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Inflation, Redistribution, and Real Activities

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August, 2013

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Empirical evidence suggests that inflation has positive effect on both output and unemployment in the long run in the United States. This paper develops a monetary model in which a higher inflation rate increases both output and unemployment. The model has two key features: (i) separation between workers and owners of firms (employers) and (ii) endogenous labor force participation. Changes in money supply redistributes consumption between employers and workers. This redistribution along with endogenous labor force participation creates a channel by which a higher inflation rate increases output, unemployment, and labor force participation. The Friedman rule does not maximize social welfare.

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Empirical evidence suggests that inflation has positive effect on both output and unemployment in the long run in the United States. This paper develops a monetary model in which a higher inflation rate increases both output and unemployment. The model has two key features: (i) separation between workers and owners of firms (employers) and (ii) endogenous labor force participation. Changes in money supply redistributes consumption between employers and workers. This redistribution along with endogenous labor force participation creates a channel by which a higher inflation rate increases output, unemployment, and labor force participation. The Friedman rule does not maximize social welfare.

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1   Introduction

There is a large empirical literature which examines the long-run relation-
ship between inflation and real activities.[1] Empirical evidence for the United
States suggests a positive long run relationship between inflation and output
(e.g. King and Watson 1992, Bullard and Keating 1995, Ahmed and Rogers
2000, Rapach 2003). At the same time, there are parallel studies which ex-
amine relationship between inflation and unemployment. These studies find
a positive relationship between inflation and unemployment in the United
States (e.g. Beyer and Farmer 2007, Berentsen, Menzio, and Wright 2010).
Taken together, these studies suggest that a higher inflation rate increases
both output and unemployment.

There is also a large theoretical literature which studies the long-run
relationship between inflation and real activities (superneutrality). The pre-
dicted relationship between inflation and real activities depends on under-
lying monetary framework. Cash-in-Advance models usually predict a neg-
ative relationship between inflation and real activities. A higher inflation
rate reduces output and employment (e.g. Stockman 1981, Cooley and
Hansen 1989). In the Money-In-Utility function models, money is superneu-
tral (Sidrauski 1967). Search theoretic monetary models which incorporate
labor markets (e.g. Kumar 2008, Berentsen, Menzio, and Wright 2010) typ-
ically predict a negative relationship between inflation and output and a
positive relationship between inflation and unemployment. The divergent
effects of inflation on output and unemployment in these models are at odds
with empirical evidence.

In this paper, I develop a search-theoretic monetary model with hetero-
geneous agents in which changes in inflation rate have positive effects on
both output and unemployment.[2] There are two key features of the model.
Firstly, only a fraction of agents, called employers, own firms. Firms require
labor to produce goods. Labor is supplied by other type of agents (nonown-

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[1]An earlier version of this paper was circulated as "Monetary Policy, Production, and
Distribution".

[2]Search theory is currently the dominant paradigm for the micro-foundation of money.
Search-theoretic models explicitly model the patterns of meetings, specialization in produc-
tion and consumption, and the information structure which lead to the 'double coincidence
of wants problem' in the goods market, and intrinsically useless (fiat) money emerges as
a medium of exchange endogenously (see Kiyotaki and Wright 1993, Lagos and Wright
2005, Shi 1995, 1997).

ers) called workers. Secondly, the labor force participation is endogenous. These two features induce novel effects of changes in money creation rate. A higher money creation rate leads to both higher output and unemployment.

The separation between workers and owners of firms is similar to Diamond and Yellin (1990) and Laing, Li, and Wang (2007) and is quite realistic. There is substantial empirical evidence that only a minority of households participate in capital markets. Mankiw and Zeldes (1991) using PSID data find that only one fourth of households owned stocks directly or indirectly. Brav, Constantinides, and Geczy (2002) using Consumer Expenditure Survey data from 1980-2000 find that only a small section $(15 - 20\%)$ of the U.S. households owned financial assets such as stocks, bonds, mutual funds, and other securities. Of those who held such financial assets, only 10-15 percent of them held assets worth more than \$2000 (in 1996 dollars). Attansaio et. al. (2002) find similar evidence for the United Kingdom.

In the model there are two markets: goods market and labor market. Both markets are characterized by search frictions. The matching process and price determination in both markets are modeled along the lines of the competitive equilibrium analyzed in Lucas and Prescott (1974), Alvarez and Veracierto (1999), Rocheateau and Wright (2005), and Kambourov and Manovskii (2009). Agents queue to enter markets. But due to frictions only a fraction of them are able to enter these markets. Agents in both markets are assumed to be price takers.

In the paper, I derive following main results, which are new to the literature. Firstly, an increase in the money creation rate increases output, labor force participation rate, and unemployment rate. These results are consistent with the empirical evidence.[3] Secondly, I find that a higher money creation rate redistributes consumption in favor of employers making employers better off and workers worse off. Finally, the Friedman rule which requires that the money creation rate be equal to the discount rate does not maximize social welfare.

The mechanism of these results is as follows. A higher money creation rate reduces consumption of workers, which induces workers to supply more labor by increasing their marginal utility of consumption. Consequently, the labor force participation and labor hours worked per employed worker rise. But at the same time a higher money creation rate erodes the value of

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[3]Section 7 provides empirical evidence on the long run relationship between inflation, output, unemployment rate, and labor force participation rate in the United States.

real wages, which induces workers to supply less labor reducing labor force participation and labor hours worked per employed worker. The net effect of a higher money creation rate depends on the relative strength of these two effects.

I find that the first effect dominates the second effect and a higher money creation rate leads to higher output, labor force participation, and labor hours worked per employed worker. At the same time, a higher labor force participation increases the number of workers searching for jobs leading to higher unemployment rate. The endogenous labor force participation plays a crucial role in generating positive association between output and unemployment in the model. With *fixed* labor force participation, the model will generate negative association between the two.

Regarding welfare results, for a *given* distribution of consumption and labor force participation rate, the Friedman rule induces optimal employment. But it does not lead to the optimal distribution of consumption and labor force participation rate.

The result that a higher money creation rate may increase output is also derived in a monetary search-theoretic framework in Shi (1998). In his model, the positive relationship between inflation and output arises due to complementarity between the household labor supply and its search intensity in the goods market. Higher inflation can encourage both work and search effort resulting in a higher output. In a very different model, Laing et. al. (2007) also find similar results.[4]

Unlike these models, I assume that the search-intensity of buyers in the goods market is *fixed*. The positive relationship between inflation and output arises due to redistribution of consumption between workers and employers rather than the complementarity between work and search effort in the goods market. In addition, in Shi model higher output is accompanied by lower unemployment. While in Laing et. al. model there is no unemployment.

My model relates to monetary search models such as Kumar (2008) and Berentsen, Menzio, and Wright (2010), which incorporate unemployment in monetary search framework. In these models, there is no separation between

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[4]Monetary models which incorporate Tobin-type effects (Tobin 1965) also predict positive relationship between inflation and output (e.g. Weiss 1980, Espinosa-Vega and Russell 1998, Heer 2003). These models rely on substitution between real money balances and capital to generate positive relationship. However, Weiss (1980) and Espinosa-Vega and Russel (1998) do not have unemployment. In Heer (2003), higher output leads to lower unemployment.

employers and workers. An agent supplies labor as well as own firms. This precludes inflation from having any redistributive effect. In addition, labor force participation in these models is exogenous. In terms of result, as mentioned earlier, these models predict a negative relationship between inflation and output, which is at odds with empirical evidence.

My results are also part of a growing literature which studies the environments with heterogeneous agents in which the Friedman rule is sub-optimal (e.g. Bhattacharya, Haslag, Martin 2005, Green and Zhou 2005, Molico 2006). Similar to these studies, I also find the Friedman rule is not efficient, though in a very different environment. These studies examine the optimality of the Friedman rule in endowment economies.

Rest of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the optimal decisions of agents. Section 4 characterizes and establishes the existence of a symmetric and stationary steady state monetary equilibrium. Section 5 analyzes the effects of changes in money creation rate on real activities. Section 6 analyzes welfare implications. Section 7 provides empirical evidence on the long run relationship between inflation, output, unemployment rate, and labor force participation rate in the United States. This is followed by concluding remarks. All proofs are in appendix.

2   The Economy

Time is discrete. Consider an economy with $K(\geq 2)$ non-storable goods consisting of two islands: H and F. Island H is inhabited by a large number (unit measure) of infinitely-lived risk-averse agents called workers. Workers desire to consume goods and supply labor. However, they cannot produce goods. Workers differ in terms of goods they desire to consume. The type of good a worker would like to consume is determined by a uniformly distributed *i.i.d.* random shock each period. Thus in any period, a particular type of good is consumed by $\frac{1}{K}$ workers.

