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ARE FACTOR BIASES AND SUBSTITUTION IDENTIFIABLE? THE CANADIAN EVIDENCE

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Keywords: normalized CES system, aggregate elasticity of substitution, biased technical change

JEL Classifications: C51, E23, E25, O30, O51

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Are factor biases and substitution identifiable? The Canadian evidence

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Abstract

Revised productivity accounts recently released by Statistics Canada are used to estimate a Klump-McAdam-Willman normalized CES supply-side system for the half-century 1961–2012. The model permits distinct rates of factor-augmenting technical change for capital and labour that distinguish between short-term versus long-term effects, as well as a non-unitary elasticity of substitution and time-varying factor shares. The advantage of the Canadian data for this purpose is that they provide a unified treatment of measurement issues that have had to be improvised in the US and European data used by previous researchers. In contrast to previous results, we find that an elasticity of substitution and distinct factor biases of technological progress are not well determined by the model. For the Canadian data, the KMW model does not appear to provide a framework that overcomes the classic Diamond-McFadden-Rodriguez non-identification result. That impossibility theorem is manifested in our findings, not overcome by them.

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Debates over the sources and direction of technical change have long been at the heart of the theory and empirics of economic growth. Uzawa's (1961) celebrated steady state growth theorem says that balanced growth requires that technical change be labour augmenting. Given that it concerns balanced growth, this does not preclude any capital bias to technological progress, only that it disappear in the long run. For a textbook treatment see Acemoglu (2009, Sec. 7.2.3), who summarizes (p. 64) the implications of Uzawa's theorem as, "... balanced growth can only be generated by an aggregate production function that features Harrod-neutral technological change. ... Suppose the production function takes the special form $F(A_K(t)K(t), A_L(t)L(t))$ balanced growth is only possible if $A_K(t)$ is constant after date T."

The empirical study of the factor biases of technical change, potentially directed at testing this implication of Uzawa's theorem, is hampered by the "non-identification" or "impossibility" theorem of Diamond, McFadden, and Rodriguez (1978) (henceforth DMR). They established that, in the absence of some assumed structure for the aggregate production technology, it is not possible to separately identify distinct factor biases of technical change and the elasticity of substitution σ between factors. In their words (pp. 125–6), there is

... a non-identifiability of the elasticity and bias in the absence of a priori hypotheses on the structure of technical change. More precisely, given the time series of all observable market phenomena for a single economy with a classical aggregate production function, one finds that the same time series *could* have been generated by an alternative production function having an arbitrary elasticity or bias at the observed points ... The identifiability of the elasticity and bias will depend on what is in fact true about the economy and on what the economist assumes a priori to be true (i.e., his maintained hypothesis, or model).

In an intriguing series of articles, Klump, McAdam, and Willman (KMW) and collaborators have proposed and implemented a model of the aggregate production sector that shows promise for providing such a structure: a constant elasticity of substitution (CES) production function estimated jointly with its marginal productivity conditions (factor demands). It is well known that in the special case $\sigma = 1$ the CES production function reduces to the Cobb-Douglas and distinct factor biases of technology are not separately identifiable. But under more general substitution possibilities $\sigma \neq 1$ the KMW model allows the estimation of factor-specific technological progress.

And indeed perhaps the only point on which empirical research agrees is that σ is unlikely to be well approximated by unity, vitiating Cobb-Douglas production specifications. The elasticity of substitution between aggregate capital and labour is, of course, of longstanding interest in macroeconomics and has been intensively studied for many years. This is at least as true today as in the past, σ being relevant to issues ranging from the effects of monetary and fiscal policies to contemporary

debates over income distribution, inequality, and factor shares. Yet, despite numerous empirical studies, "The size of the elasticity of substitution between capital and labor is much debated and still controversial." (Grossman, Helpman, Oberfield, and Sampson, 2016, p. 2.)

How much it differs from unity—and even in what direction—is less clear. In prominent work, Piketty and collaborators have argued that the decline in labour's factor share of recent decades is associated with an elasticity of substitution greater than one. By contrast, in an analysis specifically focused on extending Uzawa's theorem, Grossman et al. (2016) document that, at least for U.S. data, "... a preponderance of the evidence suggests an elasticity well below one." One such recent study is Lawrence (2015), who reconciles labour's declining share with "... estimates that corroborate the consensus in the literature that σ is less than 1." Another is the careful and novel methodology of Chirinko, Fazzari, and Meyer (2011), who find "... a precisely estimated elasticity of 0.40" for U.S. data. In an earlier survey, Chirinko (2008) concluded that "... while the estimates range widely, the weight of the evidence suggests a value of σ in the range 0.40–0.60." The lone Canadian study among those surveyed by Chirinko is Schaller (2006), who reports an estimate of 1.20.

In any case, a non-unitary elasticity of substitution offers the possibility that distinct factor biases of technology can be estimated, if the DMR impossibility theorem can be overcome by a suitable modelling structure. The seminal empirical article in the KMW literature is KMW (2007a), which uses annual US data 1953–1998. Other important empirical papers include KMW (2007b), which updates their US data to 2002 and compares the results with quarterly euro-area data 1970–2003; KMW (2008), which updates the euro-area data to 2005; and León-Ledesma, McAdam, and Willman (2015), which uses annual US data 1952–2009. However the latter articles calibrate a key parameter instead of allowing it to be freely estimated. We found no advantage to this estimation strategy, as is discussed further in Section 3, and so will mainly cite KMW (2007a) as a comparator to our findings.

Our contribution in the present article is to apply the KMW methodology to a Canadian data set that, as recently revised, seems to be ideal for this purpose. Whereas the US and euro-area data sets created by KMW were, of necessity, somewhat improvised in their construction, the Canadian data are the unified and coherent culmination of many years of work by the professional staff of Statistics Canada (StatCan). One purpose of these productivity accounts is to enable StatCan to estimate total factor productivity, estimates that provide a useful comparison with the TFP implications of our estimated CES system. To our knowledge this is the first estimation of the KMW model using a unified single-agency data set.¹ As well, we plot the loglikelihood values explicitly in order to study issues of multiple maxima that, for reasons that will become evident, are endemic to the model.

