscu \(2009\)](#), [Azzimonti, Battaglini and Coate \(2016\)](#) and [Azzimonti and Yared \(2019\)](#).

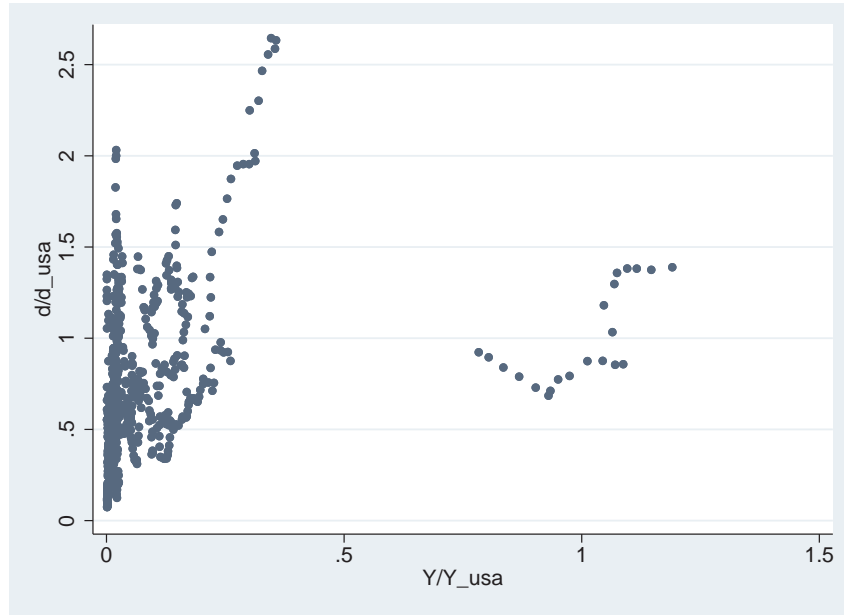


Figure 1: The Liquidity Channel Vs. the OECD Evidence, OECD data 1995-2015. The x -axis refers to the GDP of a specific country in a given year (Y) relative to the average GDP for the U.S. over the sample period (Y_{usa}). The y -axis refers to the public debt/GDP ratio (d) of a specific country in a given year relative to the average public debt/GDP ratio for the U.S. over the sample period (d_{usa}).

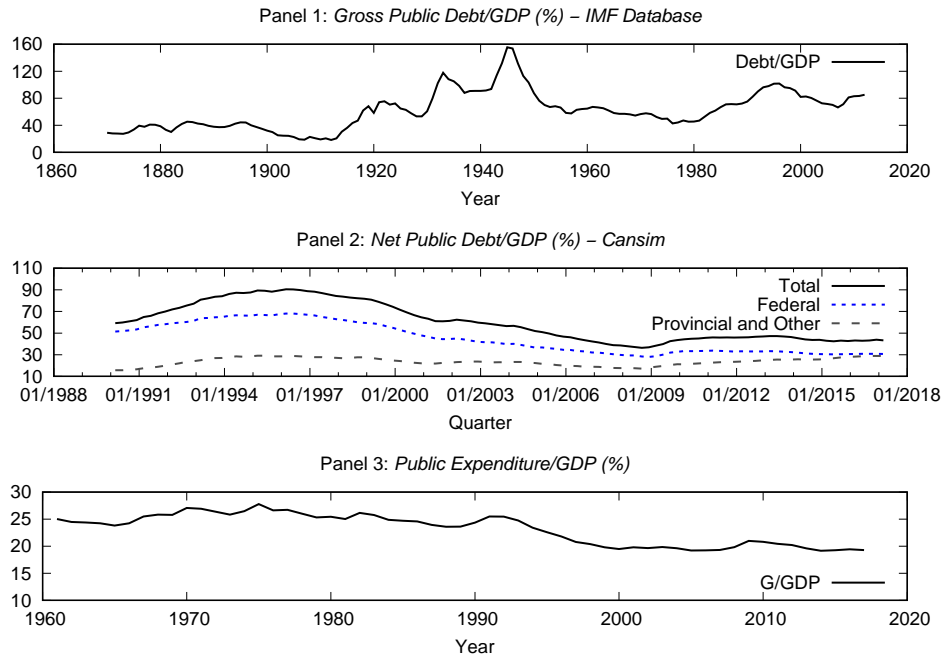


Figure 2: Time series evolution of the Canadian Gross public debt/GDP (top panel), Net public debt/GDP (middle panel), and public expenditure/GDP (bottom panel). The Gross public debt/GDP series was compiled by [Abbas, Belhocine, El-Ganainy and Horton \(2011\)](#), and it is included in a database maintained by the IMF. The Net public debt/GDP series is from Cansim data, Table 0378-0125. The Public Expenditure/GDP is computed from FRED Economic Data.

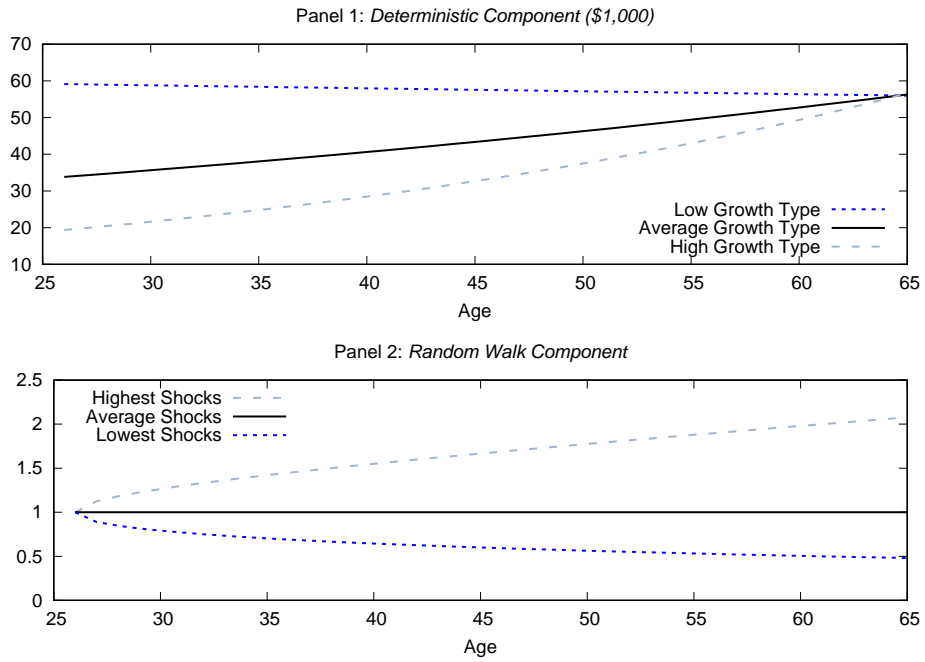


Figure 3: Pre-tax Earnings Dynamics over the Life-Cycle. The top panel plots the individual effects component of labor earnings (2010 CAD\$), while the bottom panel the stochastic random walk component.

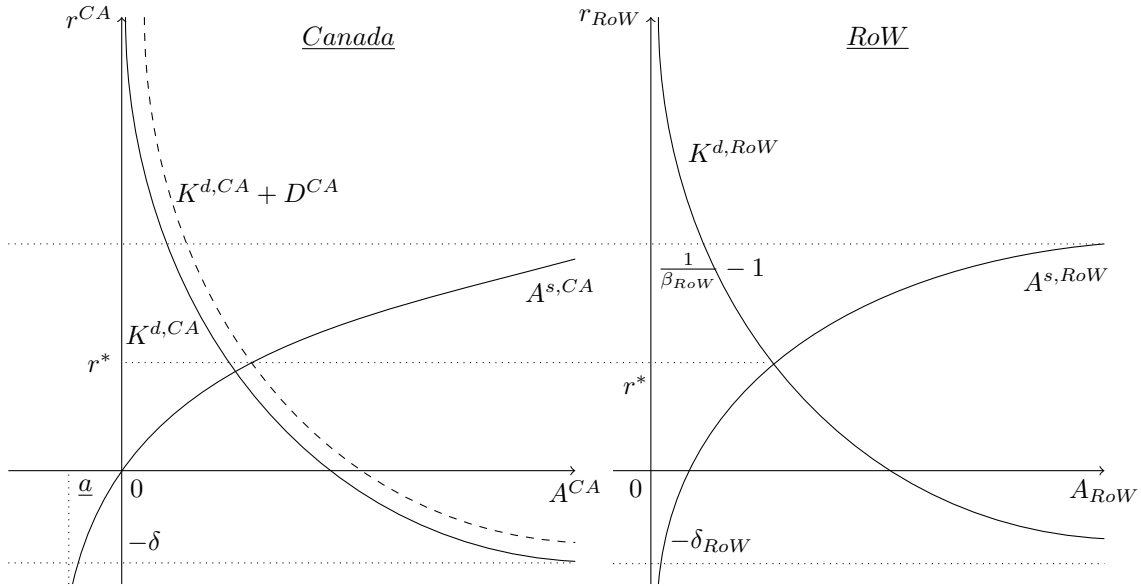


Figure 4: Equilibrium determination in the two-region economy. *CA* stands for Canada (the Home economy) and *RoW* for the Rest of the World. In the initial steady-state, under the status quo public debt, the current accounts of both regions are equal to zero.

