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The Early Bird gets the Worm? Birth Order Effects in a Dynamic Model of the Family*

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Abstract

Birth order effects are found in empirical work, but lack theoretical foundations. Our new approach to modelling children provides this. Each child has the same genetic make-up and parents do not favour a child based on its birth order. Each child's needs change as it grows, and births are sequential. At any point in time siblings are at different developmental stages, and the benefits of parental investment differ across these stages. Parental time investment in children lowers current and future wages; this opportunity cost varies across time. Birth order effects emerge from the interaction of the changing benefits and costs of parental investment.

Keywords: Birth order, children, family
JEL Classifications: D13, D91, J13

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1 Introduction

This paper contributes to theoretical models of families with children and provides useful guidelines for both empirical work on birth order effects and policy evaluation. To analyze birth order effects we develop a theoretical model that captures essential features of children as they mature. We take into account that it takes time for the children to grow and, as they develop, their needs change. Our model also explicitly incorporates the time lapse between the first and each subsequent child. The presence of a second child changes the benefits parents receive from spending time with their children. An increase in parental child care due to the arrival of the second child has different impacts on the child quality of the first-born and the second-born child. Moreover the arrival of the second child triggers more investment in the first-born compared to the case of an only child and causes a higher investment in the first-born in her second stage of life than the investment in the second-born in his second stage of life. Multiple births generate different effects: twins always receive identical treatment.

Recent empirical literature has found finding significant differences between first-born and later-born children (Ejrnæs and Pörtner 2004, Black et al. 2005, Conley and Glauber 2006, Kantarevic and Mechoulan 2006, Price forthcoming). However, theoretical models explaining birth order effects are virtually nonexistent as all of the models are atemporal when it comes to the investment in children.

While the trade-off between the number of children and the quality of each child has received much theoretical attention, starting with the seminal work by Becker and Lewis (1973) and Willis (1973), these models assume that parents produce each child of equal quality and all children are born at once. They then analyze the impact of a change in family full income on the number and the quality of children. Although a change in family full income is different from a change in family size, many empirical papers have analyzed the impact of family size on child quality inspired by the theoretical models of a quantity-quality trade off, testing the hypothesis that all else equal a bigger family size should be accompanied with a lower child quality.[1] As did many previous studies, Black et al. (2005) found little evidence of a quality-quantity trade-off due to family size, once they controlled for birth order. However, the authors found significant effects of birth order on children's educational attainment, their adult earnings, employment, and the propensity of teenage childbearing; they concluded that their "findings suggest the need to revisit economic models of fertility and child 'production,' focusing not only on differences across families but differences within families as well." (Black et al. 2005, abstract). Our work therefore fills an important gap in the theoretical literature on families with children by investigating how child quality is produced in the family and how birth order may influence the conditions of producing child quality for each child.[2]

The only empirical paper investigating a possible cause for a birth order effect is Price (forthcoming). The author looks at quality time that parents spend with each child and finds that children of higher birth order receive less quality time at any given age than do their earlier-born siblings at the same age. Non-economists concerned with birth order effects have mentioned another possible cause: Older siblings benefit from teaching the younger siblings. This explanation does not necessarily imply that younger siblings receive worse education within the family than older children, since siblings replace some of the quality time that parents would otherwise spend with them, but they lack the opportunity to be teachers from an early age on (Hall 2007, thestar.com).

[1] See e.g. Kessler (1991) for a good summary of the empirical literature up to the point of his own work as well as his own empirical work.

[2] Gugl and Welling (2004) list several important features of what children can mean to their parents. The present paper takes a much narrower but more managable approach. Birth order is not investigated in Gugl and Welling (2004).

1.1 Modelling Children

Children frequently appear in economic models as household public goods, and are arguably the most important example of these. Other models treat children as self sufficient adults, earning their own incomes, who interact with their older parents (e.g. Becker 1974).[3] While we follow the former in denying agency to children, our modelling approach differs from the existing literature in several important ways:

(1) Children's needs change as they grow. We distinguish two periods in a child's life. In the first period a child's dominant need is time spent with them; in the second period, parental time remains an essential input, but the child also needs consumption goods.

(2) Parental child care has an opportunity cost of foregone current and future earnings. The time each parent spends caring for a child is likely to have an impact on parents' future wages and, therefore, on the utility of each parent once the child is grown. Most of the literature has overlooked this aspect; an early exception is Willis (1973). We explicitly incorporate both current and future implications of this time investment in a dynamic framework.

(3) Parents or a nanny can mind the children, but parental time is seen as more valuable in producing child quality than the same amount of time provided by the nanny. There is a large empirical literature emphasizing the importance of parental (or more often maternal) time spent with the very young child.[4] However, theoretical models have not yet captured this distinction.

(4) Child bearing is sequential. In most models parents choose quality and quantity simultaneously. Sequential childbearing and different needs of children of different ages suggest that multiple children should be viewed as joint products in household production,

[3] Most recently family bargaining models have been employed to address the issue of care for the elderly (Pezzin et al. 2004, Engers and Stern 2002).

[4] See, for example, Ruhm (1998, 2000).

with the production mix varying over time. Our model makes explicit the sequential nature of decisions. It also discusses the changes in producing child quality of the first child once a second child is present by taking into account the economies of scope.[5] Ejrnæs and Pörtner (2004) model the sequential fertility decision as follows: after each child is born its genetic make-up becomes known and, based on that child's endowment, the parents decide whether they want another child. Parental investments in children occur only after child-bearing has been completed. In contrast, our model assumes that each child has the same genetic make-up, but parental investment begins at birth. Thus the first-born receives parental investment before the second child arrives, and investment in the last-born continues after the first-born is grown and self-sufficient.