Island F consists of a large number (unit measure) of K types of infinitely-lived firms. Firms possess technology to produce consumption goods but require labor supplied by workers to do so. A firm of particular type produces goods of particular type. In any period, a particular type of good is produced by $\frac{1}{K}$ firms. Similar to Diamond and Yellin (1990) and Laing et. al. (2007), I assume that each firm is owned by an infinitely-lived agents called employers (owners) and the employer desires to consume the product produced by his

own firm.[5] Employers live on island F. Assume that claims on profits of firms (or shares) cannot be used to buy goods in the goods market.[6]

There are two markets: the goods market and the labor market. Both markets are characterized by search frictions. Buyers and sellers in the goods market and workers and employers in the labor market are brought together randomly through the search process described later. Random meetings between buyers and sellers imply that a particular buyer or a seller in the goods market cannot be relocated in future. This assumption rules out credit arrangements between buyers and sellers and exchanges must be *quid-pro-quo*. Thus, money is used as a medium of exchange.

Due to random meetings, individual agents in both the goods market and the labor market face uncertainty in trading outcomes. This generates non-degenerate distributions of money holding, sales, employment status, and consumption which make the model analytically intractable and numerically challenging. In order to make these distributions degenerate and the analysis tractable, following Shi (1997, 1998), I use the construct of large worker households and firms.[7]

Each worker household is assumed to comprise of unit measure of workers and unit measure of buyers. These workers and buyers do not have independent preferences. Rather, the household prescribes their trading strategies to maximize the overall household utility. Workers and buyers share equally in the utility generated by the household consumption. With this modeling device, the decisions of different worker household types are identical in a symmetric equilibrium, except for the types of goods they consume. Thus, I can analyze the behavior of a representative worker household.

Similarly, a firm consists of unit measure of sellers who sell goods in the goods market. Just as in the case of worker households, these agents do not have independent preferences, but undertake activities in order to maximize firm's profit.[8] Large number of sellers implies that idiosyncratic risks faced

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[5]This is equivalent to assuming that employers consume goods which are different from what they produce, but they can trade in a centralized goods market without double coincidence of wants problem.

[6]This restriction is needed so that these claims do not replace money as a medium of exchange. One can assume that worker households can easily counterfeit such claims and sellers in the goods market cannot verify them (e.g. Aruba et. al. 2008).

[7]An alternative frameworks which produces degenerate distributions of money holding and prices is examined by Lagos and Wright (2005).

[8]These sellers need not be unpaid. One can assume that a *fixed number* of workers are required for sales activities. These employees are chosen randomly at the beginning of

by individual sellers are smoothed within the firm. With this construction of firms, the decisions of employers of different types are identical in a symmetric equilibrium, except for the types of goods they produce and consume. Thus, I can analyze the behavior of a representative employer (or firm).

2.1 Trading and Price Determination in the Goods and Labor Markets

I model the pricing mechanism in both markets along the lines of the competitive equilibrium analyzed in Lucas and Prescott (1974), Alvarez and Veracierto (1999), Rocheteau and Wright (2005), and Kambourov and Manovskii (2009). In this formulation, markets are assumed to be competitive. However, due to search frictions only a fraction of agents are able to enter market. Agents are assumed to be price takers. Price is determined by the condition that total demand equals total supply.

The search process in the goods market is modeled as in Rocheteau and Wright (2005). Similar to them, I assume that buyers and sellers in the goods market queue to enter the market every period and only a fraction of them are able to enter the market. Buyers and sellers who are able to enter the market trade at the given price, $\hat{p}_t(i)$, for good i. Suppose that only a *fixed* fraction ξ $(0 < \xi < 1)$ of buyers and sellers are able to enter the market.

The assumption that only a *fixed* fraction of buyers and sellers enter the market allows me to clearly contrast my results and mechanism from the studies (*e.g.* Shi 1998, Laing et. al. 2007), which focus on the complementarity between the labor supply decision and the search-intensity of households in the goods market. This issue is further discussed in the concluding section. Since, measures of buyers in a worker household and sellers in a firm are normalized to one, the measures of buyers in a worker household and sellers of a firm who are able to trade are equal to ξ in any time period.

I model search process process in the labor market along the lines of Lucas and Prescott (1974), Alvarez and Veracierto (1999), and Kambourov and Manovskii (2009). Workers queue to enter the labor market every period and only a fraction of them, χ, are able to enter the market. Workers who are able to enter the market are randomly allocated among employers. Suppose that χ is a following function of aggregate number of workers queuing for employment, $\hat{n}_t$, i.e.

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every period from the existing pool of employees.

$$\chi(\hat{n}_t) = a\hat{n}_t^{-\zeta}, \text{ where } a > 0 \ \& \ 0 < \zeta < 1. \tag{2.1}$$

I call $\chi(\hat{n}_t)$ the entry function in the labor market. Denote average price in goods market by $\hat{p}_t$ and real wage per unit of labor in the labor market by $\hat{w}_t$ at time t. Since, I am going to focus on symmetric equilibrium in which $\hat{p}_t(i) = \hat{p}_t$, I will drop the goods index i from prices of individual goods in the rest of the paper. In addition, I will denote the variables, which are taken as given by a particular worker household or an employer, with superscript "^".

2.2 Money Supply Process

Let $\hat{M}_t$ be the money supply at time t. Suppose that the government increases money supply at the constant rate g and thus $\hat{M}_{t+1} = g\hat{M}_t$. Worker households and employers receive monetary injection in lump-sum fashion. These two types of agents may receive differential transfers.

Suppose that at the initial period, an employer has an endowment of $\phi\hat{M}_0$ amount of money, where $0 \leq \phi \leq 1$. A worker household has an initial endowment of money equal to $(1-\phi)\hat{M}_0$. At the beginning of each subsequent period, each employer receives $(g-1)\phi\hat{M}_{t-1}$ units of fiat money from the government as a lump-sum transfer. Similarly, a worker household receives $(g-1)(1-\phi)\hat{M}_{t-1}$ units of fiat money from the government as a lump-sum transfer.

This formulation encompasses different possibilities. If $\phi = \frac{1}{2}$, both worker households and employers receive identical transfer as in standard monetary models. If $\phi = 1$, then employers/firms receive all the monetary injection as in limited participation models (e.g. Fuerst 1992, Christiano, Eichenbaum, and Evans 1997).

In the paper, I show that the effects of changes in money supply depends on ϕ and changes in ϕ itself has important implications. One way to thinks about ϕ is that the government transfers newly created money to financial institutions (not modeled here), and inhabitants of two islands have differential access to these institutions. Parameter ϕ captures the differential access of inhabitants of these two islands to financial institutions.

For future use, I call the parameter ϕ the distributional parameter of money supply. Note that when the money creation rate, $g < 1$ (decrease in money supply), the distributional parameter, ϕ, determines the proportion

in which withdrawal of money is distributed between worker households and employers.

3 Optimal Decisions of the Representative Worker Household and the Representative Employer

3.1 Timing

At the beginning of the period, t, both the representative worker household and the representative employer receive monetary transfers. Let M_t^h and M_t^f be units of post-transfer money with the representative worker household and the representative employer respectively at time t. After receiving transfers, the worker household chooses labor force participation rate (number of workers to participate in the labor market), n_t, and labor intensity (labor units/hours to be supplied) of an employed worker, l_t, at a given real wage $\hat{w}_t$. After receiving instructions, workers go to the labor market and fraction $\chi(\hat{n}_t)$ of workers get employment. Each employed worker then supplies l_t units of labor. Assume that once the workers go to the labor market, they remain separated from the household till the end of period t.

After workers have left the household, the worker household receives preference shock which determines the good which the household would like to consume in period t. After the preference shock, the worker household distributes available money balance, M_t^h, equally among buyers and chooses the amount of money to be spent by a buyer, m_t. After receiving instructions, buyers go to the goods market. The buyers who are able to enter the goods market (fraction ξ) buy goods at the given price, $\hat{p}_t$. For future reference, I call buyers who are able to enter the goods market '*matched buyers*'.

Similarly after receiving transfers, the employer chooses the number of labor units to hire, e_t, taking wages as given and produces output, $f(e_t)$. After production, the employer chooses quantity, q_t, to sell. He distributes the chosen quantity, q_t, equally among sellers. Since the measure of sellers is unity, each seller receives, q_t, units of goods. After receiving goods, sellers go to the goods market. The sellers who are able to enter the goods market (fraction ξ) sell goods at the given price, $\hat{p}_t$. I call sellers who are able to enter the goods market '*matched sellers*'. Since only ξ fraction of sellers are able to sell their goods, the actual sales by a firm is ξq_t. To contrast the actual amount of goods sold from the optimally chosen quantity of goods to

sell, q_t, I call q_t the *desired quantity of goods to sell*.