How successful is the KMW system in providing a structure to overcome the DMR impossibility theorem, permitting estimation of the elasticity of substitution jointly with distinct factor biases of technical change? León-Ledesma, McAdam, and Willman (2010) present Monte Carlo evidence

supporting the ability of the model to identify these distinct elements. When it comes to empirical implementation with real-world data, on the other hand, the answer to this question will inevitably be data-specific. KMW (2007a) and the other empirical papers cited above report estimation results for global maxima of the likelihood of their system that are encouraging in this respect. In contrast, we have had less success with the Canadian data. We find multiple maxima of the loglikelihood that, although yielding estimates having some sensible implications, nevertheless have difficulty distinguishing the separate effects so as to yield a compelling story about the patterns of technological progress or factor substitution.

Given that exploiting this data set is at the centre of the analysis, Section 1 begins by describing it. Section 2 summarizes the essentials of the KMW methodology as it pertains to our application. Section 3 reports our estimations results, and Section 4 consolidates the findings.

1 Data

Our data source is Table 383-0021 of the Canadian socioeconomic database (CANSIM) as released in Spring 2016, which is complete for the years 1961–2012. It defines the aggregate business sector as the whole economy less public administration, non-profit institutions, and the rental value of owner-occupied dwellings. This definition of the private sector economy appears to be broadly comparable to that used by KMW in constructing their US and euro-area data.

This edition of the data incorporates a treatment of capital cost that is improved over pre-2014 editions and, conveniently, brings it into line with the KMW construction. Previously capital cost was obtained simply as GDP less labour compensation,² so that national income was entirely attributed as payments to labour and capital. Now an external rate of return is instead used to calculate capital cost for service industries, with the result that capital income and cost differ.

This difference could be the result of imperfect competition. It could also arise because the list of inputs included in the MFP estimates is incomplete (for example, intangibles are excluded). Or it could arise because of economies of scale, so that input costs do not completely exhaust total product. (Baldwin, Gu, Macdonald, Wang, and Yan, 2014, p. 11)

This difference is referred to variously as a "residual" by Baldwin et al. (not to be confused with the Solow residual) and as a "markup" by KMW, which presumes the imperfect competition interpretation. In order to avoid both possible misinterpretations we call it *supernormal profits*.³ Beginning with the 2015 edition of Table 383-0021, this implied supernormal profits series is positive on average but can be negative in some years, consistent with KMW's markup series.

Insert Figure 1 around here

Figure 1 portrays some time series features of these aggregates. Panel (a) shows the levels of business sector real GDP and its three components: labour compensation, capital cost, and supernormal profits, all in 2007 dollars. Panels (b)–(d) plot log-differences of these variables (with the exception of supernormal profits, since it is negative in some years). These do not trend markedly, suggesting that—at least at a descriptive level—these business sector aggregates are reasonably approximated by constant long run growth processes, consistent with the neoclassical growth model. This is confirmed by the Dickey-Fuller tests in the upper portion of Table 1, which strongly reject the unit root hypothesis for these log-differences, suggesting that the annual growth rates can be treated as stationary.

Insert Table 1 around here

Consider next the factor shares: Figure 2 plots the ratios to GDP of each of labour compensation, capital cost, and supernormal profits. These average 60.9%, 36.8%, and 2.3% respectively, consistent with what growth economists commonly regard as plausible factor shares and a rate of profit in developed countries. There is some indication of a downward trend in the labour share and upward trend in the capital share, confirmed by the unit root tests in the lower portion of Table 1 that fail to reject the null of a unit root. Although modest by comparison, these trends are consistent with what KMW (2007a, 2007b) found of their US and euro-area factor shares. They are also consistent with recent international evidence. For a focus on labour's factor share see Karabarbounis and Neiman (2014); for capital's share see Piketty and Zucman (2014) who find (p. 1302) that "... capital shares have increased in all rich countries" between 1970 and 2010.

Insert Figure 2 around here

In a deterministic model balanced growth requires "great ratios" such as factor shares and the capital-output ratio to be constant in the long run. The apparent nonstationarity of factor shares over the sample period is one motivation for modelling the production sector so as to permit the short run to depart from the long run, and to allow factor shares to vary systematically with other influences in a way that is not permitted by a Cobb-Douglas specification.

Turning to a detailed consideration of the factor payments wN and qK that are the numerators of these factor shares, each of labour compensation and capital cost is the product of price and quantity. In measuring the quantity of labour N, Jorgenson has long argued the importance of accounting for changing labour force composition. We therefore use StatCan's quality-adjusted Labour Input series, consistent with KMW (2007a). Figure 3(a) plots this Labour Input series and panel (b) plots the implied real wage calculated as the ratio of real labour compensation to Labour Input. Both trend upward, as should be the case in data for a growing economy. (For this purpose labour compensation is deflated using the business sector GDP deflator, defined implicitly as the

ratio of our nominal and real business sector GDP series. So defined, it turns out that this business sector GDP deflator tracks the economy-wide GDP deflator very closely. Because *Labour Input* is an index, the units of the implied real wage on the vertical axis of Figure 3(b) have no economic interpretation, and so it is also expressed as an index.)

Insert Figure 3 around here

In the case of capital K, we use the Capital Stock series of Table 383-0021 instead of StatCan's alternative quality-adjusted Capital Input series. Capital Stock is constructed from investment and investment price indexes that are benchmarked to 2007, following a geometric depreciation pattern.

As a check on this choice, Figure 3(c) plots the ratios of each of Capital Stock and Capital Input to real GDP. (Because all these variables are indexes with 2007 = 100, the ratios are unity in 2007.) Figure 3(d) shows the implied real prices of capital services, calculated as the ratios of real capital cost (obtained by deflating with our business sector GDP deflator) to each of the Capital Input/Stock series. The quality-adjusted Capital Input ratio trends upward over time while its implied factor price trends downward. In contrast, the non-quality adjusted Capital Stock ratio and its implied price are more stable. Trending behaviour is not necessarily inconsistent with long run balanced growth—conceivably, the Capital Input variables could be converging toward their steady state values during this half-century. Nevertheless, between the two measures of capital, it happens that our favoured Capital Stock series and its implied factor price are more consistent with balanced growth behaviour.⁴

2 The KMW Framework

Estimating a production function jointly with its implied marginal productivity conditions is a well established empirical methodology with a long history, going back at least to Bodkin and Klein (1967). The approach is especially valuable for a constant elasticity of substitution (CES) production function, where the marginal productivity conditions imply factor shares that vary with other influences, in contrast to the constant factor shares implied by a Cobb-Douglas function. However CES functional forms require nonlinear estimation. As well, there is now a sizable literature suggesting that the empirical implementation of CES models benefits from normalization around, in the terminology of KMW, "fixed" or "baseline" points.