In this paper we focus on child-rearing, beginning with the birth of the first child. We take as given that spouses achieve efficiency in marriage - at least from this point forward. Spouses could have entered a binding agreement at the beginning of marriage or, starting with the arrival of the first child, bargained over life-time utility. While our model can easily be extended to incorporate family bargaining, we leave this to future research together with the analysis of changes in family policies.

We start with a simple two period model, and a single child, in order to make explicit the details of child rearing discussed above. We then add another child in the second period, extending the basic model to a total of three periods. Following that we consider various extensions of the model and briefly discuss its potential for policy evaluation. Throughout the analysis we choose explicit functional forms for parents' utility to keep the model simple and tractable; all proofs are in Appendix A.

[5] The distinction between economies of scope and economies of scale is important in our model. With sequential births the same time spent at home produces two intermediate goods: time spent with the older child and time spent with the younger child.

2 The Basic Model

A husband (denoted by h) and wife (denoted by w) face a two-period problem. They have a very young child in the first period ($k = 1$) who becomes an older child in the second ($k = 2$). Parents derive utility from child quality in period k (denoted by q_k) and from their own private consumption (denoted by x_{i1}). The utility of parent i in period k is given by $u_{ik}(q_k, x_{ik})$.

Child quality is produced differently at different ages of the child. In the first period, the child needs supervision equivalent to the time available to one adult; we normalize this time to 1. Parents can provide child care themselves, or they can hire a nanny; letting t_{i1} denote the time spent in child care by parent i in the first period, and that by the nanny t_n, the household's first-period time constraint for child care is

$$t_{w1} + t_{h1} + t_n = 1.$$

When not caring for the child, parent i is employed (for the amount of time $l_{i1} = 1 - t_{i1}$) at wage rate w_i; a nanny is available for c_n dollars per unit of time.

The private consumption good has a unit price of 1, so the household's budget constraint in the first period is given by

$$x_{w1} + x_{h1} + c_n t_n = w_w l_{w1} + w_h l_{h1}. \tag{1}$$

Child quality in the first period, depends on the time the parents and the nanny spend with the child: $q_1 = q_1(t_{h1}, t_{w1}, t_n)$. Research on early child development suggests that parental time is very important; Waldfogel (2006: 37-45) discusses the evidence. We assume parental time is more productive than nanny time by a factor of p; using (1), we rewrite the quality production function as

$$q_1 = q_1(t_{q1}), \quad t_{q1} = (p-1)(t_{w1} + t_{h1}) + 1, \quad p > 1,$$

where $q_1\left(t_{q1}\right)$ is strictly concave, at least twice differentiable and increasing; in the appendix we state further conditions sufficient to ensure an interior solution for t_{w1}.

In its second period of life, the child no longer requires constant supervision, and child quality depends on the child's consumption of private goods, x_c, and parental attention:

$$q_2 = q_2\left(t_{p2}, x_c\right),$$

where $t_{p2} = t_{h2} + t_{w2}$, and $q_2(\cdot)$ is a strictly concave, at least twice differentiable and increasing function.

Parents once again divide their time between employment and child care. Parent i's second period wage rate is increasing in time spent in the labour force in the first period: $w_{i2} = w_i(1 + \pi l_{i1}), \pi > 0$. Thus spending time with the child in the first period reduces the future wage rate. The household's second-period constraints are

$$t_{i2} + l_{i2} = 1, i = h, w; \tag{2}$$

$$x_{h2} + x_{w2} + x_c = w_w\left(1 + \pi l_{w1}\right) l_{w2} + w_h\left(1 + \pi l_{h1}\right) l_{h2}. \tag{3}$$

The husband has an absolute advantage in first-period earnings, so $w_{w1} < w_{h1}$.

Parents do not derive direct utility from time spent with the child, and care only about child quality. (See the concluding section for alternative assumptions.) In the simplest possible structure, we have

$$u_{ik} = x_{ik} + q_k, \quad i = w, h; k = 1, 2.$$

Spouse i's intertemporal utility is given by

$$U_i = u_{i1} + u_{i2}.$$

Parents have a discount factor of 1, so their utility in future periods weighs as heavily in their intertemporal decisions as the current period. Thus child quality tomorrow is as important as child quality today.[6]

3 Parental Investment in the Basic Model

Given these intertemporal utility functions, allocative efficiency is independent of distribution. Maximizing the sum of the parents' intertemporal utilities allows us to find the unique point in the parents' production possibility set that yields Pareto efficiency (Bergstrom and Cornes 1981 and 1983). As utility is additive in private consumption, we can solve for the optimal total private consumption of the parents, but not the distribution: thus we focus on $x_{pk} = x_{wk} + x_{hk}, k = 1, 2$. As child care by the husband and the wife are perfect substitutes, within each period child quality depends only on the *aggregate* parental time input (rather than the division of this time between the husband and the wife), and hence efficiency dictates that the spouse with the lower wage rate in that period is the primary caregiver, while the other parent is the primary wage earner. We assume that the parameters of the model are such that we have an interior solution for the mother while the father devotes full time to employment. By construction, in this model the wife always provides parental child care and the husband always works full-time, so $l_{hk} = 1$ and $t_{pk} = t_{wk} \in (0, 1), k = 1, 2$, and therefore $t_{q1} = (p - 1)t_{w1} + 1$.

The efficient outcome is determined by the choice of time allocations $\{t_{w1}, t_{w2}\}$ and

[6]Incorporating discounting would change our results below in predictible ways. We believe this assumption captures parents' feelings for their child.

private goods $\{x_{p1}, x_{p2}, x_c\}$ maximizing the following:

$$\sum_{k=1}^{2} u_{wk} + u_{hk} = [x_{p1} + 2q_1(t_{q1})] + [x_{p2} + 2q_2 (t_{w2}, x_c)]$$

subject to constraints $(2), (3)$, and (4).