After trading in the goods market, buyers come back to the household with the purchased goods and any residual nominal money balances. Similarly, sellers come back with their nominal sales receipts and any unsold stock of goods. The employer pays wages to employed workers in terms of money, and they return to their households.

The residual nominal money balances of buyers and wage receipts of workers are added to the worker household nominal money balance for the next period. Similarly, nominal sales receipts of sellers *net of* nominal wage payment are added to the nominal money balance of the employer to be carried to the next period. Consumption takes place. Time moves to the next period.

3.2 The Optimal Decisions of the Representative Worker Household

Assume that the representative worker household maximizes the discounted sum of utilities from the sequence of consumption less the disutility incurred from working. The household's inter-temporal utility is represented by

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[U(c_t^h) - \chi(\hat{n}_t)n_t\mu(l_t) - \tau n_t\right] \text{ with } U'(c_t^h) > 0$$

$$U''(c_t^h) < 0, \ \lim_{c_t^h \to 0} U'(c_t^h) > 0,$$

where r, $U(c_t^h)$, $\mu(l_t)$, and τ are the rate of time-preference, the utility derived from consumption, c_t^h, the disutility from supplying l_t units of labor by an employed worker and the disutility from labor force participation respectively.

Let the disutility from supplying l_t units of labor by an employed worker be the following function

$$\mu(l_t) = l_t^{\theta} \text{ with } \ \theta > 1. \tag{3.1}$$

The money spent by an individual buyer in the goods market satisfies the following inequality

$$m_t \leq M_t^h. \tag{3.2}$$

Recall that the measure of matched buyers in the household is ξ. Then consumption, c_t^h, satisfies the following inequality

$$c_t^h \leq \xi \frac{m_t}{\hat{p}_t}. \tag{3.3}$$

The budget constraint of the worker household is given by

$$M_{t+1}^h \leq M_t^h + (g-1)(1-\phi)\hat{M}_t + \chi(\hat{n}_t)\hat{p}_t\hat{w}_t n_t l_t - \xi m_t. \tag{3.4}$$

The term on the left hand side is the post-transfer money holding at the beginning of period, $t+1$. The first term on the right hand side is the nominal money balance of the household at time t, the second term is the lump-sum monetary transfer at the beginning of period $t + 1$, and the third term is the nominal wage payment received by the household. The final term is the money spent by the matched buyers at time t.

Next I set up the optimization problem of the worker household. Taking real wage, $\hat{w}_t$, prices in the goods market, $\hat{p}_t$, and the initial money holding, $(1-\phi)\hat{M}_0$, as given, the representative worker household chooses the sequence of $\{c_t,\ m_t,\ l_t,\ n_t,\ M_{t+1}^h\}$, $\forall\ t \geq 0$ to solve the following problem.

Worker Household Problem (PH)

$$\max_{c_t^h,m_t,l_t,n_t,M_{t+1}^h} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[U(c_t^h) - \chi(\hat{n}_t)n_t\mu(l_t) - \tau n_t\right]$$

subject to the constraints on money spent by an individual buyer (3.2), the household's consumption (3.3), and the budget constraint (3.4).

Turning to the optimal choices, consumption, c_t^h, is given by the equality constraint (3.3). Denote the Langrangian multipliers associated with the constraints on the nominal money balance of an individual buyer (3.2) by λ_t and on the household budget constraint (3.4) by ω_{Mt}^h. Then the first order condition for the optimal choice of M_{t+1}^h is given by

$$\omega_{Mt}^h = \frac{1}{1+r}\left[\omega_{Mt+1}^h + \xi\lambda_{t+1}\right]. \tag{3.5}$$

The first order condition has the usual interpretation. The right hand side of (3.5) is the discounted expected marginal benefit from carrying an additional unit of money next period. If the household carries one additional unit of

money next period, then it relaxes the budget constraint (3.4) as well as the constraint on the nominal balance of matched buyers (3.2). Note that only ξ fraction of buyers are able to enter the goods market.

The optimal choice of spending by an individual buyer, m_t, satisfies

$$\lambda_t = \frac{U'(c_t^h)}{\hat{p}_t} - \omega_{Mt}^h. \tag{3.6}$$

λ_t can be interpreted as the net surplus generated by a matched buyer for the household from an additional unit of expenditure. Spending of an additional unit of money increases the household's utility by $\frac{U'(c_t^h)}{\hat{p}_t}$, but at the same time it tightens the budget constraint. For a matched buyer to get positive surplus *i.e.*, $\lambda_t > 0$, the constraint on the spending of a matched buyer given in (3.2) must be binding.

The optimal choice of labor intensity, l_t, satisfies following first order condition:

$$\mu'(l_t) = \hat{p}_t \hat{w}_t \omega_{Mt}^h. \tag{3.7}$$

The left hand side of (3.7) is the marginal disutility from working. The right hand side is the marginal benefit. The household receives $\hat{p}_t \hat{w}_t$ from an additional unit of labor supply, which relaxes matched buyers expenditure constraint (3.2) and the household's budget constraint (3.4).

Finally, the optimal choice of labor force participation, n_t, satisfies following first order condition

$$\chi(\hat{n}_t)\mu(l_t) + \tau = \chi(\hat{n}_t)\hat{p}_t \hat{w}_t l_t \omega_{Mt}^h. \tag{3.8}$$

The left hand side of (3.8) is the marginal disutility from labor force participation which takes into account labor force participation cost as well as expected disutility from working. The right hand side is the expected marginal cost, which takes into account that a labor force participant finds employment with probability, $\chi(\hat{n}_t)$.

3.3 The Optimal Decisions of the Representative Employer

Assume that the representative employer maximizes the discounted sum of utilities from the sequence of consumption, c_t^f:

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$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t V(c_t^f) \text{ where } V'(c_t^f) > 0, \ V''(c_t^f) \leq 0.$$

Suppose that the representative employer has a production function given by, $f(e_t)$, with following properties

$$f(0) = 0, \ f'(e_t) > 0, \ f''(e_t) < 0, \ \lim_{e_t \to 0} f'(e_t) = \infty, \ \lim_{e_t \to 0} e_t f'(e_t) < \infty \quad (3.9)$$

where e_t is the number of labor units employed at time t. Number of labor units employed is given by the product of number of employed workers and the average number of labor units supplied by them.

Given that only ξ fraction of the sellers are able to enter the goods market, consumption by the employer, c_t^f, satisfies following equality:

$$c_t^f = f(e_t) - \xi q_t. \quad (3.10)$$

Recall that the employer can use the current sales receipt and current post transfer money holding to finance his wage bill. Thus, the employer faces following wage finance constraint:

$$\hat{p}_t \hat{w}_t e_t \leq M_t^f + \xi \hat{p}_t q_t. \quad (3.11)$$

Finally the budget constraint of the employer is given by

$$M_{t+1}^f \leq M_t^f + (g-1)\phi \hat{M}_t + \xi \hat{p}_t q_t - \hat{p}_t \hat{w}_t e_t. \quad (3.12)$$

The term on the left hand side is the post-transfer money holding at the beginning of period, $t+1$. The first term on the right hand side is the nominal money balance of the employer at time t, the second term is the lump-sum monetary transfer at the beginning of period $t+1$, and the third term nominal sales receipt. The final term is the nominal wage paid by the employer.

Next I set up the optimization problem of the employer. Taking the real wage, $\hat{w}_t$, the prices in the goods market, $\hat{p}_t$, and the initial money holding, $\phi \hat{M}_0$, as given, the employer chooses the sequence of $\{c_t^f, \ q_t, \ M_{t+1}^f, \ e_t\}$, $\forall \ t \geq 0$ to solve the following problem.

Employer's Problem (PE)

$$\max_{c_t^f, e_t, q_t, M_{t+1}^f} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t V(c_t^f)$$

subject to the constraints on the employer's consumption (3.10), wage finance constrain (3.11), and the budget constraint (3.12).