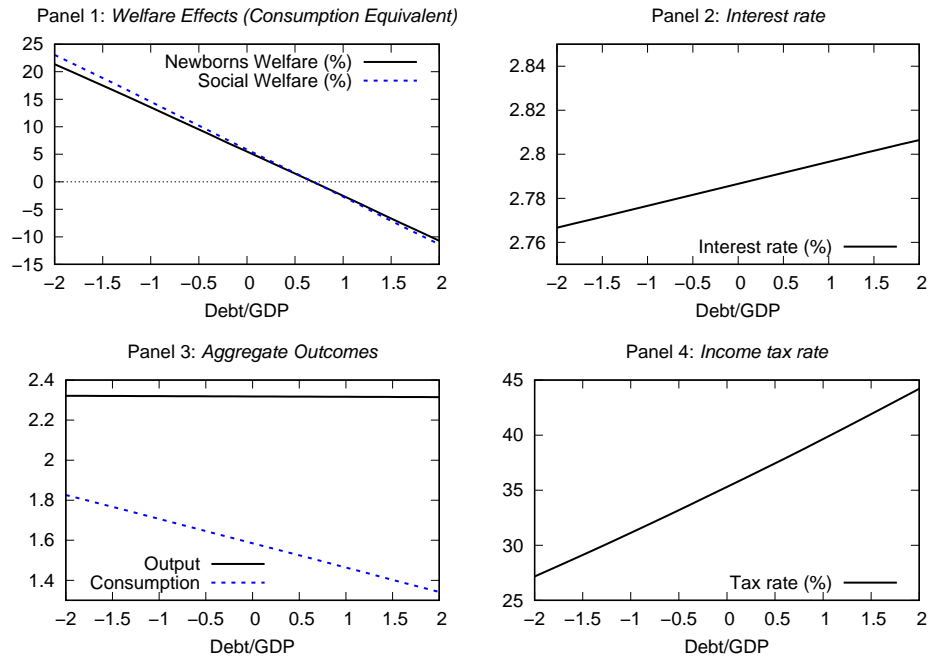


Figure 5: Equilibrium outcomes for the two-region economy.

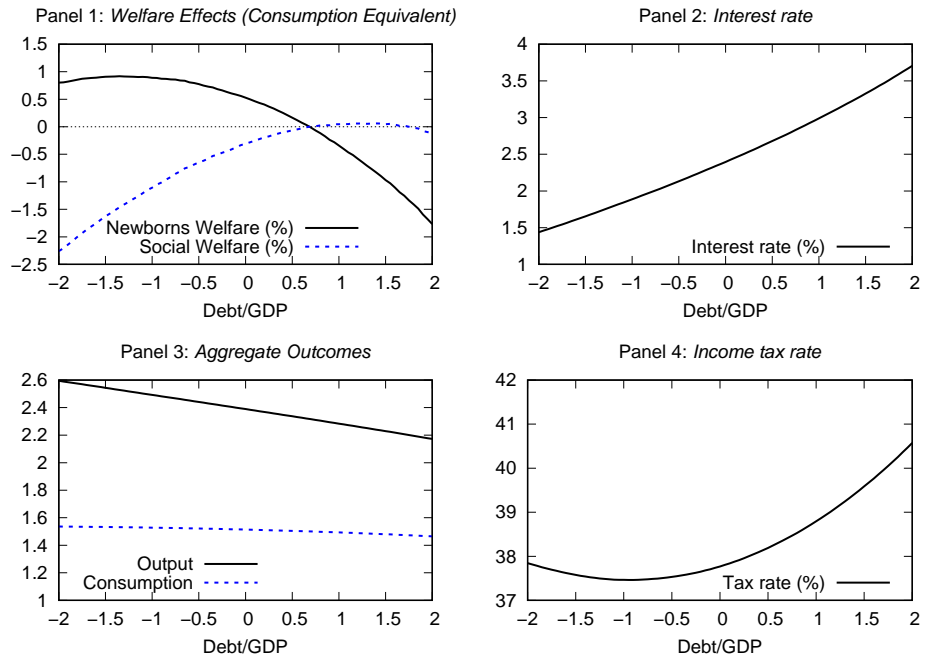


Figure 6: Equilibrium outcomes for the closed economy.

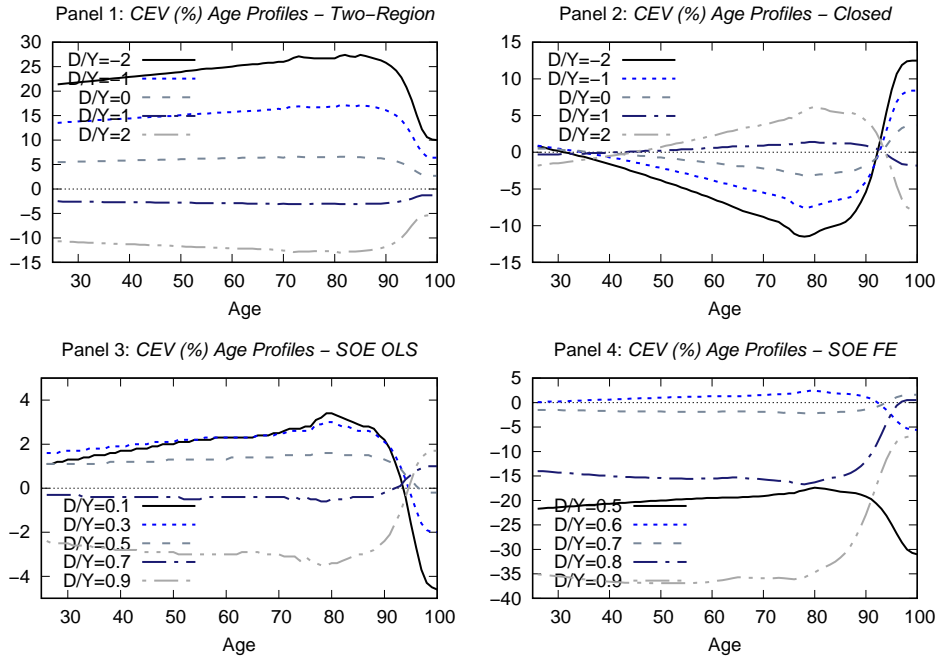


Figure 7: Age-Dependent Welfare Effects.