Inspection shows that this problem can be solved recursively: second-period choices are functions of t_{w1}, and first-period values can be found once the second-period value function is derived. These derivations are in Appendix A. Assuming an interior solution for t_{w2}, all second-period choices are functions of first-period maternal child care: $t^*_{w2} (t_{w1}), x^*_c(t_{w1}), x^*_{p2}(t_{w1})$; we have

Proposition 1 An increase in first-period maternal child care decreases the opportunity cost of the mother's time in the subsequent period (her second-period wage rate), resulting in an increase in maternal child care in the second period. Although the induced change in the child's consumption good depends on the cross partial of the second-period child quality function, the effect of an increase in first-period maternal child care on second-period child quality is always positive:

$$\partial t^*_{w2} (t_{w1}) / \partial t_{w1} = -\partial l^*_{w2} (t_{w1}) / \partial l^*_{w2} (t_{w1}) > 0;$$

$$\partial x^*_c (t_{w1}) / \partial t_{w1} \lesseqgtr 0; \qquad \partial q^*_2 (t_{w1}) / \partial t_{w1} > 0.$$

Since our utility specification rules out any income effect on the factors determining child quality in the second period, we see an increase in child quality due to the lower cost of producing it as the opportunity cost of maternal time decreases.[7]

[7] A different utility specification would yield a negative income effect on child quality due to the decrease in family full income, opposing the positive substitution effect on child quality due to the lower cost of producing it. Thus whether child quality rises or falls with a lower full income in the second period due to a lower second-period wage rate of the mother depends in a more general model on the size of both effects.

As shown by Bergstrom and Cornes (1981 and 1983), there are utility functions that lead to transferable

Obstervation 1 The maximized value of second-period parental utility is decreasing in first-period parental child care, i.e.

$$\partial\left(u_{w2}^{*}+u_{h2}^{*}\right)/\partial t_{w1}<0.$$

This result follows directly from the assumption that time spent in child care in the first period has no *direct* impact on second-period child quality.[8] Second-period utility depends on parental private goods and child quality. The more time the wife spends in child care in the first period, the smaller the full budget set available in the second: the household's time endowment is the same, but potential money income falls. Thus the maximized value of second-period parental utility must fall as t_{w1} rises. Note that this result also holds for more general utility functions: any decrease in the full income of the household in the second period will shrink the household's utility possibility set.

Given the second-period choice functions, we now characterize first-period choices. The optimality condition for the wife's division of labour in the first period takes into account the impact of child care not only on the couple's first-period utility, but also on their second-period utility. The optimal t_{w1} solves

$$2\left(p-1\right)\partial q_{1}/\partial t_{q1}=\left(w_{w}-c_{n}\right)+\left[\pi w_{w}\left(1-t_{w2}^{*}\left(t_{w1}\right)\right)\right].$$

The term on the right measures parental private consumption foregone in the first and

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utility (TU) in each period but allow for an income and substitution effect of the public good. In each period, these utility functions must take on the form of a quasi-concave function $u_{ik}\left(x_{ik},q_{k}\right)=A\left(q_{k}\right)x_{ik}+B_{i}\left(q_{k}\right)$ where $\frac{1}{A(q_k)}$ is a convex function of q_k and $\frac{B_i(q_k)}{A(q_k)}$ is a concave function in q_k.

[8] Our model assumes that the increase in child quality in the second period is independent of first period child quality: it is never too late to become a good parent. There is no "savings" component to first period time spent with the child which might provide "investment income" to supplement parents' wage income in the second period. If there were some carryover, second-period production possibilities would not necessarily diminish with the wife's first-period child care, and this partial would be ambiguous.

second periods, respectively, by increasing first-period maternal child care by one unit. The term on the left is the first-period benefit of an additional unit of maternal child care due to higher child quality. In a one-period model, the term in [ ] on the right would disappear. Since this term is non-negative (and strictly positive at an interior solution to the second-period problem), compared to the efficient time allocation were there only one period, the wife spends more time in employment (hence, less in child care) in the first period in the intertemporally efficient case: first-period child quality is traded off against second-period parental consumption. Implicit differentiation tells us that first-period maternal child care increases in both the parental productivity parameter and the cost of the nanny, and decreases in both the wife's wage rate and the wage-growth factor π, i.e.

$$\partial t_{w1}/\partial p > 0;\ \partial t_{w1}/\partial c_n > 0;\ \partial t_{w1}/\partial w_w < 0;\ \partial t_{w1}/\partial \pi < 0.$$

4 Parental Investment with Two Children

In this section we extend our basic model to incorporate more than one child. As noted above, we assume parents weigh the current and future quality of their single child equally. Extending this to two children, each child's quality receives the same weight (of unity) in each parent's utility, regardless of the sequencing of births. Before we address sequential births and birth order effects, we briefly consider the case of twins. While twins may not be explicit in much of the theoretical literature, we argue that, by ignoring the sequential nature of child bearing and its impact on child rearing, much of the existing literature implicitly assumes multiple births. Considering twin births allows us to highlight the differences implied by explicit recognition of sequential child bearing and rearing when we extend the model to three periods with a second birth in the second period.

4.1 Twin Births

In order to analyze parental investment in twins, in addition to our assumption about the utility parents derive from multiple children, we need an assumption about the nature of returns to scale in the child quality production function. We consider two extreme cases. First, suppose that both the mother and the nanny can care for two children rather than one without any extra cost, so that supervision is a pure public input.

Proposition 2 If supervision time is a pure public input, average child quality increases with more children.