Turning to the optimal choices, consumption, c_t^f, is given by the equality constraint (3.10). Denote the Langrangian multipliers associated with the constraints on the wage finance (3.11) by Ω_{wt} and on the employer's budget constraint (3.12) by ω_{Mt}^f. Then the first order conditions for the optimal choices of e_t, q_t, and M_{t+1}^f are given by

$$e_t: \ V'(c_t^f)f'(e_t) = \Omega_{wt} + \hat{p}_t \hat{w}_t \omega_{Mt}^f; \tag{3.13}$$

$$q_t: \ V'(c_t^f) = \frac{\Omega_{wt}}{\hat{w}_t} + \hat{p}_t \omega_{Mt}^f; \tag{3.14}$$

$$M_{t+1}^f: \omega_{Mt}^f = \frac{1}{1+r}\left[\omega_{Mt+1}^f + \frac{\Omega_{wt+1}}{\hat{p}_{t+1}\hat{w}_{t+1}}\right]. \tag{3.15}$$

(3.13) equates the marginal benefit of hiring an additional unit of labor, $V'(c_t^f)f'(e_t)$, with its marginal cost. The employer has to pay nominal wage, $\hat{p}_t \hat{w}_t$, which tightens the constraint on the wage finance (3.11) and the budget constraint (3.12). Ω_{wt} can be interpreted as the net surplus to the employer from hiring.

(3.14) similarly equates the marginal cost of selling an additional unit of good with the marginal benefit. The utility cost of selling an additional unit is $V'(c_t^f)$. The selling of an additional unit increases the money holding by $\hat{p}_t$, which relaxes the constraint on hiring and the budget constraint, the value of which is $\frac{\Omega_{wt}}{\hat{w}_t} + \hat{p}_t \omega_{Mt}^f$. (3.15) can be interpreted similarly. An additional unit of money carried forward relaxes the next period wage finance constraint as well as the budget constraint.

3.4   Price and Wage Determination

Since the goods market clears, the demand for goods should be equal to the supply. Recall that only fraction ξ of buyers and sellers are able to enter the

goods market. Thus, total demand for goods is $\frac{\xi \hat{m}_t}{\hat{p}_t}$ and total supply is $\xi \hat{q}_t$. Price, $\hat{p}_t$, is given by

$$\hat{p}_t = \frac{\hat{m}_t}{\hat{q}_t}. \tag{3.16}$$

Similarly, real wage, $\hat{w}_t$, is determined by the labor market market clearing condition

$$\hat{e}_t = \chi(\hat{n}_t)\hat{n}_t\hat{l}_t. \tag{3.17}$$

The left hand side of (3.17) is the total demand of labor units. The right hand side is the total supply of labor units. Due to friction, only fraction $\chi(\hat{n}_t)$ of the workers are able to enter the labor market. $\hat{l}_t$ is the average number of labor units supplied by the workers who are able to enter the labor market.

4 Symmetric Stationary Monetary Equilibrium

This paper restricts its attention to a symmetric and stationary monetary equilibrium. First, I require that all worker households have a common marginal value of money, ω_{Mt}^h, consumption, c_t^h, labor force participation, n_t, and labor intensity, l_t. Similarly, all employers have a common marginal value of money, ω_{Mt}^f, consumption, c_t^f, and employ same number of labor units, e_t. Finally, money has value $i.e.$, the marginal value of real money balance to both worker households and employers be strictly positive, $\hat{p}_t\omega_{Mt}^h, \hat{p}_t\omega_{Mt}^f > 0$.

Denote the supply of real money balance, $\hat{M} \equiv \frac{\hat{M}_t}{\hat{p}_t}$, the real money balance with a worker household, $M^h \equiv \frac{M_t^h}{\hat{p}_t}$, the real money balance with an employer, $M^f \equiv \frac{M_t^f}{\hat{p}_t}$, the real money balance with a buyer, $m = \frac{m_t}{\hat{p}_t}$, the buyer's surplus per purchase, $\hat{p}_t\lambda_t = \lambda$, the marginal value of real money balance for a worker household, $\Omega_M^h \equiv \hat{p}_t\omega_{Mt}^h$, and the marginal value of real money balance for an employer, $\Omega_M^f \equiv \hat{p}_t\omega_{Mt}^f$. From now on, I drop the subscript t from real variables.

For a symmetric stationary monetary equilibrium to exist, the surplus to a matched buyer must be positive ($i.e.$, $\lambda > 0$). If the matched buyer does not receive a positive surplus ($\lambda = 0$), then (3.5) implies that either the

marginal value of real money balance to the worker household, $p_t \omega_{Mt}^h = 0$ or $p_t \omega_{Mt}^h$ grows at the rate $(1 + r)g$. Similarly, an employer must get strictly positive surplus from hiring (*i.e.*, $\Omega_w > 0$). If $\Omega_w = 0$, then (3.15) implies that either $p_t \omega_{Mt}^f = 0$ or $p_t \omega_{Mt}^f$ grows at the rate, $(1 + r)g$. Thus in a symmetric stationary monetary equilibrium, buyers' nominal cash balance expenditure constraint (3.2) and employer's wage financing constraint (3.11) will be binding.

Using (3.5) and (3.6) one can easily show that for $\lambda > 0$, the money creation rate, g, should be greater than $\frac{1}{1+r}$. In the rest of the paper, I impose this condition.

Assumption 1: The money creation rate, $g > \frac{1}{1+r}$.

Assumption 1 also ensures that $\Omega_w > 0$ (see equation 4.6 below). Given buyers' nominal cash balance expenditure constraint (3.2) and the goods market clearing constraint (3.16), the price level is given by $\hat{p}_t \equiv \frac{\hat{M}_t^h}{\hat{q}}, \forall\, t$, in the stationary and symmetric equilibrium. Also the average price level, $\hat{p}_t$, will grow at the rate equal to the money creation rate *i.e.*, the inflation rate

$$\frac{\hat{p}_{t+1}}{\hat{p}_t} = g \quad \forall t. \tag{4.1}$$

Definition: A symmetric stationary monetary equilibrium (SSME) is defined as a collection of the worker household's choice variables $X^h \equiv \{c^h, M^h, m, l, n\}$, the employer's choice variables, $X^f \equiv \{c^f, M^f, q, e\}$, the prices in the goods market, $\hat{p}_t$, and the real wage in the labor market, $\hat{w}$, and the aggregate variables $\hat{X}^h$ and $\hat{X}^f$, such that

- Given aggregate variables, $\hat{X}^h$ and $\hat{X}^f$, and the prices in the goods market, $\hat{p}_t$, and the real wage in the labor market, $\hat{w}$, the worker household's choice variables X^h solve (PH);

- Given aggregate variables, $\hat{X}^h$ and $\hat{X}^f$, the prices in the goods market, $\hat{p}_t$, and the real wage in the labor market, $\hat{w}$, the employer's choice variables X^f solve (PE);

- price in the goods market, $\hat{p}_t$, satisfies the goods market clearing condition (3.16);

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- real wage, $\hat{w}$, satisfies the labor market clearing condition (3.17);

- aggregate variables are equal to the relevant worker household's and employer's variables, $\hat{X}^h = X^h$, $\hat{X}^f = X^f$; and

- the marginal values of real money balances to the worker household and the employer, Ω_M^h & Ω_M^f, be strictly positive and finite, $0 < \Omega_M^h, \Omega_M^f < \infty$.

Given that both goods and labor markets clear, the money market also clears and thus $\hat{M} = \hat{M}^h + \hat{M}^f$. From now on I suppress " ^ " from the aggregate variables.

(3.5) and (3.6) imply that the marginal value of real money balance to the worker household, Ω_M^h, is given by

$$\Omega_M^h = \frac{\xi}{(1+r)g - 1 + \xi} U'(c^h).\tag{4.2}$$

(3.6), (3.7) and (4.2) imply that labor intensity, l, satisfies

$$\mu'(l) = \frac{\xi}{(1+r)g - 1 + \xi} w U'(c^h).\tag{4.3}$$

(4.3) is one of the key equations of the model. It shows that for a given real wage, w, and consumption of the worker household, c^h, a higher money creation rate, g, reduces labor intensity. Intuitively, a higher money creation rate, g, reduces the value of real money balance and thus the worker household reduces labor intensity. For the similar reason, a lower real wage reduces labor supply for a given level of money creation rate, g, and worker household consumption, c^h. Finally, for a given level of real wage, w, and money creation rate, g, a lower consumption level of the worker household, c^h, increases labor intensity. A lower consumption level increases the marginal utility of consumption and thus the marginal return from working.