The many issues surrounding the specification and estimation of CES supply-side systems, including normalization, have been thoroughly exposited in this literature; see in particular the KMW (2012) survey article. Here we merely summarize the essentials needed to understand our analysis.

2.1 Growth Specifications of the Factor Efficiencies

One expression for a constant returns to scale CES production function is

$$Y_t = \left[(E_t^N N_t)^{-\rho} + (E_t^K K_t)^{-\rho} \right]^{-1/\rho}.$$
 (1)

Notation is conventional—N and K are labour and capital, ρ the substitution parameter—except for the labour- and capital-augmenting efficiency levels E_t^N and E_t^K , each of which is specified as

$$E_t^i = E_0^i \exp(g_i(t)), \quad i \in \{N, K\}.$$

The associated technology growth rates are

$$\frac{\operatorname{dln} E_t^i}{\operatorname{d} t} = \frac{\operatorname{d} g_i(t)}{\operatorname{d} t}, \qquad i \in \{N, K\}.$$
(2)

The textbook case of constant growth sets $g_i(t) = g(\gamma_i, t) = \gamma_i t$, in which case growth proceeds at constant instantaneous rate $dg_i(t)/dt = \gamma_i$. But even if this provides a good approximation to growth in the long run—and this is an open question—in the shorter run it may be unduly restrictive. Rather than impose constant growth as a maintained hypothesis, KMW permit more general growth trajectories by using the Box-Cox transformation $b(t, \lambda) = (t^{\lambda} - 1)/\lambda$ to specify $g_i(t)$ as

$$g_i(t) = g(\gamma_i, t, \lambda_i) = \gamma_i b(t, \lambda_i) = \gamma_i \left(\frac{t^{\lambda_i} - 1}{\lambda_i}\right), \quad i \in \{N, K\}.$$
 (3)

Recall the special cases of the Box-Cox transformation:

$$b(t,\lambda) = \begin{cases} t-1 & \text{when } \lambda = 1; \\ \ln t & \text{when } \lambda = 0. \end{cases}$$

Its time-derivatives are:

$$\frac{\mathrm{d}b(t,\lambda)}{\mathrm{d}t} = \begin{cases} 1 & \text{when } \lambda = 1; \\ t^{\lambda-1} & \text{in general;} \\ 1/t & \text{when } \lambda = 0. \end{cases}$$

The rates of technological progress (2) therefore depend on the curvature parameters λ_i as follows.

$$\frac{\mathrm{d}g_i(t)}{\mathrm{d}t} = \gamma_i t^{\lambda_i - 1} = \begin{cases} \to \infty \text{ as } t \to \infty & \text{if } \lambda_i > 1 & \text{(accelerating growth);} \\ \gamma_i & \text{if } \lambda_i = 1 & \text{(constant growth);} \\ \to 0 \text{ as } t \to \infty & \text{if } \lambda_i < 1 & \text{(decelerating growth).} \end{cases}$$

Although this Box-Cox specification permits accelerating growth in either or both of the efficiency levels E_t^i , accelerating growth is implausible empirically in the long run, as we (and KMW) find.

Within the general patterns of technological progress permitted by this specification, several special cases have long been of interest to growth economists.

Hicks neutrality The efficiencies of all factors improve at a common rate: $\gamma_N = \gamma_K > 0$, $\lambda_N = \lambda_K$.

Harrod neutrality Technological progress is solely labour-augmenting,

- in both the short and long run: $\gamma_K = 0, \, \gamma_N > 0, \, \lambda_N \geq 1;$
- only in the long run: $\lambda_K < 1, \, \gamma_N > 0, \, \lambda_N \ge 1.$
- If accelerating growth is ruled out a priori in these restrictions, then the constraint $\lambda_N \geq 1$ would specialize to $\lambda_N = 1$.

Solow neutrality Technological progress is solely capital-augmenting,

- in both the short and long run: $\gamma_N = 0, \gamma_K > 0, \lambda_K \ge 1$;
- only in the long run: $\lambda_N < 1, \, \gamma_K > 0, \, \lambda_K \geq 1.$
- If accelerating growth is ruled out a priori in these restrictions, then the constraint $\lambda_K \geq 1$ would specialize to $\lambda_K = 1$.

The ability to distinguish empirically between short-term versus long-term biases in technical change is important. As the opening passage of this article noted, short-run capital-augmenting technical change is not inconsistent with Uzawa's (1961) steady state growth theorem, as long as it disappears in the long run.

Of course, the CES parameterization that is the maintained hypothesis of the KMW methodology reduces to Cobb-Douglas under a unitary elasticity of substitution, in which case distinct factor efficiencies are not separately identifiable and all technological progress can be formulated as labour augmenting. Thus in the KMW framework where the elasticity of substitution is $\sigma = 1/(1+\rho)$, the restriction $\rho = 0$ or $\sigma = 1$ is also a sufficient condition for the steady state growth theorem to hold. However we find this special case to be definitively rejected, even when the technology growth rate is allowed to vary through time (that is, the Box-Cox parameter λ is unrestricted).

2.2 The Normalized System

Although the CES production function (1) is in a form similar to its typical textbook presentation, it is not suitable for empirical implementation because the substitution parameter ρ (or σ) and the parameters governing technical change are not separately identified. For this, two things are introduced: the production function is estimated jointly with the implied factor demands, and the resulting three-equation system is estimated in normalized form.