It is straightforward to check that the benefit of spending time with a child now increases because of the public input character of child care, and so the time the mother devotes to child care in both periods also increases. Note further that given the same amount of maternal child care in both periods, the marginal benefit of maternal child care increases (it doubles) in each period while leaving the marginal benefit of child consumption for each child the same as in the model with a single child. With no minimum level of parental consumption in each period, more children do not lead to a quality-quantity trade off. The more children we add, the higher the benefit of maternal child care in producing the sum of child quality and thus the more child quality per child is produced.

Second, assume that maternal child care and time spent with the nanny are both private inputs in the child quality function. If the wage rate in each period were independent of past labour supply, the cost of producing a child of a certain quality in each period would be exactly the same as with an only child. With twins we would have twice the cost and twice the pleasure. However, the wage rate in the second period is not independent of first-period labour supply; hence child quality changes.

Proposition 3 When child care is rival for the mother and the nanny, average child quality increases with more children unless the mother's time constraints bind.

To see this, first assume that both child consumption and maternal child care are doubled compared to the case of the only child. This implies the same marginal benefit from each child for each of the variables as in the case of the only child, but because maternal time is doubled in both periods, the opportunity cost of maternal child care decreases in both periods. Second-period opportunity cost is lower because of the second-period earnings implications of first-period labour supply. First-period opportunity cost is lower because the second-period benefit of first-period labour supply decreases with an increase in second-period maternal child care. Hence average child quality increases. However, if the only child receives more than half of the maternal time in each period, then doubling this time for twins is not feasible. In this case either both parents are at a corner with the mother spending all her time caring for the twins and the father working full time, or the mother spends all her time caring for the twins while the father now divides his time between employment and child care. Thus if the time constraints of the mother bind, child quality per twin may be lower than in the case of an only child.

Investment in twins probably falls between the two cases above. Our model thus predicts that twin births would not cause a dramatic fall in average child quality and, indeed, Angrist et al. (2005) find no quantity-quality trade off when they compared otherwise similar families with and without multiple births.[9]

Twin siblings always receive equal treatment. The same is typically not the case for two children born sequentially. In the next section we extend our basic model to a third period to allow sequential births and child rearing, and derive the implications of this sequencing

[9] With different preferences and rival time inputs, the average quality per twin can be lower than the child quality of the only child. This is the case when an exogenous increase in the number of children reduces parental consumption by less than what is needed to maintain the same child quality as in the case of the only child. In order to arrive at such a result it is necessary to introduce a decreasing marginal rate of substitution of parental consumption with respect to the number of children, n. For example

$$u_{ik} = x_{ik} + v(nq),$$

where $v'(\cdot) > 0$ and $v''(\cdot) < 0$ would do the trick.

on parental investments and child quality.

4.2 A Three-Period Model with Two Children

4.3 Adding a third period to the basic model

Suppose now we add another period to the end of the basic model above. By the beginning of this third period, the child who was an infant in the first period has grown, and is self-sufficient: the grown child requires neither time nor money transfers from its parents. Thus both parents work full time in the third period. Parents still care about their grown child, but the child enters their utility function as a constant; this constant is suppressed in our subsequent analysis. Clearly, if third-period wage rates depend on previous labour force experience, as second-period wage rates do in the model above, inclusion of this third stage in which parents work full-time increases the opportunity cost of parental child care in both previous periods. All else equal, this will decrease parental child care in the first and second periods, increase t_n and decrease $q_k, k = 1, 2$.

4.4 Adding a second child

Now suppose that in the second period, parents have another child. Then in the second period the household consists of two parents, an infant, and an older child. Let superscript a denote the child born in the first period and b the child born in the second period. Moreover let q_k^a denote the child quality of the first-born in her kth period of life and let q_k^b denote the child quality of the second-born in his kth period of life. That is, q_1^a enters the parents' utility in the first period, q_2^a and q_1^b enter the parents' utility in the second period, and q_2^b enters the parents' utility in the third period.

Assuming a discount factor of one is especially appealing in this context, because it guarantees that parents will not favour their first-born because they value current utility more than future utility. Of course, that does not guarantee equal treatment of each child, because the costs of producing child quality depend on the changing opportunity cost of time of the parents as well as the child's developmental stage.

A key factor in this analysis is the assumption made regarding economies of scope in child care: is a given amount of parental child care as productive if it is shared by two children, rather than devoted to one? Can parents purchase child care for two children at the same rate as for one? The presence in the second period of two children, of different ages and with different needs, both requiring some supervision time, requires further allocation decisions on the part of the parents. Economies of scope in supervision time implies that any detrimental effects of sharing a caregiver with a sibling are compensated for by the benefits from sibling interaction.

We consider two possibilities: first, economies of scope for both mother and nanny, and second, no economies of scope for the nanny but economies of scope for the mother. The latter case is the obvious extension of our two-period model, but the first case will provide a benchmark and help us identify what happens in the latter case.

4.4.1 Full economies of scope for both nanny and mother

With full economics of scope in child care for both nanny and mother, either can provide full care for both children simultaneously. Since the infant requires full time supervision, this implies that in the second period the older child also interacts with an adult full-time; with a slight abuse of notation we have "t^a_{w2}"$= 1$. The full objective function for parents is then

$$\sum_{k=1}^{3} u_{wk} + u_{hk} = \left[x_{p1} + 2q^a_1\left(t^a_{q1}\right)\right] + \left[x_{p2} + 2q^b_1\left(t^b_{q2}\right) + 2q^a_2\left(1, x^a_c\right)\right]$$
$$+\left[x_{p3} + 2q^b_2(t^b_{w3}, x^b_c)\right]$$

Parents choose the following variables to maximize this function

1. private goods for the parents in each of periods 1, 2, and 3 ($x_{pk}, k = 1, 2, 3$);

2. private goods for the first child in period 2 (x_c^a);

3. private goods for the second child in period 3 (x_c^b) ;

4. division of the required unit of child care between the wife and the nanny in periods 1 and 2;

5. division of wife's time between child care and labour market participation in period 3.