Given that $\mu(l) = l^\theta$ and $\chi(n) = an^{-\varsigma}$, (3.7) and (3.8) imply the labor force participation rate, n, satisfies

$$1 + \frac{\tau}{al^\theta n^{-\varsigma}} = \frac{\mu'(l)l}{\mu(l)} \equiv \theta.\tag{4.4}$$

(3.3) and the goods market clearing condition (3.16) imply that consumption of the worker household, c^h, is given by

$$c^h = \xi q. \tag{4.5}$$

From (3.14) and (3.15) I have

$$\Omega_w = \frac{(1+r)g - 1}{(1+r)g} V'(c^f)w. \tag{4.6}$$

Then from (3.13) and (4.6), I get an expression for the marginal value of real money balance to the employer, Ω_M^f, given by

$$\Omega_M^f = \frac{1}{(1+r)g} V'(c^f). \tag{4.7}$$

(3.13) and (3.14) imply that labor demand is given by

$$f'(e) = w. \tag{4.8}$$

The employer equates the marginal product of labor to the real wage.

(3.10) and (4.5) imply that consumption of employer, c^f, is given by

$$c^f = f(e) - \xi q = f(e) - c^h. \tag{4.9}$$

The market clearing condition for the goods market, $p_t = \frac{M_t^h}{q_t}$, imply that the desired quantity of goods to sell, q, satisfies

$$q = M^h. \tag{4.10}$$

Using the employer's budget constraint (3.12), the wage finance constraint (3.11), and (4.10), I derive relationship between the desired quantity of goods to sell, q, and the real wage bill, ew,

$$ew = \frac{g\xi + (g-1)\phi(1-\xi)}{g - (g-1)\phi} q. \tag{4.11}$$

For a given real wage bill, ew, (4.11) traces a negative relationship between g and q for any $\phi > 0$. This happens because the real wage bill can be financed in two ways: monetary injection and sales. An increase in g increases the share of real wage bill financed by monetary injection. Thus, the employer has to sell less goods to finance his real wage bill leading to fall in q. When $\phi = 0$, employers do not receive monetary injection and changes in g do not affect q.

Using (4.5), (4.8) and (4.11), I get an alternative expression for the consumption of the worker household, c^h, given by

$$c^h = \xi q = \frac{g - (g-1)\phi}{g\xi + (g-1)\phi(1-\xi)}\xi f'(e)e. \tag{4.12}$$

(4.12) shows that consumption, c^h, and the desired quantity of goods to sell, q, are strictly positive only when, $g\xi + (g-1)\phi(1-\xi) > 0$.

Assumption 2 The parameter values are such that $g\xi + (g-1)\phi(1-\xi) > 0$.

(4.12) is the key equation of the model and captures the distributional consequences of changes in money supply. It shows that for any $\phi > 0$, changes in g affect consumption of worker households *directly*. In particular, for a given employment, e, an increase in g reduces c^h, for any $\phi > 0$.[9] Thus, a higher g redistributes consumption in favor of employers. This happens because a higher g reduces the desired quantity to sell, q.

The direct effect of monetary policy on c^h is the consequence of the separation between the worker households and employers. In a model in which worker households and employers are identical and the representative household receives both wages and dividends, consumption is independent of ϕ and changes in g do not have distributional consequences and consumption is not affected directly by changes in g.[10] Thus, for a given employment level, e, consumption of the household, c^h, is also fixed. As shown below, the distributional effects can significantly change the effects of changes in g on output, unemployment and labor force participation.

Note also that for any given g, changes in ϕ also affects consumption of worker households, c^h, by changing the proportion of real wage bill financed by monetary injection. In particular, if $g > 1$, an increase in ϕ reduces c^h for a given e as a greater proportion of real wage bill is financed by monetary injection. On the other hand, if $g < 1$ (monetary withdrawal), an increase in ϕ increases c^h for a given e as a greater proportion of monetary withdrawal is borne by employers.

By putting (4.8) and (4.12) in (4.3), I get

$$\theta l^{\theta-1} = \frac{\xi}{(1+r)g - 1 + \xi} f'(e) U'\left(\frac{g - (g-1)\phi}{g\xi + (g-1)\phi(1-\xi)}\xi f'(e)e\right). \tag{4.13}$$

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[9]Let $X(g) \equiv \frac{g-(g-1)\phi}{g\xi+(g-1)\phi(1-\xi)}$. Then, $X'(g) = -\frac{\phi}{(g\xi+(g-1)\phi(1-\xi))^2} < 0$.
[10]Proof is available on request.

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(4.13) together with (4.4) and the labor market clearing condition $e = \chi(n)nl$ determine equilibrium e, n, and l and characterize the SSME. Combining (4.4), (4.13), and $e = \chi(n)nl$, the three equations characterizing SSME can be reduced to one equation in one unknown, namely, equilibrium employed labor units, e. For ease of expression, I normalize $\frac{\tau}{a(\theta-1)} = 1$.

Under the normalization that $\frac{\tau}{a(\theta-1)} = 1$, the equilibrium employed labor units, e, satisfies following equation:

$$\theta \left(\frac{e}{a}\right)^{\frac{\zeta(\theta-1)}{\theta(1-\zeta)+\zeta}} = \frac{\xi}{(1+r)g - 1 + \xi} f'(e) U'\left(\frac{g - (g-1)\phi}{g\xi + (g-1)\phi(1-\xi)} \xi f'(e)e\right).$$
(4.14)

Also equilibrium n and l are given by

$$n = \left(\frac{e}{a}\right)^{\frac{\theta}{\theta(1-\zeta)+\zeta}} \quad \& \quad l = \left(\frac{e}{a}\right)^{\frac{\zeta}{\theta(1-\zeta)+\zeta}}.$$
(4.15)

(4.15) shows that both n and l are increasing functions of e.

Assumption 3. The production function is such that $\frac{df'(e)e}{de} > 0$.

An example of the production function which satisfies the above assumption is $f(e) = e^{\beta}$.

Proposition 1. Under assumptions 1, 2, and 3, there exists a unique SSME characterized by equations (4.1)-(4.13).

For the analysis of the effects of monetary policy, it is useful to illustrate the monetary equilibrium in terms of demand and supply of labor units. (4.8) traces a downward sloping demand (LD) curve in (e, w) space. The supply of labor units is given by:

$$\theta \left(\frac{e}{a}\right)^{\frac{\zeta(\theta-1)}{\theta(1-\zeta)+\zeta}} = \frac{\xi}{(1+r)g - 1 + \xi} w U'\left(\frac{g - (g-1)\phi}{g\xi + (g-1)\phi(1-\xi)} \xi f'(e)e\right).$$
(4.16)

It is easy to show that (4.16) traces an upward sloping labor supply (LS) curve in (e, w) space for a given values of g and ϕ. (4.16) also shows that the supply of labor units is directly affected by both the parameters of money supply g and ϕ. The existence of equilibrium is illustrated below:

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Figure 1
Graphic Portrait of Equilibrium
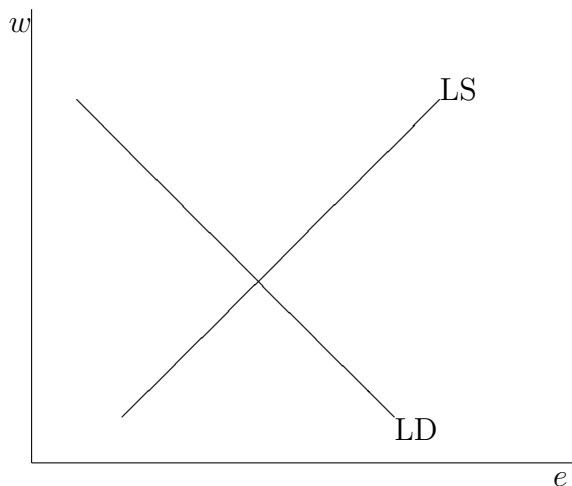
Before analyzing the effects of changes in the money creation rate, g, it is instructive to analyze the effects of changes in the distributional parameter of money supply, ϕ. The following proposition summarizes the effects of changes in the distributional parameter, ϕ, for a given money creation rate, g, on output, unemployment, labor force participation, and real wage.

Proposition 2. If the money creation rate $g > 1$, then a higher value of the distributional parameter of money supply, ϕ, increases labor force participation, n, labor intensity, l, employed labor units, e, output, $f(e)$, and unemployment rate, $1 - \chi(n)$, and reduces real wage, w. If the money creation rate $g < 1$, then a higher value of the distributional parameter of money supply, ϕ, reduces labor force participation, n, labor intensity, l, employed labor units, e, output, $f(e)$, and unemployment rate, $1 - \chi(n)$, and increases real wage, w.