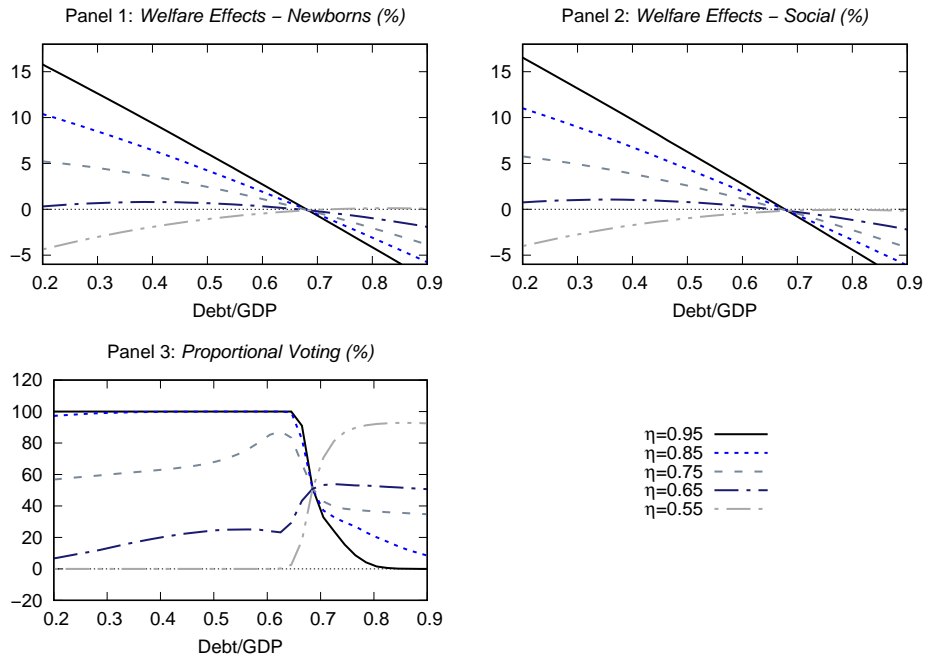


Figure 8: Welfare effects and proportional votes responses to different values of the parameter η .

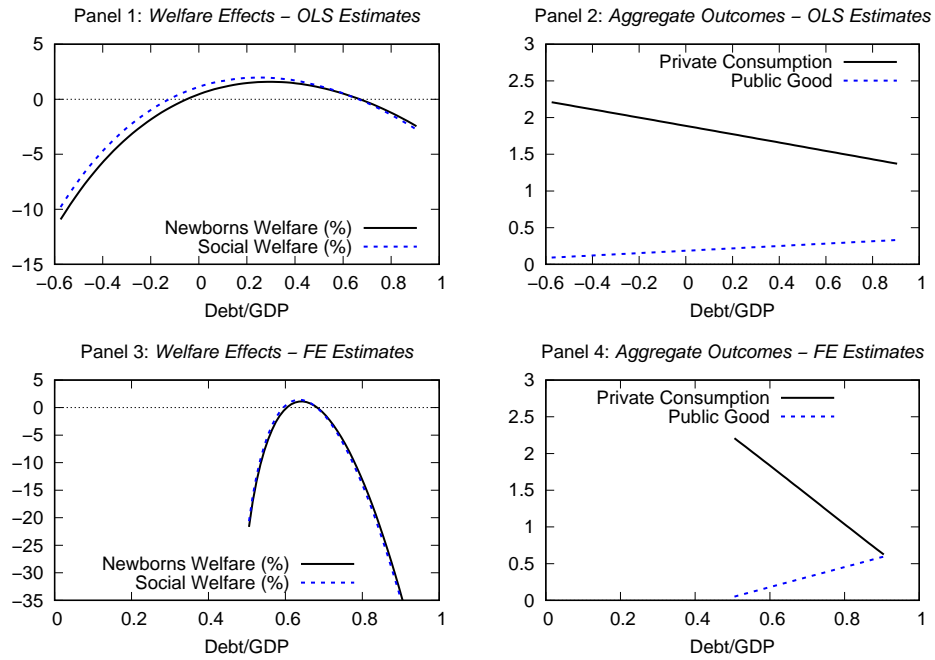


Figure 9: Equilibrium outcomes for the Small Open Economy with Public Goods. The top panels refer to the analysis done with the partial correlation between public debt and public expenditure estimated with OLS on the time-averages by country. The bottom panels refer to the Fixed Effects estimates on the Panel data (see the OLS and FE columns in Table 4).

<i>Parameter</i>	<i>Value</i>	<i>Target</i>
<i>Set Externally - Home Economy:</i>		
<i>Model Period</i>	<i>Year</i>	<i>Frequency of T-4 Tax Return Data</i>
<i>J - Maximum Age</i>	76	<i>Certain death at age 101</i>
<i>J_R - Maximum Working Age</i>	41	<i>Retirement at age 66</i>
<i>π_j^d - Death probabilities</i>	-	<i>Canadian Life Tables, average for 1990-2007, see Appendix D</i>
<i>δ - Capital depreciation rate</i>	0.0422	<i>Capital depreciation estimates, PWT9.0</i>
<i>α - 1-Labor share</i>	0.345	<i>Labor share of output = 65.5%, PWT9.0</i>
<i>θ - Risk Aversion</i>	1.5	<i>Elasticity of Intertemporal Substitution = 0.75</i>
<i>ρ_ε - Persistence of the AR(1) income shocks</i>	0.540	Baker and Solon (2003)
<i>σ_ε - S.d. of the AR(1) income shocks</i>	0.358	Baker and Solon (2003)
<i>σ_s - S.d. of the Random Walk income shocks</i>	0.0837	Baker and Solon (2003)
<i>σ_{f_α} - S.d. of the level fixed effect</i>	0.366	Baker and Solon (2003)
<i>σ_{f_β} - S.d. of the slope fixed effect</i>	0.0095	Baker and Solon (2003)
<i>σ_{f_α,f_β} - Covariance between fixed effects</i>	-0.0031	Baker and Solon (2003)
<i>m_{f_α} - Mean of the level fixed effect</i>	10.102	<i>Intercept of a Log-earnings regression on LFS data</i>
<i>m_{f_β} - Mean of the slope fixed effect</i>	0.0131	<i>Gradient of a Log-earnings regression on LFS data</i>
<i>g - Government Consumption share of GDP</i>	0.214	<i>G/GDP = 21.4%, average for 1990-2007</i>
<i>τ_c - Consumption Tax Rate</i>	0.124	<i>Average GST and PST, Mendoza, Razin and Tesar (1994)</i>
<i>φ_R - Pension Replacement Rate</i>	0.479	<i>Average pension/average earnings ratio, OECD (2015)</i>
<i>d - Government Long-Run Debt/GDP ratio</i>	0.685	<i>Debt/GDP = 68.5%, average for 1990-2007</i>
<i>ᾱ - Borrowing limit</i>	-0.310	<i>Households can borrow up to 20% of average labor earnings</i>

Table 1: Calibration of the Two-Region Model, Parameters Set Externally for Canada

<i>Parameter</i>	<i>Value</i>	<i>Target</i>
<i>Set Externally - RoW:</i>		
δ_{RoW} - Capital depreciation rate	0.0432	Capital depreciation estimates, PWT9.0
α_{RoW} - 1-Labor share	0.441	Labor share of output = 55.9%, PWT9.0
θ_{RoW} - Risk Aversion	1.5	Elasticity of Intertemporal Substitution = 0.75
$\rho_{\varepsilon, RoW}$ - Persistence of the AR(1) income shocks	0.949	Cross-country average, see text
$\sigma_{\varepsilon, RoW}$ - S.d. of the AR(1) income shocks	0.276	Cross-country average, see text
L_{RoW} - Labor input	152.6	Size of the RoW labor force
$\bar{\mu}_{RoW}$ - Measure of RoW	126.8	Total RoW Employment/Average Employment ratio
<i>Calibrated in Equilibrium:</i>		
β - Discount factor	1.0123	Current Account = 0, average for 1990-2007
β_{RoW} - Discount factor	0.958	$r = 2.79\%$, T-bills real interest rate, average for 1990-2007
Z_{RoW} - TFP	0.31	Canadian Capital/World Capital = 2.26%, PWT9.0
ξ - Government Transfers	0.19	Income inequality ratio, pre-post Gov. $\frac{Var[\log(y^{Post})]}{Var[\log(y^{Pre})]} = 46\%$
<i>Robustness Analysis:</i>		
ψ - Costly intermediation	0.02	Borrowing and lending interest rates gap = 2%
ζ - Interest rate elasticity wrt public debt/GDP	0.827	Estimate on OECD data, see Table 5

Table 2: Calibration of the Two-Region Model, Parameters Set Externally for the RoW and Parameters Set in Equilibrium

<i>Parameter</i>	<i>Value</i>	<i>Target</i>
<i>Set Externally:</i>		
r_D - Interest rate on public debt	0.0279	T-bills real interest rate, average for 1990-2007
γ_G - Public Good share of GDP	0.128	$c_G/GDP = 12.8\%$, 2017 value
$\rho_{g,d}$ - G/GDP and D/GDP Correlation	{-}	OLS and FE estimates, see Table 4
<i>Calibrated in Equilibrium:</i>		
β - Discount factor	1.0138	Current Account = 0, average for 1990-2007
ξ - Government Transfers	0.19	Income inequality ratio, pre-post Gov. $\frac{\text{Var}[\log(y^{Post})]}{\text{Var}[\log(y^{Pre})]} = 46\%$
η_{OLS} - Consumption weight in Utility	0.674	Max votes for the status-quo D/Y, see text
θ_{OLS} - Curvature of Utility Function	1.742	Relative Risk Aversion = 1.5
<i>Alternative Calibration:</i>		
η_{FE} - Cons. Weight in Utility	0.704	Max votes for the status-quo D/Y, see text
θ_{FE} - Curvature of Utility Function	1.710	Relative Risk Aversion = 1.5

Table 3: Calibration of the Public Goods Model, all the other parameters are set to their benchmark values reported in Table 1.