So expressed, the KMW (2007a, equs. (6), (7), (8)) normalized system is as follows.⁵

$$\log\left(\frac{w_t N_t}{p_t Y_t}\right) = \log\left(\frac{1-\pi}{1+\mu}\right) + \frac{1-\sigma}{\sigma} \left[\log\left(\frac{Y_t/\bar{Y}}{N_t/\bar{N}}\right) - \log\zeta - g_N(t,\bar{t})\right]$$
(4a)

$$\log\left(\frac{q_t K_t}{p_t Y_t}\right) = \log\left(\frac{\pi}{1+\mu}\right) + \frac{1-\sigma}{\sigma} \left[\log\left(\frac{Y_t/\bar{Y}}{K_t/\bar{K}}\right) - \log\zeta - g_K(t,\bar{t})\right]$$
(4b)

$$\log\left(\frac{Y_t}{N_t}\right) = \log\left(\frac{\zeta \bar{Y}}{\bar{N}}\right) + g_N(t,\bar{t})$$

$$-\frac{\sigma}{1-\sigma}\log\left\{\pi \exp\left[\frac{1-\sigma}{\sigma}\left(g_N(t,\bar{t}) - g_K(t,\bar{t})\right)\right] \left(\frac{K_t/\bar{K}}{N_t/\bar{N}}\right)^{(\sigma-1)/\sigma} + (1-\pi)\right\}$$
(4c)

The first two equations are the marginal productivity conditions in factor share form while, in the third equation, the maintained hypothesis of constant returns to scale permits the production function to be expressed in labour intensive form. The expressions $g_i(t, \bar{t})$ are the normalized versions of the Box-Cox growth terms (3), defined as

$$g_i(t,\bar{t}) = \frac{\bar{t}\gamma_i}{\lambda_i} \left[\left(\frac{t}{\bar{t}} \right)^{\lambda_i} - 1 \right] \qquad i \in \{N,K\},$$

where \bar{t} is the arithmetic mean of the time trend series. This normalization does not alter the economic interpretations we have given for the growth parameters γ_i , λ_i . Notice, for example, that $\lambda_i = 1$ still yields constant growth, $g_i(t,\bar{t}) = \gamma_i(t-\bar{t})$, just with a redefined time index.

In addition to being parameterized in terms of the elasticity of substitution σ instead of ρ , several parameters appear in the system (4) that do not appear in the original production function (1). The "distribution parameter" π governs the distribution of factor incomes. Although it does not literally equate to any simple expression for relative factor payments, in the KMW framework π is typically well approximated by the sample mean of $q_t K_t/(w_t N_t + q_t K_t)$, which is 0.377 in our sample. The parameter μ corresponds to the rate of supernormal profits and so should roughly equal the average of $(p_t Y_t - w_t N_t - q_t K_t)/p_t Y_t$ which, from Table 1, is 0.0226. And the parameter ζ captures the difference between observed output and its geometric mean, at the baseline input levels (the geometric means). Because this difference should be small, ζ should be close to unity.

The Cobb-Douglas Special Case

The imposition on the maintained model (4) of restricted values for the parameters γ_N , λ_N , γ_K , and λ_K corresponding to the various growth hypotheses is, for the most part, straightforward and yields the nested testing structure shown in Figure 5. Less straightforward is the limiting Cobb-Douglas case of a unitary elasticity of substitution, $\sigma \to 1$, which cannot be obtained simply by setting $\sigma = 1$ in the system (4) (because $1 - \sigma$ appears in a denominator in (4c)). In normalized form, allowing for general Box-Cox growth $g(t, \bar{t})$ in technology, the Cobb-Douglas version of the system is

$$\log\left(\frac{w_t N_t}{p_t Y_t}\right) = \log\left(\frac{1-\pi}{1+\mu}\right) \tag{5a}$$

$$\log\left(\frac{q_t K_t}{p_t Y_t}\right) = \log\left(\frac{\pi}{1+\mu}\right) \tag{5b}$$

$$\log\left(\frac{Y_t}{N_t}\right) = \log\left(\frac{\zeta\bar{Y}}{\bar{N}}\right) + \pi\log\left(\frac{K_t/\bar{K}}{N_t/\bar{N}}\right) + \frac{\bar{t}\gamma}{\lambda}\left[\left(\frac{t}{\bar{t}}\right)^{\lambda} - 1\right]$$
 (5c)

The further special case of constant growth at rate γ sets $\lambda=1$ in the production function, so the Box-Cox term in the third equation reduces to the normalized constant-growth expression $g(t,\bar{t})=\gamma(t-\bar{t})$.

These Cobb-Douglas special cases of the KMW system appear in Figure 5 as the most restricted models of the nested testing structure. Both are strongly rejected, as we are about to discuss,

consistent with KMW's findings for their US and euro-area data. This rejection motivates interest in the CES model, to which we now turn.

3 Estimation Results

Like previous researchers in this literature, we estimated the CES system (4) as a nonlinear system of seemingly unrelated regressions.⁶ The results are presented in Table 2, which is constructed for ready comparability with the US estimates in Table 1 of KMW (2007a).

Insert Table 2 around here

3.1 The Maintained Model

An important feature of the model is that the singularity at $\sigma = 1$ induces multiple maxima in the likelihood function. This would not be particularly important were these maxima clearly dominated by a unique global maximum, so that the data plainly favour one set of estimates. But for the Canadian data this turns out not to be the case.

This is illustrated in Figure 4, which provides a scatter plot of the loglikelihood values against the σ estimates yielded by alternative starting values.⁷ Interspersed with these local and global maxima are the loglikelihood values yielded by estimations across a grid of σ values. The local maxima at $\hat{\sigma} = 0.919, 1.033$, and 1.179 are in the neighborhood of the point of singularity at $\sigma = 1$, while the global maximum is at the much larger $\hat{\sigma} = 2.215$. However the scatter plot makes it easy to see that this global maximum is not well determined relative to alternative estimations over a broad range of σ estimates. As well, its loglikelihood value is only slightly above those for the two local maxima in the range $\sigma > 1$; in fact even the local maximum at $\sigma = 0.919$ is little below the others. Consequently, for these data the maintained model does not clearly yield a favoured estimate of σ .