The above proposition follows from equation (4.14) and (4.15). The mechanism is as follows. As discussed earlier, a higher value of ϕ for $g > 1$ reduces the worker household consumption, c^h, for a given level of employed labor units, e. The fall in c^h by increasing the marginal return from working induces higher supply of labor units from the worker household. Essentially, changes in ϕ do not affect the demand for labor units curve (equation 4.8). But for $g > 1$, a higher ϕ shifts the supply of labor units curve (equation 4.16) downward to the right in the (e, w) space. Thus, equilibrium employed

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labor units and output increase and real wage falls. Higher employment increases labor force participation rate and labor intensity. A higher labor force participation rate reduces the entry probability of workers in the labor market leading to higher unemployment rate. Opposite happens when the money creation rate, $g < 1$. In this case, the supply of labor units curve shifts upward to the left leading to lower equilibrium employed labor units and higher real wage.

Next, I analyze the effects of changes in the money creation rate, g, for a given distributional parameter of money, ϕ, on real activities.

5   Effects of Changes in the Money Creation Rate

(4.16) shows that a higher g affects supply of labor units directly by reducing the marginal value of real money balance and indirectly through consumption of worker households, c^h. I call the first effect *substitution effect of inflation* and the second effect *income effect of inflation* on supply of labor units.

The substitution effect of inflation on supply of labor units is negative, but the income effect is positive. The substitution effect captures the traditional inflation tax effect. For a given worker household consumption, c^h, and real wage, w, a higher money creation rate, g, reduces the return from working inducing the worker household to supply less labor units.

The positive income effect of inflation on supply of labor units is new to the literature and is the consequence of the separation between worker households and employers. In the model, for a given level of real wage, w, and $\phi > 0$, a higher money creation rate, g, reduces worker household consumption, c^h, which by increasing the return from working induces a larger supply of labor units from the worker household.

The above analysis shows that a higher money creation rate may increase or reduce supply of labor units depending on the relative strength of income and substitution effects. The relative strength of these two effects depends on the curvature properties of the worker household's utility function, rate of discount, r, and the distributional parameter of monetary policy, ϕ.

When $\phi = 0$ (worker households receive all the monetary injection), the income effect is absent and thus a higher money creation rate, g, unambiguously reduces supply of labor units. Since, the demand for labor units curve

remains unaffected, a higher g leads to lower employment and output and higher unemployment rate as in standard monetary models.

To show the effects of changes in the money creation rate, g, when $\phi > 0$, I assume that the worker household has a CRRA preference: $U(c^h) = \frac{c^{h(1-\alpha)}}{1-\alpha}$. Then the following proposition summarizes the effects of higher money creation rate, g, on employment and output.

Proposition 3. Suppose that the worker household has a CRRA preference: $U(c^h) = \frac{c^{h(1-\alpha)}}{1-\alpha}$. If at the initial equilibrium the values of parameters are such that

$$\frac{\alpha\phi[(1+r)g - 1 + \xi]}{[g\xi + (g-1)\phi(1-\xi)][g - (g-1)\phi]} > 1 + r \qquad (5.1)$$
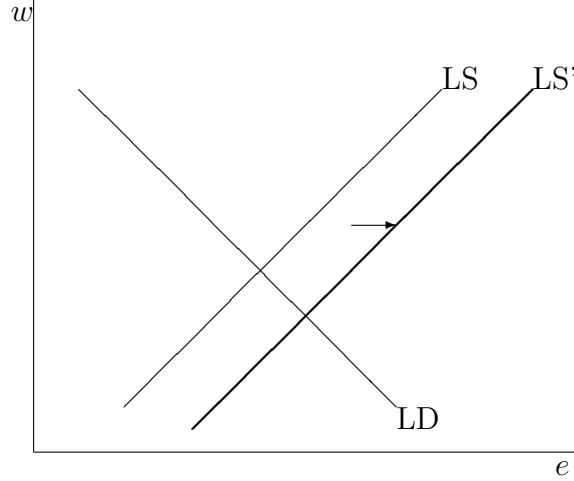
then a higher money creation rate, g, increases employed labor units, e, labor force participation, n, labor intensity, l, output, $f(e)$, and unemployment rate, $1 - \chi(n)$, and reduces real wage, w.

The above condition ensures that the positive income effect of inflation on supply of labor units dominates the negative substitution effect. Thus, supply of labor units curve shifts downward to the right in the (e, w) space. In general, since the left hand side of (5.1) is increasing in α, higher the coefficient of relative risk-aversion more likely (5.1) will be satisfied. This happens because higher is α greater is the income effect. In section 7, I show that this condition is satisfied for the U.S. economy for reasonable values of parameters.

In the case, (5.1) is not satisfied, a higher g reduces employed labor units, e, labor force participation, n, labor intensity, l, output, $f(e)$, and unemployment rate, $1 - \chi(n)$, and increases real wage, w. The effect of changes in g on equilibrium variables is illustrated in the following figure.

Figure 2
Effects of Increase in g when Condition 5.1 is Satisfied

The result that a higher money creation rate, g, can increase output, labor force participation, and unemployment through the redistribution of consumption between workers and employers is new to the literature.

In the next section, I examine the issue of the optimality of market allocations.

6   Welfare

I first begin with characterizing social optimal allocations. The social planner maximizes

$$\max_{c_t^h, n_t, l_t, e_t} \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t [\pi[U(c_t^h) - an_t^{1-\zeta}l_t^\theta - \tau n_t] + (1-\pi)V(c_t^f)]$$

subject to resource constraints

$$c_t^h + c_t^f = f(e_t) \ \& \tag{6.1}$$

$$e_t = an_t^{1-\zeta}l_t \tag{6.2}$$

where π is the relative weight put by the social planner on the utility of worker households.

In the steady state, it can be shown that the first order conditions reduce to

$$c^h : \pi U'(c^h) = (1-\pi)V'(c^f); \qquad (6.3)$$

$$l : \theta l^{\theta-1} = U'(c^h)f'(e); \qquad (6.4)$$

$$n : \theta[1-\zeta] = -\zeta + 1 + \frac{\tau}{al^\theta n^{-\zeta}}. \qquad (6.5)$$

(6.3) characterizes the condition for the socially optimal distribution of consumption. The social planner equates the social marginal utilities of consumption of the worker household and the employer. (6.4) determines the social optimal level of labor intensity. It equates the social marginal cost of supplying one extra unit of labor with its social marginal benefit. (6.5) characterizes the condition for the socially optimal level of labor force participation. While choosing labor force participation, the social planner takes into account the effect of having extra labor market participants on the entry probability of workers in the labor market.

Proposition 4. Under the assumption that $\lim_{c^h \to 0} \pi U'(c^h) = \lim_{c^f \to 0} (1-\pi)V'(c^f)$, there exists a unique allocation, $\{c_s^h, c_s^f, e_s\}$, which satisfies (6.1)-(6.5).

From the comparison of (6.3)-(6.6) with (4.4), (4.12), and (4.13), it is immediately clear that market allocations do not coincide with social optimal allocations. To analyze the optimality properties of market allocations, it is useful to first consider the case of exogenous labor force participation, $\overline{n}$, and thus exogenous matching rate of workers, $\chi(\overline{n})$. In this case, (4.4) and (6.5) do not apply.

Given the identity $e = \chi(\overline{n})\overline{n}l$, the comparison of (6.4) and (4.13) immediately shows that for a given worker household consumption, c^h, the market level of employed labor units, e, approaches its socially optimal level, e_s, as the money creation rate, g, approaches (from above) the Friedman rule, $(1+r)g = 1$. In other words, for a *given* distribution of consumption, the Friedman rule induces optimal level of employment. However, the comparison of (6.3) with (4.12) immediately shows that the Friedman rule does not lead to the optimal distribution of consumption.

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Only when the government has access to other tax and transfer instruments it can induce optimal distribution of consumption. Indeed, if the government can choose the distributional parameter of money supply, ϕ, then together with the Friedman rule it can achieve the socially optimal allocations. In this case, one can easily show that the optimal money creation rate is given by the Friedman Rule, $g_s = \frac{1}{1+r}$, and the optimal distributional parameter of money supply, ϕ_s, is given by

$$\phi_s = \frac{\xi}{r}\left[\frac{f'(e_s)e_s - c_s^h}{\xi f'(e_s)e_s + (1-\xi)c_s^h}\right]. \qquad (6.6)$$

The choice of ϕ allows the government to redistribute wealth from agents with relatively low marginal utility of consumption to agents with relatively high marginal utility of consumption.

Now, I consider the case in which labor force participation is endogenous. Suppose that the government can choose ϕ to induce optimal distribution of consumption. Then the comparison of (6.5) and (4.4) shows that the Friedman rule and the optimal choice of ϕ do not induce optimal labor force participation rate and unemployment rate. In fact, with the Friedman rule and the optimal choice of ϕ, the market equilibrium results in too much labor force participation and unemployment relative to the social optimum. The reason is the externality inherent in the search process in the labor market. One additional labor market participant reduces the entry probability of workers in the labor market. Market does not take into account this externality, leading to too many labor market participants.