Proposition 4 With full economies of scope for the mother and the nanny, both children receive the same amount of maternal child care when they are young. Parents produce higher child quality of the first-born in her second period of life than of the second-born child in his second period of life.

We show in the appendix that t_{w1} and t_{w2} are symmetric to each other in this constrained optimization problem and therefore the optimal amount of maternal child care will be the same in both periods. This implies both children receive the same amount of time when they are young. The time input in q_2^a is free up to one unit, but the time input has a positive opportunity cost in the third period. Therefore the cost of producing child quality of the first-born in her second period of life is cheaper than the cost of producing the same level of child quality for the second-born in his second period of life. Thus the child born in the first period has a higher child quality in the second period than the second-born has in the third period.

One may wonder why nanny time is a perfect substitute for parental time in the second period of a child's life, but not in the first, and why then parents cannot hire a nanny in the third period as well. We now return to our previous assumption that only time spent with the mother (or father) but not the nanny enters the child quality function of a child in its second period of life.[10]

4.4.2 Full economies of scope for mother only

If there are perfect economies of scope with maternal child care but not for the nanny, then the budget constraints are the same but the parental utility function becomes

$$\sum_{k=1}^{3} u_{wk} + u_{hk} = [x_{p1} + 2q_1^a(t_{q1}^a)] + [x_{p2} + 2q_1^b\left(t_{q2}^b\right) + 2q_2^a\left(t_{w2}, x_c^a\right)]$$
$$+[x_{p3} + 2q_2^b(t_{w3}^b, x_c^b)]$$

Parents maximize this function by choosing the same variables as in section 4.4.1.

Proposition 5 With economies of scope for the mother only, the first-born receives less maternal child care in her first period of life than does the second-born child in his first period of life. Assuming $\partial q_2^a/(\partial x_c^a \partial t_{w2}) \geq 0$ and $\partial q_2^b/(\partial x_c^b \partial t_{w3}) \geq 0$ is sufficient to ensure that the first-born receives more attention and child quality in her second period of life than the second-born child does in his second period of life.

Maternal child care is higher in the second period, because only maternal child care produces economies of scope. Compared to the previous case, both children receive more maternal child care in their first period of life: even though the first-born receives less maternal care in the first period of her life than the second-born, the cost of maternal child care in the first period is lower given the additional benefit of staying at home with the children in the second period. Because of economies of scope for the mother in the second period, maternal child care is highest in the second period compared to the first and the third period. Assuming $\partial q_2^a/(\partial x_c^a \partial t_{w2}) \geq 0$ and $\partial q_2^b/(\partial x_c^b \partial t_{w3})0$ guarantees that the

[10] An alternative explanation why nanny time is a private input while maternal child care produces economies of scope is that nanny-care is exclusive - as would be the case, for instance, if the infant was placed in a child care centre rather than with an in-house nanny when not with the mother.

first-born receives a higher child quality in her second period of life than the second-born receives in his second period, because higher maternal child care goes hand in hand with weakly increased child consumption.

Using data from the American Time Use survey and restricting the age range of children from 4 to 13, Price (forthcoming) finds that many parents spend equal time with their children at any given point in time and that time spent with children typically decreases from one period to the other (as the children grow older). This is consistent with an extension of our model in which what is now called the second period of a child's life is split up into more periods in which children's needs for parental time play a lesser and lesser role as they grow. One could also test our assumption of economies of scope by checking how often parents spend time with one child while the other sibling is present.

Compared to an only child in the three-period model, the first-born in a family with two children will have higher child quality in both of her periods as a dependent – a prediction that is consistent with empirical findings (e.g. Black et al. 2005). Two words of caution when it comes to the only-child vs. first-born comparison: First, our results crucially depend on the assumption that the utility weights on each child remain the same as we add more children. Second, the model does not answer the question of why there exist families with just one child, all else equal. Explaining different levels of fertility is an important issue that we will address in future research building on the present model. The first extension we discuss in the next section provides a probabilistic explanation for different family sizes, and why the spacing of siblings also varies across families.

5 Discussion and Extensions

This paper examines birth order effects. Parents do not favour one child over the other: each child in each of its developmental stages enters their intertemporal utility function with the same utility weight. However, we do not require that parents devote the same resources to each child in each developmental stage. Central to our findings of birth order effects is the assumption that investments in children must begin at birth. Moreover children's needs change as they grow, so that parents deal with different production functions for child quality in any period where children of different ages are present in the household.

5.1 Probabilistic Second Birth and Spacing of Children

Suppose the birth of the second child in the second period is probabilistic rather than certain. The household's intertemporal objective function is now the expected sum of parental utilities: parents maximize intertemporal utility knowing that they have one child in the first period, but are uncertain whether they will have a second child or not. Thus parents who, in the second period, have one child, and those with two children, will have made identical first- period investments. In the second period, the uncertainty is resolved, and second-period maternal care for the first-born will depend on the existence of the second child: the first child will enjoy both more maternal care and child quality if the second child is born. Our model also predicts that if parents can keep trying to have children in later periods, since the older child moves more and more towards self-sufficiency, a child born in the third period would receive less attention than if the child were born in the second period. Price (forthcoming) finds that the longer the spacing between siblings the bigger the gap in quality time reported for each child at the same age. Price speculates that parents divide family resources equally across children at each point in time out of a sense of fairness. Our model offers a different explanation for the same observed phenomenon: Parents spend equal time with their children due to the public nature of maternal child care; as the spacing of child births increases the benefit of maternal child care for the older child decreases and hence less time is spent with both children.