Insert Figure 4 around here

Although the local maxima are in the neighborhood of, and so evidently to some extent induced by, the singularity at $\sigma = 1$, the special case of the model associated with that point of singularity—the Cobb-Douglas system (5)—is strongly rejected. Figure 5 shows its position within the larger nested testing structure that we will consider shortly. Likelihood ratio tests rejecting this special case are presented in Table 3.

Insert Figure 5 around here

In addition to their similar loglikelihood values, other criteria provide little basis for selecting among the alternative maxima of the maintained model, which share the following features.

- All four maxima yield estimates of π , $1 + \mu$, and ζ that are consistent with the values discussed in Section 2.2. The distribution parameter π is estimated to be close to the sample mean of 0.377. The estimate of $1 + \mu$ implies a value for μ that is close to the mean rate of supernormal profits of 0.0226. And the normalization constant ζ is close to unity, indicating that the geometric means used for normalization are suitable baseline values.
- All four maxima imply an average TFP growth rate of about 1%. This value is, of course, entirely plausible, although it is higher than the 0.44% average growth rate over this period of StatCan's TFP series in Table 383-0021.

Our estimate of around 1% TFP growth makes an interesting comparison with previous estimates that arose from earlier versions of the data set and alternative methodologies for constructing the series related to capital services. Specifically, StatCan conforms with the internationally-accepted "bottom up" methodology for treating capital services which, based on earlier versions of Table 383-0021, yielded an estimate of 0.28% for TFP growth 1961–2011. Diewert and Yu (2012) contrasted this with an estimate of 1.03% yielded by their "top down" approach. (For a comparison of the two methodologies, see the exchange between Gu (2012) and Diewert (2012).) By taking Table 383-0021 as published, our analysis accepts the StatCan construction of the capital services series; nevertheless we obtain TFP growth rates closer to that of Diewert and Yu.

Turning to comparisons with other countries, KMW (2007a, Table 1; 2007b, Tables 3, 4) find rates of TFP growth of 1.2-1.4% for the US and 0.28–0.31% for the euro area. In relation to these US estimates, our Canadian estimates are consistent with the broader empirical TFP literature, which typically finds lower rates for Canada than for the US.

The goodness of fit of the model, as measured by the equation R^2 's, is similar across the maxima.

The equations have stationary residuals, as judged by augmented Dickey-Fuller (ADF) tests.

Hence, regardless of the maximum, the equations of the model capture the relationships among the nonstationarity growth trajectories of the variables.

These similarities across the maxima do not, however, extend to the growth parameters γ_N , λ_N , γ_K , and λ_K , the first three of which vary widely in their point estimates and significance. The main respect in which the estimates are uniformly sensible is that, in the few instances where the Box-Cox parameters λ_N and λ_K exceed unity, their standard error does not reject $\lambda_i = 1$. Hence none of the estimates yield the implausible implication of accelerating growth.

Considering the growth parameters individually, the most uniform results emerge for $\hat{\lambda}_K$ which, although varying in the range 0.356–1.078 across the alternative maxima, is always significantly

positive. As well, for three of the four maxima $\hat{\lambda}_K$ is well within one standard error of unity, implying constant growth in capital-augmenting technology at rate γ_K .

The overarching conclusion is that, although the maintained model yields estimation results that are in many respects sensible, for these data it has difficulty estimating the elasticity of substitution joint with factor-specific technology biases. That is, the DMR non-identification theorem appears to be revealed in these estimation results, not overcome by them.

Variations on the Analysis

Is there any alternative version of the model or approach to estimation that might shed light on these ambiguities?

Calibrating the parameter π . In some of their work, KMW (2007b, 2008) and León-Ledesma, McAdam, and Willman (2015) find that the nonlinear estimation of the model can sometimes be aided by setting the parameter π to its approximate sample value, which in our data is 0.377. We tried this, but the results were qualitatively much the same. There were five maxima yielding estimates of σ in the range 0.632–1.744. Although the global maximum was at the highly plausible $\hat{\sigma} = 1.033$, its loglikeihood function value was only slightly above those for quite different estimates of σ . Estimates of the growth parameters γ_N , λ_N , γ_K , and λ_K varied widely. And—despite the global maximum at $\hat{\sigma} = 1.033$ —the Cobb-Douglas special case was strongly rejected.

Hence this variation on the analysis resolves nothing. In fact, the strong sensitivity of the location of the global maximum to what should be an innocuous change—setting $\pi = 0.377$ when its freely-estimated value is essentially that—bears out the weak identification of the substitution elasticity.

The Kmenta linearization Another possibility is to make use of the fact that implied TFP growth rates like those reported in Table 2 are calculated using a linearization of the production function due to Kmenta (1967). This linearization can be used, not just for the behind-the-scenes TFP calculation, but as an alternative parameterization of the production function equation in the supply-side system. KMW call this the *Kmenta approximation* of the model.

Unfortunately, applied to our data the Kmenta approximation manifests much the same ambiguities as the regular version. Its loglikelihood is qualitatively similar to that for the regular model in Figure 4. There are three maxima at $\sigma = 1.03296$, 1.18702, and 1.79664. As before, the largest of these is on a very flat portion of the loglikelihood. Although it is now the intermediate estimate $\sigma = 1.18702$ that is the global maximum, as before this loglikelihood value is only marginally above those of the local maxima. It also turns out the estimates of the growth parameters γ_N , λ_N , γ_K , and λ_K display wide variation over the three maxima, similar to the regular model of Table 2.

In short, the Kmenta approximation does not resolve any of the ambiguities arising from the regular model, and so we do not report detailed estimation results for it.

3.2 Restricted Models

Another route to progress is if additional information in the form of parameter restrictions can be introduced.

Figure 5 shows the nested testing structure that arises from successive imposition on the maintained model of parameter restrictions having economic interpretations. Based on the likelihood ratio tests of Table 3, only a few of the restricted models are supported by the data, namely M3 $(\gamma_N = \gamma_K)$, M4 $(\lambda_N = 1)$, M5 $(\lambda_K = 1)$, and M35 $(\gamma_N = \gamma_K, \lambda_K = 1)$. These supported restrictions are broadly consistent with the evidence in the individual coefficient estimates across the alternative maxima of the maintained model reported in Table 2. However only one of these supported special cases has a global maximum with a plausible elasticity of substitution: M4 $(\lambda_N = 1)$ yields $\sigma = 1.03483$. The others (models M3, M5, and M35) yield estimates of σ in the range of 2.12302–3.45771.