Now, I consider the question whether the Friedman rule leads to Pareto improvement when the government does not have other instruments. To answer this question, I analyze the effects of monetary policy on consumption and its distribution. Denote the elasticity of production with respect to employment by $\eta(e) \equiv \frac{f'(e)e}{f(e)}$. Then from (4.12), it follows that the share of the worker household consumption in output, $\frac{c^h}{f(e)}$, is given by

$$\frac{c^h}{f(e)} = \frac{g-(g-1)\phi}{g\xi + (g-1)\phi(1-\xi)}\xi\eta(e). \qquad (6.7)$$

The share of the employer's consumption in output, $\frac{c^f}{f(e)}$, is given by

$$\frac{c^f}{f(e)} = 1 - \frac{c^h}{f(e)}. \qquad (6.8)$$

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The following proposition summarizes the distributional consequences of changes in g.

Proposition 5. Suppose that the elasticity of production function with respect to employment, $\eta(e) \equiv \frac{f'(e)e}{f(e)}$, is constant and the value of distributional parameter of money supply, $\phi > 0$. Then,

a. a higher money creation rate, g, reduces the share of worker household consumption in output, $\frac{c^h}{f(e)}$, and increases the share of employer's consumption in output, $\frac{c^f}{f(e)}$;

b. a higher money creation rate, g, reduces worker household consumption, c^h; and

c. if the condition stated in (5.1) is satisfied, a higher g increases employer's consumption, c^f. In addition, a higher g makes employers better-off and worker households worse-off.

An example of the production function which satisfies the condition that the elasticity of production function with respect to employment, $\eta(e) \equiv \frac{f'(e)e}{f(e)}$, is constant is $f(e) = e^\beta$. If the condition stated in (5.1) is not satisfied, a higher g may increase or lower employer's consumption, c^f. Also, it has ambiguous effect on the welfare of employers and worker households.

From the proposition, it is immediately clear that different money creation rates may produce Pareto optimal but noncomparable market allocations. Moving from a higher g to the Friedman rule does not lead to Pareto improvement. If the condition in (5.1) is satisfied, then it reduces the consumption of employers. If the social planner puts sufficient weight on the utilities of employers, such a move may not even increase the social welfare.

In the next section, I provide empirical evidence on the long run effects of changes in inflation on output, unemployment rate and labor force participation rate in the United States. I also show that the model generates positive relationship among inflation rate, output, unemployment, and the labor force participation rate as observed in the U.S. data for reasonable values of parameters.

7 Quantitative Analysis

As discussed earlier, empirical evidence suggests a positive long-run relationship between inflation and output in the U.S. (King and Watson 1992, Bullard and Keating 1995, Ahmed and Rogers 2000, and Rapach 2003) and a negative long-run relationship between unemployment and inflation (Beyer and Farmer 2007, Berentsen, Menzio, and Wright 2010). Using cointegration technique, I provide further evidence on the long run relationship between output, unemployment, and inflation in the United States. In addition, I provide evidence on the long run relationship between labor force participation rate and inflation rate in the United States.

To test for cointegration between two variables x_t and y_t, I estimate following relationship:

$$y_t = b_1 + b_2 x_t + b_3 t + \gamma_t \tag{7.1}$$

where γ_t is the error term. According to Engel-Granger (1987) representation theorem if both y_t and x_t are integrated of order 1, $I(1)$, but the residual, γ_t, is integrated of order zero, $I(0)$, then two variables are said to be cointegrated. Cointegration can also be established through the error-correction modeling technique outlined below:

$$\Delta y_t = \sum_{i=1}^{n_1} h_i \Delta y_{t-i} + \sum_{i=0}^{n_2} k_i \Delta x_{t-i} + d\gamma_{t-1} + \Gamma_t \tag{7.2}$$

where Γ_t is the error term.

Equation 7.2 incorporates the short-run dynamics into the adjustment process. When the two variables are adjusting towards their long-run equilibrium, the gap between y_t and x_t decreases. Since the gap between the two variables is measured by γ_t, cointegration is established if the coefficient of γ_{t-1}, d, is negative and significant. Kremers *et. al.* (1992) have shown that this approach towards cointegration is a more efficient method. To establish cointegration, I use both approaches.

For estimation, I use annual data of real GDP (base 2000), $f(e_t)$, unemployment rate (UR), $1 - \chi(n_t)$, labor force participation rate (LFPR), n_t, and inflation rate, inf_t, for the period 1950-2007 for the U.S. taken from *International Financial Statistics*. I derive inflation rate by taking logrithmic difference of GDP deflator, $gdpfla_t$, *i.e.* ($inf_t = \ln gdpfla_t - \ln gdpfla_{t-1}$). First, I perform unit root test based on Augmented Dicky-Fuller (ADF)

statistics on the difference and the level of inflation rate, log of real GDP, unemployment rate and labor force participation rate. The test indicates that all four variables are integrated of order one, $I(1)$. Then to test for cointegration, I estimate following equations:

$$\log f(e_t) = 2.9048 + 0.8033 inf_t + 0.0325t, \ \overline{R}^2 = 0.99, \ ADF = -2.96; \ (7.3)$$

$$UR_t = 0.0418 + 0.2311 inf_t + 0.0002t, \ \overline{R}^2 = 0.17, \ ADF = -3.68; \quad (7.4)$$

$$LFPR_t = 0.3440 + 0.2608 inf_t + 0.0029t, \ \overline{R}^2 = 0.89, \ ADF = -3.21. \ (7.5)$$

All the estimated coefficients are significant at 1%. In the estimated equations, ADF indicates the value of ADF statistics for the unit root test on the residuals with no constant and trend included in the regression. The unit root test on the residuals of estimated equation rejects the null of unit root at 1% in all the regressions.[11] The results suggest that inflation rate is cointegrated with output, unemployment rate, and labor force participation rate. In addition, inflation rate has a positive and significant effect on output, unemployment, and labor force participation.

To further, confirm the existence of cointegration, I estimate the error-correction model outlined in (7.2) for output, unemployment, and labor force participation. The estimated models are given in appendix 2. The estimated models show that in all cases, the coefficients on the lagged values of residuals are negative and significant.

Next, I show that the model generates positive relationship among inflation, output, unemployment, and labor force participation for reasonable values of parameters. Assume that both worker households and employers have CRRA utility function:

$$U(c) = \frac{c^{1-\alpha}}{1-\alpha}. \tag{7.6}$$

As shown in (5.1), the effects of changes in inflation rate depends on the values of r, g, ξ, α and ϕ.

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[11]The critical values at the significance levels of 1% and 5% are -2.61 and -1.98 respectively (Mackinnon 1991).

To begin with I set time period to be a year as in Lucas (2000) and Rocheateau and Wright (2008). Then, I set the rate of discount, $r = 0.03$ as in Lucas (2000) and Rocheteau and Wright (2008) and $\xi = 0.5$ as in and Rocheteau and Wright (2008). To set the value of g, I estimate the average inflation rate per annum in the U.S. for the period 1950-2007. The estimated value is 3.5%. I set $g = 1.035$ to match the observed average inflation rate.

Now, I consider two values of $\phi = 0.5$ & 1. When $\phi = 0.5$, both worker households and employers receive same amount of monetary injection as in standard monetary models. When $\phi = 1$, firms receive all the monetary injection as in limited participation models (e.g. Fuerst 1992, Christiano, Eichenbaum, and Evans 1997).

Suppose now that $\phi = 0.5$, then (5.1) is satisfied for any $\alpha > 1.92$. In the macro literature, the value of $\alpha = 2$ is commonly used (e.g. Shi and Wen 1999, Alvarez, Atkeson, and Kehoe 2002, Erosa and Ventura 2002, Heer 2003).[12] If $\phi = 1$ then (5.1) is satisfied for any $\alpha > 0.97$. In general, lower is the ϕ, higher is the α required to generate positive relationship between inflation rate, output, unemployment, and labor force participation. For example, if $\phi = 0.4$, then α should be greater than 2.45.

Now I vary ξ and consider a very high value of $\xi = 0.99$. This implies that ninety-nine percent of buyers and sellers in the goods market are able to do their desired trading. This changes results only marginally. In the case, when $\phi = 0.5$, the model generates positive relationship between inflation rate, output, unemployment, and labor force participation for any $\alpha > 1.99$. For $\phi = 1$, the required $\alpha > 1$.