	FE	OLS
gtotgdp	1.017 (0.293)	8.557 (2.558)
ssgdp	3.553 (0.564)	7.088 (3.175)
taxgdp	0.733 (0.477)	-6.773 (2.413)
shortinrate	-1.722 (0.465)	-2.787 (3.373)
longinrate	1.291 (0.451)	-6.159 (7.497)
Time Dummies	Yes	-
N	503	28
R^2	0.611	0.384

Table 4: The (D/Y, G/Y) Correlation in the OECD Data. Standard errors in parentheses. The dependent variable is the public debt/GDP ratio. The explanatory variables are the Total Public Outlays/GDP ratio (gtotgdp), the Social Security Benefits/GDP (ssgdp), the Total Tax Revenues/GDP (taxgdp), and two interest rate measures for the short-run (shortinrate) and for the long run (longinrate).

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Appendix A - The OLG Model and its Recursive Representation

A - Stationary Equilibrium

The economy is a production economy with an endogenous asset distribution, where a government collects taxes to finance both an exogenously given stream of public expenditures and a pension scheme. The model is an extension of the [Huggett \(1996\)](#) economy, appropriately modified to allow for several sources of heterogeneity in labor income. Beside the workers, there is a representative firm that produces the final output relying on a Cobb-Douglas production function on aggregate labor and aggregate capital.

Time is discrete. The economy is populated by finitely lived agents facing an age-dependent death probability π_j^d . Age is denoted with j and there are J overlapping generations, each consisting of a continuum of agents. At age J_R all agents that are still alive become retirees. Regarding the mortality risk, I consider two different environments. In the benchmark case, there is a perfectly competitive annuity market, where the agents share their mortality risk. It follows that in this version of the model there are no accidental bequests, as the average asset holdings of the individuals that die are zero. In an alternative case, there are accidental bequests denoted by b , which are distributed uniformly to all the surviving agents, and the newborns.

Preferences: Agents' preferences are assumed to be time-separable and represented by the utility function $U(\cdot)$. Agents' utility is defined over stochastic consumption $\{c_j\}_{j=1}^J$ sequences: their aim is to choose how much to consume (c_j) and how much to save in an interest bearing asset (a_{j+1}) in each period of their lives, in order to maximize their objective function. The agents' problem can be defined as:

$$\max_{\{c_j, a_{j+1}\}_{j=1}^J} \mathbb{E}_1 U(c_1, \dots, c_J) = \max_{\{c_j, a_{j+1}\}_{j=1}^J} \mathbb{E}_1 \sum_{j=1}^J \beta^{j-1} \left[\prod_{s=1}^j (1 - \pi_s^d) \right] u(c_j)$$

where \mathbb{E}_1 represents the expectation operator over the idiosyncratic sequences of shocks, and $\beta > 0$ is the subjective discount factor. In the benchmark formulation, I assume that $u(c_j) = \frac{c_j^{1-\theta}}{1-\theta}$, that is the per-period utility function is strictly increasing in consumption, strictly concave, satisfies the Inada conditions, and has a constant relative risk aversion. The labor supply is fixed and equal to the time endowment multiplied by the value of the efficiency units.

Endowments: Agents differ in their labor endowments $\epsilon_{j,s,\varepsilon,f_\alpha,f_\beta}$. There are five channels that contribute to the determination of the total efficiency units that the workers supply in the labor market. First, there is an age component j , which interacts with one of the fixed effects, namely the growth rate of individual productivity. Second, there are two correlated fixed effects, f_α and f_β , that are jointly normally distributed and are observed at labor market entry. Finally, there are two time-varying shocks, one being a stochastic component ε that follows a stationary $AR(1)$ process with normally distributed shocks ν^ε , and the other one being a stochastic component s that follows a random walk process RW with normally distributed shocks ν^s . The income dynamics of the life-cycle for a generic individual i are represented as follows:

$$\left\{ \begin{array}{l} \epsilon_{i,j} = f_{\alpha,i} + f_{\beta,i} * (j - 1) + \varepsilon_{i,j} + s_{i,j} \\ \varepsilon_{i,j} = \rho_{\varepsilon} \varepsilon_{i,j-1} + \nu_{i,j}^{\varepsilon} \\ s_{i,j} = s_{i,j-1} + \nu_{i,j}^s \\ (f_{\alpha,i}, f_{\beta,i}) \stackrel{iid}{\sim} N(0, 0; \sigma_{f_{\alpha}}^2, \sigma_{f_{\beta}}^2, \sigma_{f_{\alpha}, f_{\beta}}), \nu_{i,j}^{\varepsilon} \stackrel{iid}{\sim} N(0, \sigma_{\varepsilon}^2), \nu_{i,j}^s \stackrel{iid}{\sim} N(0, \sigma_s^2) \end{array} \right.$$

Since the regression analysis is performed on the log transformation of labor earnings, the total efficiency units a worker is endowed with is the exponential transformation of the product of all the components described above. It follows that labor earnings are $y_j^w = w \exp(\epsilon_{j,s,\varepsilon,f_{\alpha},f_{\beta}})$. After the common retirement age J_R , the labor endowment drops to zero, and the agents receive a pension $\bar{y}_R(f_{\alpha}, f_{\beta})$ paid for with the contributions of the economic active agents. The pension is a fixed replacement rate ϕ_R of the type-dependent average labor earnings, and agents pay proportional taxes (τ_R) to contribute to the balanced-budget pension scheme. Income taxes are levied, with a proportional tax rate (τ), to finance interest payments on public debt rD , public expenditure G , and public transfers TR . Agents can insure against their mortality risk. As a consequence, on average agents die with zero wealth. Newborns enter the economy with a zero asset endowment and with the average realization of the stochastic component of labor earnings, which is normalized to 1.

Now the problem of the agents in their recursive representation is defined, then I provide a formal definition of the equilibrium concept used in this model, the recursive competitive equilibrium. The individual state variables are: age $j \in \mathcal{J} = \{1, \dots, J\}$, the level fixed effect $f_{\alpha} \in \mathcal{F}_{\alpha} = \{f_{\alpha,\min}, \dots, f_{\alpha,\max}\}$, the slope fixed effect $f_{\beta} \in \mathcal{F}_{\beta} = \{f_{\beta,\min}, \dots, f_{\beta,\max}\}$, the permanent stochastic component of the labor endowment $s \in \mathcal{S} = \{s_{\min}, \dots, \bar{s}, \dots, s_{\max}\}$, the auto-regressive stochastic component of the labor endowment $\varepsilon \in \mathcal{E} = \{\varepsilon_{\min}, \dots, \bar{\varepsilon}, \dots, \varepsilon_{\max}\}$ and asset holdings $a \in \mathcal{A} = [\underline{a}, \bar{a}]$. Notice that a is discretized using an exponentially growing grid with either 101 or 201 points, depending on the version of the model. ε is discretized with the Rouwenhorst method, using a 5-state Markov chain. s is discretized using a 5-state grid whose elements are functions of the s.d., and are changing over the working age, as the variance of the random walk increases linearly. This method is an extension for a life-cycle set-up of the popular Tauchen/Hussey procedure. The transition function of the auto-regressive component of the labor endowment shock is represented by the matrix $\Pi(\varepsilon', \varepsilon) = [\pi(v, z)]$, where each element $\pi(v, z)$ is defined as $\pi(v, z) = \Pr\{\varepsilon_{j+1} = z | \varepsilon_j = v\}$, $v, z \in \mathcal{E}$. In every period the exogenous labor endowments are given by $\epsilon_{j,s,\varepsilon,f_{\alpha},f_{\beta}} = f_{\alpha} \times (f_{\beta} * (j - 1)) \times \varepsilon \times s$. The stationary distribution of working-age agents is denoted by $\mu_j(a, s, \varepsilon, f_{\alpha}, f_{\beta})$ while that of retirees with $\mu_j^R(a, f_{\alpha}, f_{\beta})$. Φ_j denotes the share of each cohort j in the total population. These satisfy the recursion $\Phi_{j+1} = \left(\frac{1 - \pi_j^d}{1 + g_n}\right) \Phi_j$, and are normalized to add up to 1.