5.2 Changing Parents' Preferences or the Child Quality Function

5.2.1 Early childhood bonding

In this extension to the basic model of section 2, we provide a rationale why mothers may care more about their children's well being over time as a result of them spending more time with the child. We use a model similar to rational addiction (Becker and Murphy, 1988): spending more time with the child in the first period increases the utility of having the child in the second period. Both parents take this effect into account when they choose their time investments. We modify second period utility as follows,

$$u_{i2} = x_{i2} + (1 + t_{i1})\, q_2.$$

We retain symmetry between the time inputs of spouses in the production of child quality in both periods. Notice that although the allocation of parental time matters to each parent, parental time inputs are perfect substitutes in the sum of parents' intertemporal utility functions. So long as the wife's opportunity cost of time is always lower than the husband's, intertemporal efficiency may still demand that the husband work full-time in employment, while the wife spends some time at home with the child. While both parents have the same preferences in the first period, in the second period the mother has a higher preference for child quality than the father because the mother has spent more time with the child in the first period. Although this assumption makes only a quantitative difference in the basic model, it is a neat way of introducing differences in preferences of the parents for the child over time. With respect to birth order, such an assumption may be interesting to pursue, especially if the arrival of the second child makes it necessary for the father to spend time with the older child. The reason for such a division of child care may be that time of parents is not a pure public input when both children are quite young and the mother is slightly more productive in dealing with a child in its first period of life.

5.2.2 Parents pay for early parenting "mistakes"

As emphasized before, our model does not incorporate a feedback of earlier child quality on current child quality. That is, children are not punished for early parenting mistakes in all future periods and so it is never too late to become a good parent. Here we consider another extension that also shows a positive impact on second-period utilities of first-period investment in the child as does the previous extension. However, the transmission mechanism is different: While above parents love their child more as they have spent more time with it in the first period, but first-period investment in the child does not enter the child quality function in the second period, in this extension first-period investment in the child has an impact on second-period child quality. With the same investment into the child in its second period of life, having spent less time in the first period will lead to a lower marginal productivity of second-period investment.[11] Both extensions have in common that they reduce the negative impact of first-period parental child care on the sum of second-period utility of the parents.

5.2.3 Child spends time with both parents

We observe couples where both individuals divide their time between child care and employment. A model that treats parental child care as limited substitutes, or in which each parent values the time he or she spend with the child as well as child quality (the existence value of the child), would more likely lead to an interior solution for both parents.

[11] See also Heckman (2007).

5.2.4 Children spend time with their siblings

We have assumed that whenever there are economies of scope any detrimental effects of sharing a caregiver with a sibling are just compensated for by the benefits from sibling interaction. This symmetry can be changed so that one sibling may benefit more from sibling interaction than the other. Indeed, both Price (forthcoming) and Hall (2007) mention sibling interaction as an important factor in the production function of a child. Hall even points to it as a possible cause for the empirical finding that first-born children have higher IQ's than their siblings.

Our model has focused on the changing opportunity cost of maternal child care and economies of scope as the main forces in producing birth order effects. In contrast, if it is true that older siblings boost their IQ by teaching younger siblings what they have learned, as suggested by Hall, providing these teaching opportunities to children of all ages within the educational system may enhance the IQ of later born siblings as well.

5.3 Household Decision Making

A common assumption is that marriage embodies an implicit contract, determined at the time of marriage and played out over its duration. We assume spouses achieve efficiency, at least with the arrival of the first child; whether this occurs through bargaining over life-time utility from that point or some other contractual agreement is left open.

As we are concerned only with child quality in this paper and, given our utility functions, allocative efficiency is independent of distribution, we need not specify the threat point in the intertemporal bargaining problem of the parents or the details of their contractual arrangement.[12] However, for policy analysis the welfare of each parent becomes

[12] Extensions of the model that endogenize the decision of having the first child and/or decisions of human capital investment would need to deal with this question.

important apart from the impact of the policy on child quality. From either a perspective of gender equality or a political economy view the impact of family policies on each individual member of the household should not be neglected. Appendix B solves for the symmetric Nash Bargaining solution in our model to give the reader a sense of how to analyze the distributional effects of a policy change using the most common bargaining solution.

5.4 Policy Evaluation

Our model lends itself to the analysis of parental leave policies. By taking into account the sequential nature of birth it can also be used to distinguish between the impacts of parental leave policies that pay only a lump-sum, on the one hand, and parental leave policies under which payments are based on previous earnings of the parent taking leave, on the other hand. The model can also be extended to analyze the impact of parental leave policies on the spacing of children. Analyzing the impacts of parental leave policies is important from a public policy perspective concerned with child-wellbeing and gender equity. It is also important from a political economy point of view: political parties often make policies towards the family a campaign issue and it is therefore worthwhile to analyze whether fathers and mothers would be adversely affected by parental leave policies.

Child care policies that affect the cost or quality of child care can also be evaluated in our model. For example, the Canadian province of Quebec provides universal child care for Cdn $7 a day to parents; many families have taken advantage of the program. Baker et al. (2005) presents findings consistent with our model: By lowering the cost of non-parental child care parents lower their child care in the first period and so produce lower child quality. Our model also suggests a remedy: The negative outcome might be avoided if non-parental child care can be offered at a quality similar to that of parental child care.

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7 Appendix A

Here we derive and prove the results in the text. We first provide the notation and definitions of the model in succinct form to facilitate reading through the appendix. We present only variables and parameters that appear in the appendix. Unless specifically mentioned, the husband spends all his time in employment in each period, that is $t_{hk} = 0, k = 1, 2, 3$.