Insert Table 3 around here

Other restrictions are rejected at conventional significance levels, including combinations of the supported restrictions. For example, although the restrictions $\lambda_N = 1$ and $\lambda_K = 1$ (models M4 and M5) are not individually rejected, combining them as $\lambda_N = \lambda_K = 1$ (model M45) or even just $\lambda_N = \lambda_K$ (model M6) is rejected. Similarly, whereas the restrictions $\gamma_N = \gamma_K$ (model M3) and $\lambda_N = 1$ (model M4) are not individually rejected, combining them (model M34) is rejected.

The only special case model that is both not rejected and has a global maximum with a plausible elasticity of substitution ($\hat{\sigma} = 1.03483$) is M4, which imposes $\lambda_N = 1$ (sustained growth in labour-augmenting technology at a constant rate). The estimation results are shown in the final column of Table 2. However neither γ_N nor γ_K is significantly different from zero; in fact the point estimate of γ_N is negative. As well, λ_K is more than two standard errors less than 1, so any capital-augmenting technical change is not sustained. Hence this model does not point to either labour or capital-augmenting technical change as sources of sustained growth. In fact this implication is similar to the original findings of KMW (2007a) based on their US data set. As shown in that column of Table 2, their estimates of $\lambda_N = 0.439$ and $\lambda_K = -0.118$ are both significantly less than unity.

4 Conclusions

We have estimated a supply side system for the Canadian business sector—essentially, the aggregate private sector economy—for the half-century 1961–2012. There are reasons to believe that the

Canadian data that have recently become available are, for this purpose, superior to the US and euro-area data that have been available to previous researchers.

By using a CES production function, the elasticity of substitution is not constrained to unity and factor shares can vary with other influences, in contrast to a Cobb-Douglas system. The more general substitution possibilities and factor share behaviour that this permits are supported empirically, both by the estimated model and, in the case of factor shares, by the univariate nonstationarity that is evident over the sample period.

In principle this framework permits distinct rates of factor-augmenting technical change to be identified joint with the substitution parameter. Consequently hypotheses of classic interest concerning the direction of technical change, such as Harrod neutrality, are potentially testable. As well, the use of Box-Cox specifications for growth rates makes it possible to distinguish between short-term versus long-term biases in technical change. The empirical model is supported by, among other things, plausible implied rates of TFP growth.

Nevertheless, applied to the Canadian data the model exhibits endemic multiple maxima of the loglikelihood that fail to yield a compelling portrayal of factor biases or substitution. The alternative maxima of the maintained model are little different in their loglikelihood values, yet encompass a broad range of estimates of the growth parameters and the elasticity of substitution. Another indication that the elasticity of substitution is not well determined is that the global maximum is highly sensitive to the imposition of supported parameter restrictions and other variations on the analysis. Instead of providing a structure that overcomes the Diamond-McFadden-Rodriguez (1978) non-identification theorem, the KMW model applied to the Canadian data yields results that seem to manifest that impossibility result.

Conjectures as to why, in this respect, our Canadian data differ from the US and euro-area data studied by KMW are inevitably speculative. One possibility that could be investigated is the distinction between gross output and value added production functions. When, as here, the domestic macroeconomy is modeled as a single sector, the two production functions are the same in a closed economy but differ in an open economy. In estimating the supply side in gross-output form, the KMW model treats domestic value-added and imported intermediate goods as a single aggregate. This abstraction may not be important for the U.S., but may be for Canada. Future research on the Canadian supply side may wish to recast it in value-added form, distinguishing between the domestic and international sectors.

Notes

¹Herrendorf, Herrington, and Valentinyi (2015) estimate a model very similar to the KMW system using published US data in order to study sectoral transformations between agriculture, manufacturing, and services 1947–2010. They are able to use conventional economy-wide data by including government in the service sector. However, among other variations, their model imposes constant growth in factor-augmenting technology (the special case of $\lambda_N = \lambda_K = 1$ in the Box-Cox expression (3)) instead of permitting time-varying growth rates.

²Given the internal consistency of the StatCan methodology, we accept its treatment of self-employment income rather than attempting any adjustment of the kind suggested by Gollin (2002) and performed by KMW (2007a, equ. (10)).

³We thank an anonymous referee for suggesting this as the best choice of terminology. Because, in the StatCan terminology, capital cost excludes supernormal profits, it is analogous to the "capital income" of KMW (2007a) as they use the term in, for example, their Figure 1.

⁴This divergent behaviour between the two implied real capital price series is not sensitive to the use of the GDP deflator to obtain the real series. We experimented with the alternative of using a capital deflator constructed from CANSIM Table 031-0002. The behaviour of the resulting real capital prices differs little from Figure 3(d). In any case, the implied real factor prices (of both labour and capital) are not used in the model estimation, only as descriptive evidence justifying our use of the associated factor quantity series.

⁵This corrects a few typesetting errors in the third equation as it appears in KMW (2007a) where, most importantly, the closing brace is misplaced. The system appears correctly in the working paper version (KMW 2004, equs. (9), (10), (11)) and in KMW (2007b, equs. (3), (4), (5)).

⁶Past experience with nonlinear systems estimation leads us to favour TSP, the numerical properties of which have been favourably evaluated by McCullough (1999). However we began by verifying that our TSP routines successfully replicate the estimation results of KMW (2007a). We thank Alpo Willman for providing the data and RATS code that made this possible.

An obvious limitation of estimating this model as a seemingly unrelated system is that it ignores possible endogeneity of some explanatory variables. The motivation for doing so is presumably the lack of any natural instrumental variables in this context. As is now well understood, when weak instruments are used to treat endogeneity the cure may be worse than the disease. The only attempt of which we are aware to tackle this issue in the specific context of aggregate supply-side systems is the working paper by Luoma and Luoto (2011), who consider a Bayesian approach to estimation.