A.1 - Problem of the Agents

The model is solved backwards, starting from the terminal age J and with the assumption that the terminal utility value is zero, i.e. $V_{J+1} = 0$.

Problem of the retirees. The value function of an age- j retired agent whose current asset holdings are equal to a is denoted with $V_j^R(a, f_\alpha, f_\beta)$. The problem of these agents can be represented as follows:

$$V_j^R(a, f_\alpha, f_\beta) = \max_{c, a'} \{u(c) + \beta(1 - \pi_j^d) V_{j+1}^R(a', f_\alpha, f_\beta)\} \quad (1)$$

s.t.

$$(1 + \tau_c)c + a' = [(1 + r)/(1 - \pi^d)]a + (1 - \tau)\bar{y}_R(f_\alpha, f_\beta) + tr_G + b$$

$$c \geq 0, \quad a' \geq 0$$

The value functions are indexed also by the two individual effects, as the pension benefits are a fixed replacement rate of the average labor earnings. Because of the related computational burden, the transitory shocks do not enter in the computation of the pension, as they would require keeping track of another continuous state variable. Instead, for the pension computation I consider the average labor earnings arising from the permanent components of the labor efficiency units and the unconditional means of the other stochastic elements. Since the AR1 component is mean reverting, and the dispersion of the RW component is not too wide, this simplification should not heavily affect the results. In the budget constraint notice the presence of the after-tax pension payment $(1 - \tau)\bar{y}_R(f_\alpha, f_\beta)$, the public transfer tr_G and (possibly) the accidental bequest b . In the model with annuities, there are no accidental bequest, but the gross interest rate is adjusted by the average survival probability $(1 - \pi^d)$.

Problem of the workers. The value function of a working-age agent whose current asset holdings are equal to a , whose current efficiency units shock is ε and whose fixed effects are f_α and f_β is denoted with $V_j(a, s, \varepsilon, f_\alpha, f_\beta)$. The problem of these agents can be represented as follows:

$$V_j(a, s, \varepsilon, f_\alpha, f_\beta) = \max_{c, a'} \left\{ u(c) + \beta(1 - \pi_j^d) \sum_{\varepsilon', s'} \pi(\varepsilon', \varepsilon) \pi(s', s) V_{j+1}(a', s', \varepsilon', f_\alpha, f_\beta) \right\} \quad (2)$$

s.t.

$$a \geq 0 : (1 + \tau_c)c + a' = [(1 + r)/(1 - \pi^d)]a + (1 - \tau_R - \tau)w\varepsilon_{j,s,\varepsilon,f_\alpha,f_\beta} + tr_G + b$$

$$\underline{a} < a < 0 : (1 + \tau_c)c + a' = [(1 + r^b)/(1 - \pi^d)]a + (1 - \tau_R - \tau)w\varepsilon_{j,s,\varepsilon,f_\alpha,f_\beta} + tr_G + b$$

$$a_0 = 0, \quad c \geq 0, \quad a' > \underline{a}$$

Non-retired agents have to set optimally their consumption/savings plans. They enjoy utility from consumption, and face some uncertain events in the future. In the next period they can still be alive, and with joint probability $\pi(\varepsilon', \varepsilon) \pi(s', s)$ they transit from their current efficiency units shocks (ε, s) to the value (ε', s') . These agents pay income taxes $\tau w \epsilon_{j,s,\varepsilon,f_\alpha,f_\beta}$ to finance total government outlays. They also pay a proportional tax τ_R on their labor earnings to finance the pension scheme. Finally, they are born with the average shock $(\bar{\varepsilon}, s = 0)$, with no wealth and are subject to an exogenous borrowing constraint, $\underline{a} \leq 0$.

Problem of the foreign workers. The value function of a foreign agent whose current asset holdings are equal to a , whose current efficiency units shock is ε is denoted with $V(a, \varepsilon)$. The problem of these agents can be represented as follows:

$$V_{RoW}(a, \varepsilon) = \max_{c, a'} \left\{ u(c) + \beta_{RoW} \sum_{\varepsilon'} \pi(\varepsilon', \varepsilon) V_{RoW}(a', \varepsilon') \right\} \quad (3)$$

s.t.

$$c + a' = (1 + r)a + w_{RoW} \epsilon_\varepsilon$$

$$c \geq 0, \quad a' > \underline{a}_{RoW} = 0$$

Foreign agents have to set optimally their consumption/savings plans. They enjoy utility from consumption, and face some uncertain events in the future. In the next period, with probability $\pi(\varepsilon', \varepsilon)$ they transit from their current efficiency units ε to the value ε' .

A.2 - Recursive Stationary Equilibrium

Definition 1 For given public policies $\{\tau, \tau_c, \phi_R, G/Y, TR/Y, D/Y\}$ a recursive stationary equilibrium is a set of home decision rules, $\{c_j(a, s, \varepsilon, f_\alpha, f_\beta), a'_j(a, s, \varepsilon, f_\alpha, f_\beta)\}_{j=1}^{J_R-1}$ and $\{c_j^R(a, f_\alpha, f_\beta), a_j^{R'}(a, f_\alpha, f_\beta)\}_{j=J_R}^J$, foreign decision rules, $\{c_{RoW}(a, \varepsilon), a'_{RoW}(a, \varepsilon)\}$, value functions, $\{V_j(a, s, \varepsilon, f_\alpha, f_\beta)\}_{j=1}^{J_R-1}$ and $\{V_j^R(a, f_\alpha, f_\beta)\}_{j=J_R}^J$, foreign value functions, $V_{RoW}(a, \varepsilon)$, prices $\{r, w, w_{RoW}\}$, a set of stationary distributions, $\{\mu_j(a, s, \varepsilon, f_\alpha, f_\beta)\}_{j=1}^{J_R-1}$ and $\{\mu_j^R(a, f_\alpha, f_\beta)\}_{j=J_R}^J$, and foreign stationary distributions, $\mu_{RoW}(a, \varepsilon)$, such that:

- Given relative prices $\{r, w\}$, taxes and pension benefits $\bar{y}_R(f_\alpha, f_\beta)$, the individual home policy functions $\{c_j(a, s, \varepsilon, f_\alpha, f_\beta), a'_j(a, s, \varepsilon, f_\alpha, f_\beta)\}_{j=1}^{J_R-1}$, $\{c_j^R(a, f_\alpha, f_\beta), a_j^{R'}(a, f_\alpha, f_\beta)\}_{j=J_R}^J$ solve the household problems (1)-(2), and $\{V_j(a, s, \varepsilon, f_\alpha, f_\beta)\}_{j=1}^{J_R-1}$, $\{V_j^R(a, f_\alpha, f_\beta)\}_{j=J_R}^J$ are the associated value functions.
- Given relative prices $\{r, w_{RoW}\}$, the individual foreign policy functions $\{c(a, \varepsilon), a'_{RoW}(a, \varepsilon)\}$, solve the household problem (3).
- Given relative prices $\{r, w\}$, K/L solves the domestic firm's problem.
- Given relative prices $\{r, w_{RoW}\}$, K_{RoW}/L_{RoW} solves the foreign firm's problem.
- The labor markets are in equilibrium, and the labor inputs L and L_{RoW} correspond to the total supply of labor efficiency units in both regions

$$L = \sum_{j=1}^{J_R-1} \Phi_j \int_{\mathcal{A} \times \mathcal{S} \times \mathcal{E} \times \mathcal{F}_\alpha \times \mathcal{F}_\beta} \epsilon_{j,s,\varepsilon,f_\alpha,f_\beta} d\mu_j(a, s, \varepsilon, f_\alpha, f_\beta)$$

$$L_{RoW} = \int_{\mathcal{Q}} \int_{\mathcal{A} \times \mathcal{E}} \epsilon_\varepsilon d\mu_{RoW}(a, \varepsilon) dq$$