7.1 Notation and Definitions

The utility of parent i in period k is given by $u_{ik}(x_{ik}, q_k) = x_{ik} + q_k$.

Parental consumption in period k is given by $x_{pk} = x_{hk} + x_{wk}$.

The per-unit cost of the nanny's child care is denoted by c_n.

The wage rate of parent i in the first period is given by w_i.

The wage rate of parent i in the second period is given by $w_{i2} = w_i \left(1 + \pi \left(1 - t_{i1}\right)\right)$.

The wage rate of parent i in the third period is given by $w_{i3} = w_{i2} \left(1 + \pi \left(1 - t_{i2}\right)\right)$.

The child quality function in the first period for a child born in the first period is a strictly concave, at least twice differentiable function: $q_1 = q_1 \left(t_{q1}\right)$, where $q_1'(t_{q1}) > 0$ and $t_{q1} = \left(p - 1\right) t_{w1} + 1, p > 1$.

If there is more than one child, superscripts a and b distinguish between children. For sequential births, child a is born in the first period and child b is born in the second period.

For a child born in the second period the child quality functions are given by, $q_1^b = q_1^b \left(t_{q2}\right)$ for the child's first period of life, where $t_{q2} = \left(p - 1\right) t_{w2} + 1, p > 1$, and $q_2^b = q_2^b(t_{w_3}, x_c^b)$ for the child's second period of life.

7.2 The Basic Model

The parents' problem is

$$\max_{t_{w1},t_{w2},x_c} (w_w - c_n)(1 - t_{w1}) + w_h + w_w (1 + \pi(1 - t_{w1})) (1 - t_{w2}) + w_h (1 + \pi) - x_c$$
$$+2 [q_1(t_{q1}) + q_2 (t_{w2}, x_c)]$$

where the budget constraints have been substituted in to rewrite the optimization problem in terms of the three ultimate choice variables. First order conditions (FOCs) are given by

$$\frac{\partial}{\partial t_{w1}} = -[(w_w - c_n) + w_w\pi (1 - t_{w2})] + 2(p - 1)(\partial q_1/\partial t_{q1}) = 0$$
$$\frac{\partial}{\partial t_{w2}} = -w_w (1 + \pi(1 - t_{w1})) + 2(\partial q_2/\partial t_{w2}) = 0 \quad (4)$$
$$\frac{\partial}{\partial x_c} = -1 + 2(\partial q_2/\partial x_c) = 0 \quad (5)$$

7.2.1 Proof of Proposition 1

The second and third equations can be solved for second-period variables (t_{w2}, x_c) as functions of t_{w1}. Note that first-period maternal child care is sunk in the second period and so enters the optimization of second-period utility as a parameter. Moreover, given first-period maternal child care, we can solve first for the second-period variables by maximizing second-period parental utility and then solve for first-period maternal child care once we have solved for the second-period choices as functions of first-period maternal child care.[13] The FOCs for this problem are of course given by (4) and (5). Using the solutions for second-period choice variables as functions of first-period maternal child care, we can show that optimal child quality increases with an increase in first-period maternal child care. The second period problem reduces to one of maximizing the expression

[13]In order for t^*_{w2} to be within the permissable range of $(0, 1)$, the wage rate of the mother in the second period must be greater than some $\underline{w}_{w2}$. The specific lower bound will depend on the functional form of q_2.

$2q_2(x_c, t_{w2}) - (x_c + w_w(1 + \pi(1 - t_{w1}))t_{w2})$, in which the first term is the total benefit of a combination of child consumption and maternal child care while the second is the total cost of producing this benefit. In order to see what happens to child quality as t_{w1} changes, it is useful to replace the cost of producing child quality as given by $x_c + w_w(1 + \pi(1 - t_{w1}))t_{w2}$ with the cost function of child quality, $\psi(q_2)$. Since $q_2(x_c, t_{w2})$ is strictly concave, $\psi(q_2)$ is strictly convex. A decrease in the price of one of the inputs in the production of child quality at the same level of child quality as before the price change, decreases the cost of producing child quality without changing the benefit . Hence a decrease in w_{w2}, as would be the case as t_{w1} decreases, causes an increase in child quality to restore equality between the marginal benefit of child quality and marginal cost of child quality once again. At the same time, a decrease in w_{w2} must lead to a higher amount of maternal child care in the second period: In order to produce the same amount of child quality as before the decrease in w_{w2}, cost efficiency requires more maternal child care and less child consumption. Thus t_{w2} increases even at the lower level of q_2. Since we have already determined that q_2 increases with a decrease in w_{w2}, t_{w2} must increase. Hence t^*_{w2} goes up due to a decrease in t_{w1}. To sum up,

$$\partial q^*_2(t_{w1})/\partial t_{w1} > 0;\ \partial t^*_{w2}(t_{w1})/\partial t_{w1} > 0.$$

What happens to child consumption in the second period depends on the cross-partial derivative of the child quality function. The following conditions are derived from the second partial above and inspection of the first order condition (5).

$$\text{If } \partial^2 q_2/(\partial x_c \partial t_{w1}) > 0, \quad \partial x^*_c(t_{w1})/\partial t_{w1} > 0;$$
$$\text{if } \partial^2 q_2/(\partial x_c \partial t_{w1}) = 0, \quad \partial x^*_c(t_{w1})/\partial t_{w1} = 0;$$
$$\text{if } \partial^2 q_2/(\partial x_c \partial t_{w1}) < 0, \quad \partial x^*_c(t_{w1})/\partial t_{w1} < 0.$$