Finally, I consider the case in which time period is equal to a quarter rather than a year. In this case, I set $r = 0.008$ and $g = 0.009$. Change in time period does not alter the results much. For example, with $\xi = 0.5$ and $\phi = 0.5$, the required $\alpha > 1.97$ and for $\phi = 1$, the required $\alpha > 0.99$. Similar is the case when $\xi = 0.99$. The above discussion shows that the model generates a positive relationship among inflation rate, unemployment, output, and labor force participation for reasonable values of parameters.

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[12]Empirical estimates of α varies quite considerably. For example, Kydland and Presscott (1982) use value of $\alpha = 1.5$ and Cooley and Hansen (1989) use logarithmic preferences ($\alpha = 1$). Hodrick et. al. (1991) use values in the range of 0-9.5, Jones et. a. (1993) use values in the range of 1.0-2.5 in their endogenous growth model, and Wang and Shi (2006) use value of 4.

8   Conclusion

In this paper, I studied the effects of changes in the inflation rate in a model with endogenous labor force participation and separation between workers and employers (owners) of the firms. In the model, a higher inflation rate increases output, labor force participation, and unemployment. These results are consistent with the empirical evidence.

In the model, a higher inflation rate increases output and employment, but it also makes workers worse off and employers better off. The market equilibrium is inefficient. The Friedman rule for a given distribution of consumption and labor force participation leads to efficient employment. However, the Friedman rule does not ensure efficient distribution of consumption and labor force participation.

In the model developed, I assumed that the number of buyers and sellers are fixed. Also only a fixed fraction of buyers and sellers are able to enter the goods market in any time period. One can endogenize the number of buyers or their search-intensity (as in Shi 1998, Liang *et. al.* 2007) as well as the number of sellers (as in Rocheteau and Wright 2005, 2008). These extensions will also allow me to endogenize the entry rates for both buyers and sellers in the goods market. The endogeneity of number of buyers or their search-intensity is likely to magnify the positive effects of inflation on output and employment as in Shi (1998) and Liang *et. al.* (2007). On the other hand, the endogeneity of number of sellers is likely to weaken the positive effects of inflation on output and employment as in Rocheteau and Wright (2005).

Appendix 1: Proofs

Proposition 1. The equilibrium employment, e, is given by

$$\theta e^{\frac{\zeta(\theta-1)}{\zeta+\theta(\zeta-1)}} = \frac{\xi f'(e)}{(1+r)g - 1 + \xi} U'\left(\frac{\xi f'(e)e}{(1+r)((g-1)\phi + \xi)}\right). \qquad \text{A1}$$

The LHS of A1 is increasing in e. Simple differentiation of the RHS of A1 with respect to e shows that it is decreasing in e if $\frac{f'(e)e}{de} \geq 0$. In addition, given the assumptions that $\lim_{c^h \to 0} U'(c^h) > 0$, $\lim_{e \to 0} f'(e) \to \infty$, and $\lim_{e \to 0} f'(e)e < \infty$

$$\lim_{e \to 0} \text{LHS} = 0 < \lim_{e \to 0} \text{RHS}.$$

Under above conditions, if we plot the LHS and the RHS of A1 against e, then we get a unique intersection of the LHS and the RHS at some $0 < e < \infty$. Thus there exists a unique and finite e which solves A1.

Proposition 2. Follows from discussion in the text.

Proposition 3. With CRRA utility function A1 becomes

$$\theta e^{\frac{\zeta(\theta-1)}{\zeta+\theta(\zeta-1)}} = \frac{\xi^{1-\alpha}}{(1+r)g - 1 + \xi} \frac{f'(e)}{(f'(e)e)^\alpha} \left[\frac{g\xi + (g-1)\phi(1-\xi)}{(g-(g-1)\phi)}\right]^\alpha. \qquad \text{A2}$$

The effect of the money creation rate, g, on equilibrium employment depends on how it affects $T(g) \equiv \frac{1}{(1+r)g - 1 + \xi} \left[\frac{g\xi + (g-1)\phi(1-\xi)}{(g-(g-1)\phi)}\right]^\alpha$. Simple differentiation of $T(g)$ with respect to g shows that $\frac{dT(g)}{dg} > 0$ if

$$\frac{\alpha\phi[(1+r)g - 1 + \xi]}{[g\xi + (g-1)\phi(1-\xi)][g-(g-1)\phi]} > 1 + r. \qquad \text{A3}$$

In the case, condition A3 is satisfied a higher g shifts up the RHS to the right for a given e. Thus equilibrium employment, e rises. An increase in e increases n, l, $f(e)$, and $1 - \chi(n)$ and reduces wage, w.

Proposition 4.

The social planner allocations are characterized by following two equations:

$$\pi U'(c^h) = (1-\pi)V'(c^f) \equiv (1-\pi)V'(f(e) - c^h) \quad \& \qquad \text{A4}$$

$$\theta(1-\zeta)^{\frac{(c-1)(\theta-1)}{\zeta+\theta(1-\zeta)}} e^{\frac{\zeta(\theta-1)}{\zeta+\theta(1-\zeta)}} = U'(c^h)f'(e). \qquad \text{A5}$$

A4 gives a relationship between c^h and e. The total differentiation of A4 shows

$$\frac{dc^h}{de} > 0. \qquad \text{A6}$$

Since, $f(0) = 0$ and $c^f \geq 0$, under the assumption that $\lim_{c^h \to 0} U'(c^h) = \lim_{c^f \to 0} V'(c^f)$, A4 implies that $c^h = 0$, when $e = 0$.

A5 gives another relationship between c^h and e. The total differentiation of A10 shows

$$\frac{dc^h}{de} < 0. \qquad \text{A7}$$

Given the assumptions that $\lim_{e \to 0} f'(e) = \infty$, and $\lim_{c^h \to 0} U'(c^h) > 0$, A5 implies that $c^h \to \infty$ when $e = 0$. The above discussion implies that the curves traced by A4 and A5 intersect only once in the (e, c^h) space.

Proposition 5.

Part a. Part a. follows from simple differentiation of (6.7) with respect to g which shows that

$$\frac{d(c^h/f(e))}{dg} = -\frac{\phi}{(g\xi + (g-1)\phi(1-\xi))^2}\xi\eta(e) < 0. \qquad \text{A8}$$

Thus the share of consumption of worker's household in GDP is decreasing in g and that of employers is increasing in g.

Part b. Case I: Equilibrium output is declining in g. In this case, part b. of the proposition follows from part a.

Case II: Equilibrium output is increasing in g.

The equilibrium employment is given by

$$\theta\left(\frac{e}{a}\right)^{\frac{\zeta(\theta-1)}{\theta(1-\zeta)+\zeta}} = \frac{\xi f'(e)}{(1+r)g - 1 + \xi}U'\left(c^h\right). \qquad \text{A9}$$

Note that LHS of A9 is increasing in e. The first term in the RHS is decreasing in both g and e. This term falls with an increase in g. Thus, for a new equilibrium to be achieved, it must be the case that the second term in RHS, $U'(c^h)$ rises, which requires c^h to fall.

Part c. (6.8) and part a. imply that a higher g increases c^f and the welfare of employers, when (5.1) is satisfied. Also, higher l, n, and lower c^f imply that a higher g reduces the welfare of worker households.

Appendix 2

Table below shows the estimated error-correction models for output and employment respectively. I chose lag lengths on the basis of Akaike and Schwarz information criteria. I chose the specification which minimized these statistics.

Error-Correction Models

Variables	$\Delta \ln f(e_t)$	ΔUR_t	$\Delta LFPR_t$
γ_{t-1}	-0.2430^{**}	-0.3693^*	-0.0965^*
Δinf_{t-1}	-1.1213^*	-0.1139	-0.0658^{**}
Δinf_{t-2}			-0.1074^*
Δinf_{t-3}			-0.0646^{**}
$\Delta \ln f(e_{t-1})$	0.3527^*		
$\Delta \ln f(e_{t-2})$	0.4189^*		
$\Delta \ln f(e_{t-3})$	0.1770^{**}		
ΔUR_{t-1}		0.1765^*	
$\Delta LFPR_{t-1}$			0.2446^{**}
$\Delta LFPR_{t-2}$			0.2625^{**}
$\Delta LFPR_{t-3}$			0.4569^*
R^2	0.18	0.45	0.68
DW	2.13	1.98	2.05

Note: * and ** indicate significance levels of 1% and 5% respectively.

The estimated equations show that the coefficients associated with lagged residuals are negative and significant, suggesting cointegrating relationship among output and inflation, unemployment and inflation, and LFPR and inflation.

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