- The global asset market clears

$$K + K_{RoW} + D = \sum_{j=1}^{J_R-1} \Phi_j \int_{\mathcal{A} \times \mathcal{S} \times \mathcal{E} \times \mathcal{F}_\alpha \times \mathcal{F}_\beta} a'_j(a, s, \varepsilon, f_\alpha, f_\beta) d\mu_j(\cdot) + \sum_{j=J_R}^J \Phi_j \int_{\mathcal{A} \times \mathcal{F}_\alpha \times \mathcal{F}_\beta} a_j^{R'}(a, f_\alpha, f_\beta) d\mu_j^R(\cdot) + \int_{\mathcal{Q}} \int_{\mathcal{A} \times \mathcal{E}} a'_{RoW}(a, \varepsilon) d\mu_{RoW}(\cdot) dq$$

- The pension system is budget-balanced, and the payroll tax τ_R is set to satisfy this condition.
- The government's intertemporal budget constraint is satisfied, that is total tax revenues T (obtained from taxing labor income, asset income, pensions, and consumption) minus total government expenditure (the sum of government purchases G and public transfers TR) are equal to the interest costs of public debt, namely public debt is constant over time

$$r \frac{D}{Y} = \frac{(T - G - TR)}{Y}$$

- The stationary distributions $\{\mu_j(a, s, \varepsilon, f_\alpha, f_\beta), \mu_j^R(a, f_\alpha, f_\beta), \mu_{RoW}(a, \varepsilon)\}$ satisfy

$$\mu_{j+1}(a', s', \varepsilon', f_\alpha, f_\beta) = \int \nu(a, s, \varepsilon, f_\alpha, f_\beta, j, a', s', \varepsilon') d\mu_j(a, s, \varepsilon, f_\alpha, f_\beta) \quad (4)$$

$$\mu_{j+1}^R(a', f_\alpha, f_\beta) = \int \nu^R(a, f_\alpha, f_\beta, j, a') d\mu_j^R(a, f_\alpha, f_\beta) \quad (5)$$

$$\mu_{RoW}(a', \varepsilon') = \int \nu_{RoW}(a, \varepsilon, a', \varepsilon') d\mu_{RoW}(a, \varepsilon) \quad (6)$$

In equilibrium the measure of agents in each state is time invariant and consistent with individual decisions, as given by the above three equations (4), (5), and (6), where $\nu(\cdot)$, $\nu^R(\cdot)$ and $\nu_{RoW}(\cdot)$ are the transition functions.

A.3 - Welfare Measures

- The ex-ante welfare measure W is the expected utility of a new-born in the steady-state, namely the value function of the age 1 individuals, evaluated at the initial conditions and integrate with respect to the fixed effect:

$$W = \int_{\mathcal{F}_\alpha \times \mathcal{F}_\beta} V_1(a=0, s=\bar{s}, \varepsilon=\bar{\varepsilon}, f_\alpha, f_\beta) d\mu_1(a=0, s=\bar{s}, \varepsilon=\bar{\varepsilon}, f_\alpha, f_\beta) \quad (7)$$

- The social welfare measure W^S is utilitarian, i.e. it weights the agents' lifetime utilities by their mass in the steady-state

$$W^S = \sum_{j=1}^{J_R-1} \Phi_j \int_{\mathcal{A} \times \mathcal{S} \times \mathcal{E} \times \mathcal{F}_\alpha \times \mathcal{F}_\beta} V_j(a, s, \varepsilon, f_\alpha, f_\beta) d\mu_j(a, s, \varepsilon, f_\alpha, f_\beta) + \sum_{j=J_R}^J \Phi_j \int_{\mathcal{A} \times \mathcal{F}_\alpha \times \mathcal{F}_\beta} V_j^R(a, f_\alpha, f_\beta) d\mu_j^R(a, f_\alpha, f_\beta) \quad (8)$$

- The consumption based welfare measure ϖ is the percentage increase in consumption in all states of the world that makes welfare in the counterfactual economy $W^1(\varpi)$ equal to welfare in the baseline one W^0

$$W^0 = W^1(\varpi)$$

$$\varpi = \left(\frac{W^1}{W^0} \right)^{\frac{1}{1-\theta}} - 1 \quad (9)$$

Appendix B - Data, Estimation and Empirical Targets

- The estimation was performed with STATA SE 15.1.
- The public version of the Labour Force Survey data can be downloaded from the following website:

<http://dli-idd-nesstar.statcan.gc.ca/webview/>

- The Canadian Life Tables can be downloaded from the following website:

<https://www150.statcan.gc.ca/n1/pub/84-537-x/84-537-x2018002-eng.htm>

- The public debt and gdp data were obtained from the OECD and the PWT9.0 databases.
- The CPI and the net public debt data were obtained from the CANSIM database.
- The public expenditure/GDP was computed dividing the Government Final Consumption Expenditure series by the GDP series. Both series were retrieved using the FRED website: the id of the former is NAEXKP03CAA189S and for the latter is NAEXKP01CAA189S.

<https://fred.stlouisfed.org/>

- The national net lending or net borrowing was obtained from the CANSIM database, series v62305900, and was adjusted for inflation using the GDP deflator, series v1997756.
- The real interest rate on Treasury Bills for Canada was computed subtracting the inflation rates (obtained from the CPI) from the nominal interest rates. Both series were retrieved using the FRED website: the id of the interest rates series is INTGSTCAM193N and the id for the CPI is CANCEPIALLMINMEI. Both series are monthly, but the first series reports an annual rate. This was transformed to its monthly counterpart, and the resulting average real monthly interest rate was then converted to its annual counterpart.

Appendix C - The Solution Algorithm

This algorithm represents the computational procedure used to solve the two-region model:

1. Set the home public debt/GDP ratio to its long-run value.
2. Generate a discrete grid over the asset space for the home economy $[\underline{a}, \dots, a_{\max}]$ and for the RoW $[\underline{a}_{RoW}, \dots, a_{RoW, \max}]$.
3. Generate a joint discrete grid over the two individual effects $[f_{\alpha, \min}, \dots, f_{\alpha, \max}] \times [f_{\beta, \min}, \dots, f_{\beta, \max}]$.
4. Generate a discrete grid over the AR(1) income shocks with the Rouwenhorst method for the home economy $[\varepsilon_{\min}, \dots, \varepsilon_{\max}]$ and for the RoW $[\varepsilon_{RoW, \min}, \dots, \varepsilon_{RoW, \max}]$.
5. Generate a discrete grid over the random walk income shocks with a life-cycle adaptation of Tauchen/Hussey's method $[s_{\min}, \dots, s_{\max}]$.
6. Guess the interest rate r_0 .
7. Guess the home income tax rate τ_0 .
8. Guess the home pension benefits $\bar{y}_{R,0}(f_\alpha, f_\beta)$.
9. Get the home capital demand K_0 and wages w_0 .
10. Get the home saving functions $a'_j(a, s, \varepsilon, f_\alpha, f_\beta)$, $a_j^{R'}(a, f_\alpha, f_\beta)$ and the home value functions $V_j(a, s, \varepsilon, f_\alpha, f_\beta)$, $V_j^R(a, f_\alpha, f_\beta)$.
11. Get the home stationary distributions $\mu_j(a, s, \varepsilon, f_\alpha, f_\beta)$, $\mu_j^R(f_\alpha, f_\beta)$.
12. Get the RoW capital demand $K_{RoW,0}$ and wages $w_{RoW,0}$.
13. Get the RoW saving functions $a'_{RoW}(a, \varepsilon)$.
14. Get the RoW stationary distributions $\mu_{RoW}(a, \varepsilon)$.
15. Get the aggregate world capital supply and check the world asset market clearing; Get r_1 .
16. Update r'_0, τ'_0 and $\bar{y}'_{R,0}(f_\alpha, f_\beta)$ (with a relaxation method).
17. Iterate until the global asset market clearing and aggregate consistency of the pensions are satisfied.
18. Compute the home ex-ante welfare W of a new-born and the social welfare W^S .
19. Get the home consumption functions $c_j(a, s, \varepsilon, f_\alpha, f_\beta)$, $c_j^R(a, f_\alpha, f_\beta)$, the RoW consumption functions $c_{RoW}(a, \varepsilon)$ and check the final good market clearing.
20. Adjust the parameters $\beta, \beta_{RoW}, A_{RoW}, \xi$ until the empirical targets are matched.
21. Set the public debt/GDP ratio to a counterfactual value on an appropriate grid and repeat steps 2-18 for all the points in the grid.