Herrendorf et al. (2015) use three stage least squares to estimate their disaggregated sectoral model.

⁷Figure 4 is similar in spirit to the scatter plots of local and global optima in the bottom right panel of Graphs 1.1–4.4 of the KMW (2004) working paper.

⁸For a more explicit discussion of gross output vs. value added production functions in closed vs. open economies, see Herrendorf et al. (2015, p. 108) who remark:

In a closed economy, GDP equals value added by definition. ... In an open economy, GDP is in general not equal to domestic value added because some intermediate inputs are not produced domestically but are imported from other countries. ... While imported intermediate inputs are often abstracted from, they can be quantitatively important, in particular in small open economies that import most of the resources and many of the agricultural and manufactured intermediate goods that they use.

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Table 1: Univariate Statistics

		ADF tests $(p\text{-values})^{a}$		
Variable	Mean	constant	constant+trend	
Log-difference of:				
real GDP	0.0347	0.0017	0.0012	
real labour compensation	0.0328	0.0030	0.0013	
real capital cost	0.0356	0.0003	0.0010	
Share in GDP of:				
labour compensation	0.6090	0.3611	0.1274	
capital cost	0.3684	0.1040	0.0141	
supernormal profits	0.0226	0.0000	0.0000	

^a Schwarz's Bayesian information criterion generally suggested either 0 or 1 augmenting lags in the ADF regressions and so, in addition to the indicated specifications of the deterministic component, we used a single lag in all regressions.

Table 2: Supply-side System, Canadian Business Sector, 1961–2012^a

	Maintained model M0			KMW	Restricted	
					US	model
.	First local	Second local	Third local	Global	results ^b	M4
Parameter	maximum	maximum	maximum	maximum	1953–1998	$\lambda_N = 1$
_	0.919	1.033	1.179	2.215	0.556	1.035
σ	(0.061)	(0.027)	(0.024)	(0.439)	(0.018)	(0.027)
0/27	0.033	-0.053	-0.000	0.006	0.015	-0.048
γ_N	(0.018)	(0.051)	(0.000)	(0.001)	(0.000)	(0.044)
λ_N	0.773	1.166	13.8215	0.136	0.439	(0.044)
λ_N	(0.119)	(0.198)	(8.684)	(0.072)	(0.076)	1
0/**	-0.034	0.198) 0.107	0.020	0.009	0.004	0.097
γ_K	(0.029)	(0.083)	(0.020)	(0.003)	(0.004)	(0.072)
λ_K	0.029) 0.935	0.877	0.356	1.078	-0.118	(0.072) 0.747
\wedge_K	(0.268)	(0.235)	(0.061)	(0.295)	(0.336)	(0.108)
π	0.203) 0.377	0.377	0.379	0.372	0.221	0.377
Λ	(0.003)	(0.003)	(0.003)	(0.003)	(0.009)	(0.002)
$1 + \mu$	1.024	1.024	1.024	1.024	1.042	1.024
$1 + \mu$	(0.003)	(0.003)	(0.003)	(0.003)	(0.011)	(0.003)
ζ	1.028	1.027	1.029	1.020	1.029	1.026
ζ	(0.005)	(0.005)	(0.004)	(0.005)	(0.006)	(0.005)
Loglikelihood	(0.003)	(0.003)	(0.004)	(0.005)	(0.000)	(0.003)
Logiikeiiilood ${\mathscr L}$	326.351	328.695	328.638	329.134		328.289
Average TFP	320.331	326.093	320.030	329.134		320.209
growth rate	0.009	0.010	0.009	0.010	0.013	0.008
growth rate	0.009	0.010	0.009	0.010	0.013	0.008
R_N^2	0.578	0.635	0.626	0.726		0.625
$R_N^2 \ R_K^2 \ R_Y^2$	0.401	0.373	0.244	0.224		0.355
R_{Y}^{2}	0.977	0.978	0.978	0.979		0.978
- • 1		0.000	0.0.0	0.000		0.0.0
$\mathrm{ADF}_N{}^\mathrm{c}$	-2.932	-3.093	-2.960	-4.323	-4.310	-3.036
	[0.042]	[0.027]	[0.039]	[0.000]		[0.032]
$\mathrm{ADF}_K{}^\mathrm{c}$	-4.330	-4.099	-3.729	-3.192	-3.580	-4.068
	[0.000]	[0.001]	[0.004]	[0.020]		[0.001]
$\mathrm{ADF}_{Y}{}^{\mathrm{c}}$	-2.845	-2.853	-2.943	-2.905	-3.960	-2.879
	[0.052]	[0.051]	[0.041]	[0.045]		[0.048]
	-	- ·				- •

^a Standard errors in parentheses.

b Reproduces column 1.4 of KMW (2007a) Table 1. c p-values in brackets. ADF regressions include an intercept, no trend, and (in view of the data being annual) one augmenting lag. For 50 observations the associated ADF critical values are -2.93 (5%) and -2.60 (10%). The null of a unit root is rejected at conventional significance levels, indicating that the equations of the model are generally successful in yielding stationary residuals.

Table 3: Likelihood Ratio Tests of Selected Restricted Models^a

Restricted model	Restrictions under test	\mathscr{L} at global max	LR statistic	Number of restrictions	p-value ^b
M3	$\gamma_N = \gamma_K$	328.179	1.910	1	0.167
M4	$\lambda_N = 1$	328.289	1.690	1	0.194
M5	$\lambda_K = 1$	329.105	0.058	1	0.810
M6	$\lambda_N = \lambda_K$	325.550	7.168	1	0.007
M34	$\gamma_N = \gamma_K, \lambda_N = 1$	311.541	35.186	2	0.000
M35	$\gamma_N = \gamma_K, \lambda_K = 1$	327.964	2.340	2	0.310
M45	$\lambda_N = \lambda_K = 1$	322.438	13.392	2	0.001
Cobb-Douglas $(\lambda \neq 1)$	$\sigma = 1$	305.618	47.032	3	0.000
Cobb-Douglas	$\lambda=1,\sigma=1$	277.746	102.776	4	0.000

^a Relative to the global maximum of the maintained model at $\mathcal{L}=329.134$.
^b Right tail area of a χ^2 distribution with degrees of freedom equal to the number of restrictions.