Appendix D - Additional Results and Plots

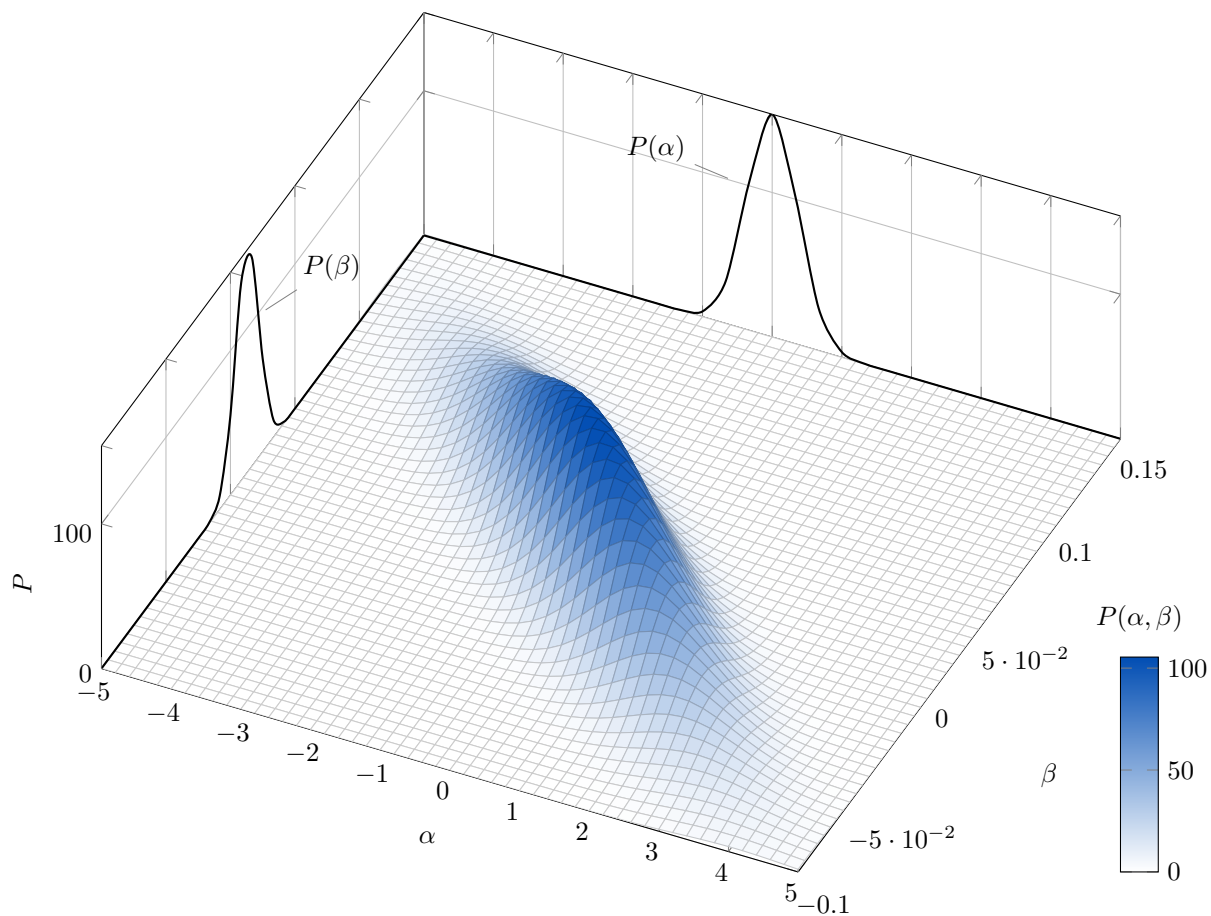


Figure 10: Joint bivariate normal density of the correlated level (α) and slope (β) fixed effects. For ease of comparison with the joint density $P(\alpha, \beta)$, the marginal densities $P(\alpha)$ and $P(\beta)$ are multiplied by an appropriate normalizing factor.

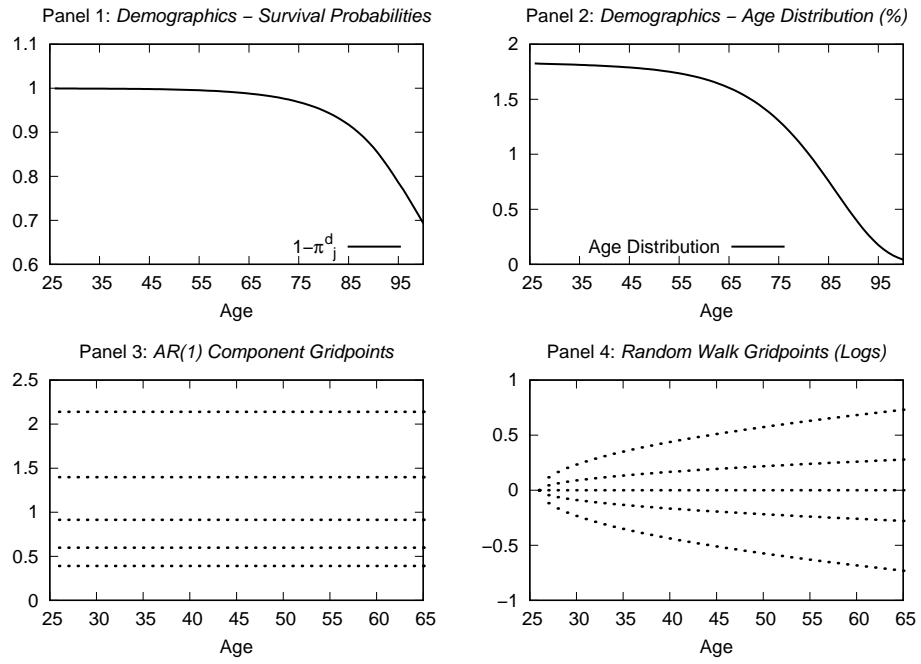


Figure 11: Demographics and discretization of the income process.

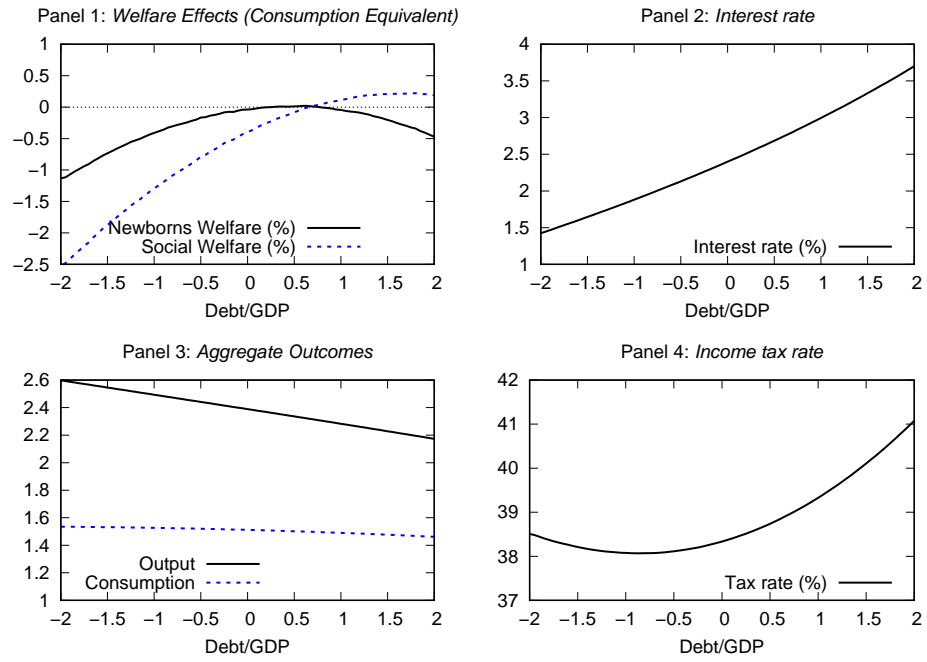


Figure 12: Equilibrium outcomes for the closed economy (with accidental bequests).

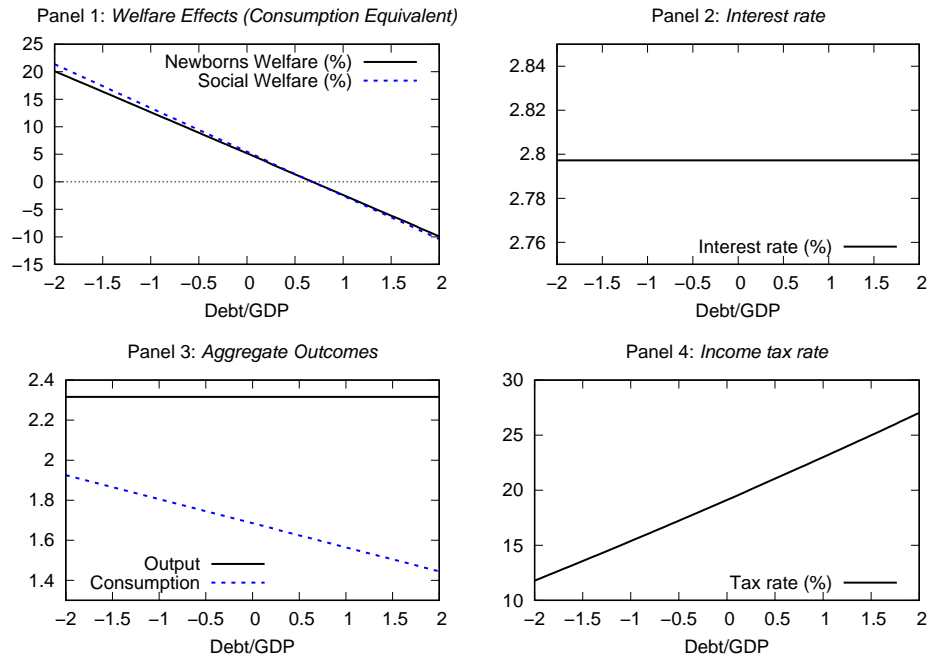


Figure 13: Equilibrium outcomes for the SOE model.

	(1)	(2)
	FE	FE-IV
logdgdgdp	0.827 (0.174)	0.629 (0.187)
logrgdgp	-2.026 (0.486)	-2.568 (0.482)
logcpi	-1.122 (0.661)	-1.596 (0.659)
Time Dummies	Yes	Yes
N	531	481
R^2	0.398	0.394

Standard errors in parentheses

Table 5: Partial Correlation between Interest rates and public debt/GDP.

Appendix D.1 - The SOE Model with an Exogenous Interest Rate Rule

As a robustness analysis, I solved the SOE version of the model, closing it by assuming an ad-hoc rule for the determination of the domestic interest rate. In order to assign an interest rate in the counterfactual cases, with a changing public debt, I exploit the empirical relationship estimated on a Panel of OECD Countries over the 1990-2015 period. In particular, I estimated the interest rate elasticity to public debt using a reduced form regression reported in Table 5. As for the dependent variable, I consider the long-run interest rate on Treasury bills (a variable available in the OECD database), while the main variable of interest is the log of the public debt/GDP ratio (logdgdgdp). In order to reflect the features of the theoretical model, I consider a parsimonious regression, which tries to control for aggregate shocks (with a set of time dummies), and other macroeconomic outcomes, such as the log of GDP (logrgdgp) and log of the price level (logcpi).³³ Arguably, there are sources of endogeneity that might bias the estimates. In particular, current public debt mechanically increases with the current interest rate, as the cost of servicing the debt increases, which can potentially be financed with more debt. This leads to an obvious issue of reverse causality. In order to circumvent this problem, I instrument the current public debt with its lagged value. The past value of public debt is predetermined, hence it does not respond to the future value of the rate of return of treasury bills. At the same time, the high autocorrelation of debt ensures that this is not an instance of a weak instrument: the correlation between the endogenous regressor and the instrument is 0.988. The system GMM estimation of the panel model (FE-IV) leads to my preferred estimate of 0.629, which is somewhat lower compared to the basic fixed effects (FE) estimate.

The results of this version of the model are quite different from what is discussed in the main body of the

³³Notice that I cannot consider the log of the inflation rate, because there are some deflationary events. I consider the CPI instead, which provides the same information without suffering from this problem.

paper. However, the model has some counterfactual implications, hence it can be summarily dismissed. The welfare effects show a peculiar non-monotonic behavior. Their spike at a public debt/GDP ratio of -200% is caused by the extremely low cost of renting capital. The interest rate is negative and close to $-\delta$, a value that would lead the demand for capital to diverge. A low and negative interest rate leads to extremely high wages and output, with rates of change that are orders of magnitude larger than in the other versions of the model, and unrealistic. Moving from the status quo to lower values of the public debt/GDP ratio has non-linear effects, as the positive effects of an increase in output are partially offset by the lower rate of return of saving. Since the elasticity of the interest rate to changes in the public debt/GDP ratio is estimated from data on countries that have accumulated public debt, the predictions of the model in the region with Public Wealth cannot be trusted. Furthermore, according to this model, comparing the 1990's to the 2000's the average Canadian aggregate consumption in two subsequent decades should have decrease by approximately 20%. After the observed decrease in the public debt/GDP ratio, such a drastic adjustment in consumption has not materialized, casting doubts on the reliability of this version of the model to perform counterfactual analysis.

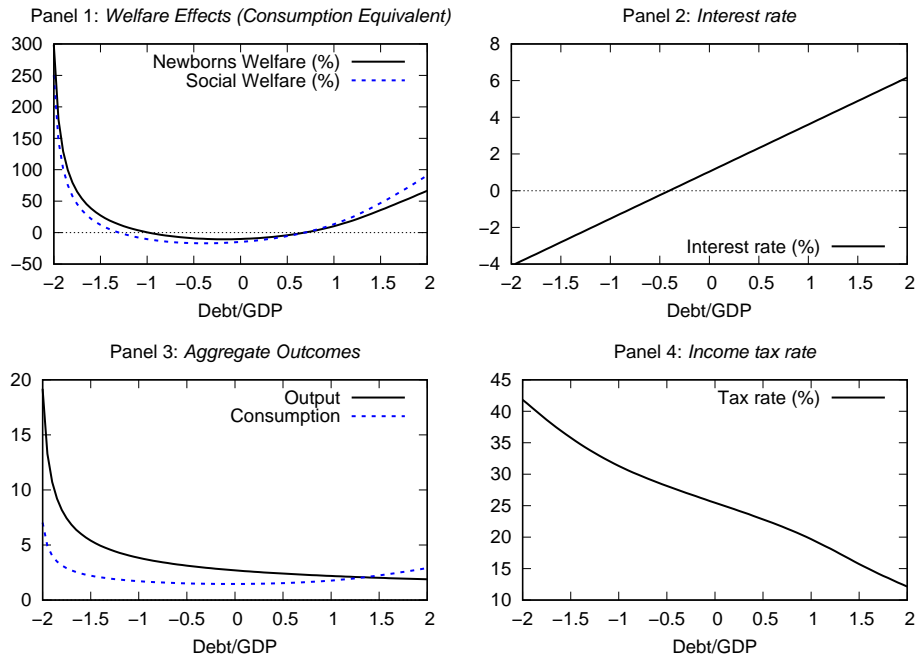


Figure 14: Equilibrium outcomes for the SOE model with an exogenous interest rate rule.