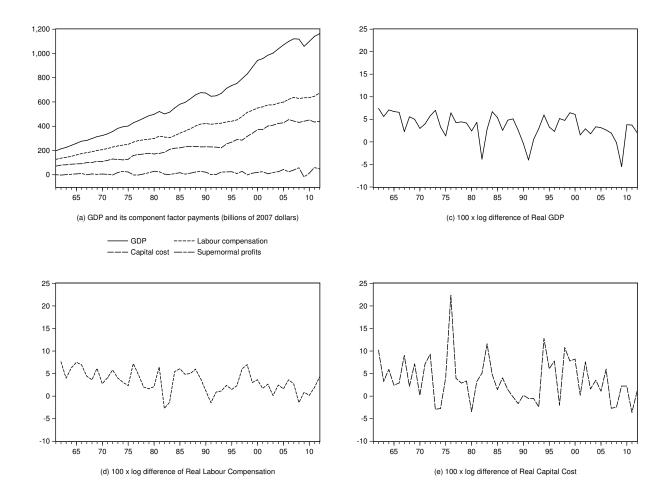
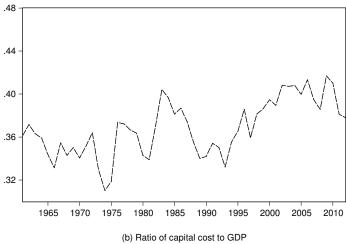


Figure 1: Canadian business sector GDP, its component factor payments, and their growth rates, 1961-2012





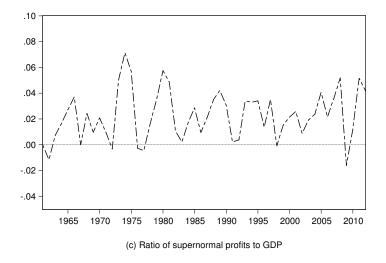


Figure 2: Business sector factor shares and rate of profit, 1961-2012

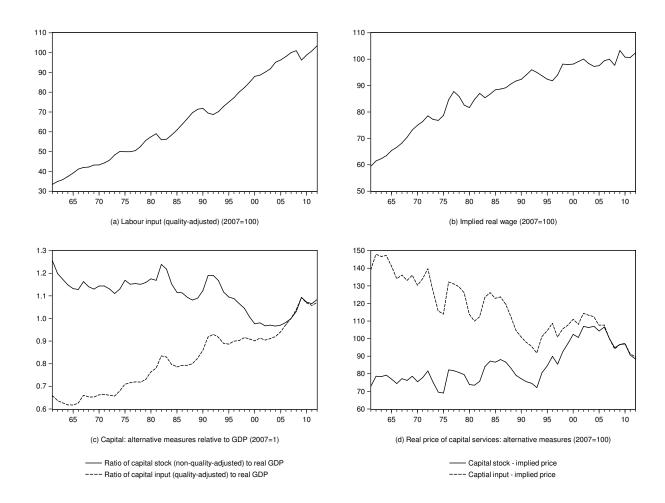


Figure 3: Business sector labour, capital, and implied real factor prices, 1961–2012

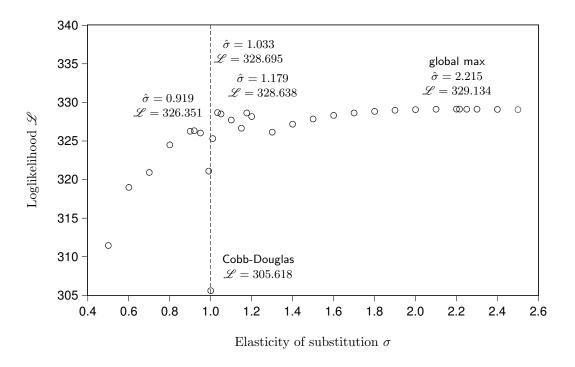


Figure 4: Scatter plot of loglikelihood values for the maintained model M0: four maxima

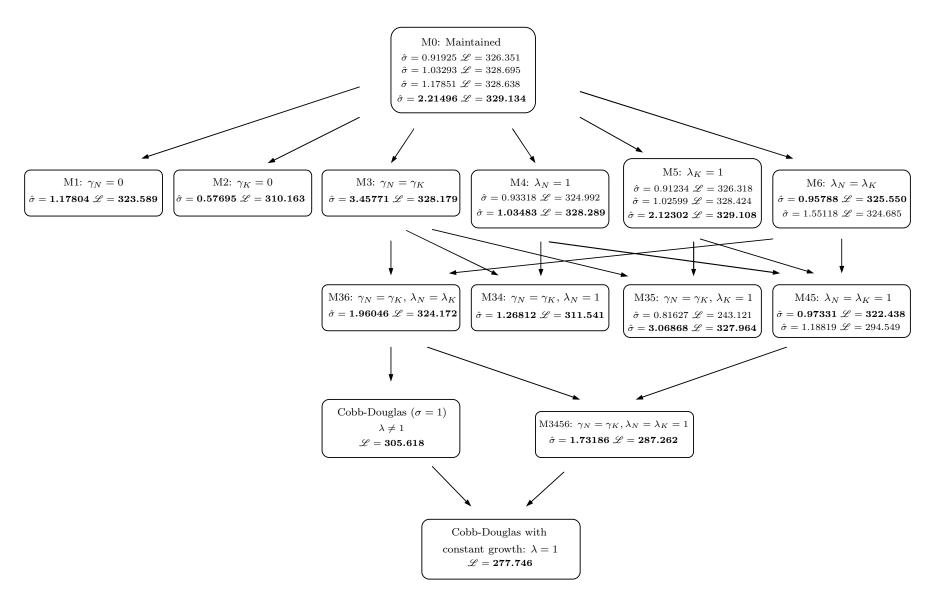


Figure 5: The KMW nested testing structure (unique or global maxima are bolded) [In hardcopy, the readability of this Figure can be improved by setting Orientation to Auto portrait/landscape when printing the paper, which will print this page in landscape format.]