7.2.2 Proof of Observation 1

Intertemporal utility maximization requires that second-period decisions maximize second-period utility given first-period choices. Therefore the envelope theorem applies when we determine the impact of first-period maternal child care on second-period utility. It is given by

$$\partial \left(u_{w2}^{*}+u_{h2}^{*}\right)/\partial t_{w1}=-\pi w_{w}[1-t_{w2}^{*}(t_{w1})]<0. \tag{6}$$

First-period maternal child care, t_{w1}, maximizes $x_{p1}+2q_1(t_{q1})+u_{w2}^{*}+u_{h2}^{*}$.For an interior solution, the FOC for t_{w1} yields

$$-(w_w-c_n)+2(p-1)\partial q_1/\partial t_{q1}=-\partial \left(u_{w2}^{*}+u_{h2}^{*}\right)/\partial t_{w1},$$

or by equation (6)

$$-(w_w-c_n)+2(p-1)(\partial q_1/\partial t_{q1})=-\pi w_{w}[1-t_{w2}^{*}(t_{w1})]. \tag{7}$$

The left hand side of (7) is the net benefit of an additional unit of first-period maternal child care; the right hand side of (7) is the net second-period cost of that unit.[14]

7.2.3 Comparative Statics-First Period Maternal Child Care

Assuming a unique interior solution for t_{w1}^{*}in $(0,1)$, we derive the following:

1. Optimal maternal child care in the first period increases with an increase in the parental quality parameter p.

Implicit differentiation gives $\partial t_{w1}/\partial p=2[t_{w1}^{*}(p-1)(\partial^2 q_1/\partial t_{q1}^2)+\partial q_1/\partial t_{q1}]/(-SOSC)$,where

[14] While FOCs for the second-period variables are necessary and sufficient, the FOC for first-period maternal child care is only necessary but not sufficient.

the denominator is the absolute value of the second derivative of the intertemporal value function $(u^*_{w1} + u^*_{h1} + u^*_{w2} + u^*_{h2})$ with respect to t_{w1}, and thus for an interior solution $SOSC < 0$. An additional assumption on the child quality function guarantees that the numerator is positive: As $q_1\left(t_{q1}\right)$ is a strictly concave and increasing function $\partial^2 q_1/\partial t^2_{q1} < 0$. Then $\partial q_1/\partial t_{q1} + t^*_{q1}(p-1)(\partial^2 q_1/\partial t^2_{q1}) = \partial q_1/\partial t_{q1} + ((-1)t^*_{w1} + 1)(\partial^2 q_1/\partial tp^2_{q1}) > 0$ so long as the elasticity of the marginal product of quality units of child care is smaller or equal to unity at the optimal t_{w1}. If this assumption holds, $\left|\partial^2 q_1/\partial t^2_{q1} \times (p-1)t^*_{w1}\right| < \partial q_1/\partial t_{q1}$ and hence the numerator is positive.

2. Optimal maternal child care in the first period increases in the nanny's wage rate $\partial t_{w1}/\partial c_n = (-SOSC)^{-1} > 0$;

3. Optimal maternal child care in the first period decreases in both the mother's wage rate, and the return to labour market experience

$$\partial t_{w1}/\partial w_w = (-1 - \pi\left(1 - t^*_{w2}\left(t_{w1}\right)\right) + w_w \pi (\partial t^*_{w2}(t_{w1})/\partial w_{w2})\left(1 + \pi\left(1 - t_{w1}\right)\right))(-SOSC)^{-1} < 0.$$

The last term in the numerator is negative because the demand for maternal child care in the second period is decreasing in its opportunity cost: $\partial t^*_{w2}(t_{w1})/\partial w_{w2} < 0$. Analogously,

$$\partial t_{w1}/\partial \pi = (-w_w\left(1 - t^*_{w2}\left(t_{w1}\right)\right) + w^2_w \pi \frac{\partial t^*_{w2}(t_{w1})}{\partial w_{w2}}\left(1 - t_{w1}\right))(-SOSC)^{-1} < 0.$$

7.3 Twin Births

7.3.1 Proof of Proposition 2: Mother's and nanny's time are public inputs

Denote one twin by superscript a and the other by superscript b. With economies of scale for both the mother and the nanny, i.e. $t^a_{q1} = t^b_{q1} = t_{q1}, t^a_{w2} = t^b_{w2} = t_{w2}$ the parents' problem – with constraints already incorporated – can be written as

$$\max_{t_{w1},t_{w2},x_c^a,x_c^b} \left\{ \begin{array}{c} (w_w - c_n)(1 - t_{w1}) + w_h + 2\left[q_1^a(t_{q1}) + q_1^b(t_{q1})\right] \\ +w_w(1 + \pi(1 - t_{w1}))(1 - t_{w2}) + w_h(1+\pi) - x_c^a - x_c^b + 2\left[q_2^a(t_{w2}, x_c^a) + q_2^b\left(t_{w2}, x_c^b\right)\right] \end{array} \right\}.$$

The FOCs are:

$$\frac{\partial}{\partial t_{w1}} = -[(w_w - c_n) + w_w\pi(1 - t_{w2})] + 4(p-1)\partial q_1/\partial t_{q1} = 0 \tag{8}$$

$$\frac{\partial}{\partial t_{w2}} = -w_w(1 + \pi(1 - t_{w1})) + 2[(\partial q_2^a/\partial t_{w2}) + (\partial q_2^b/\partial t_{w2})] = 0 \tag{9}$$

$$\frac{\partial}{\partial x_c^a} = -1 + 2(\partial q_2/\partial x_c^a) = 0 \tag{10}$$

$$\frac{\partial}{\partial x_c^b} = -1 + 2(\partial q_2/\partial x_c^b) = 0 \tag{11}$$

Clearly both twins will receive the same amount of the consumption good in their second period of life, i.e. $x_c^a = x_c^b$. In comparison to the only child, note that given the same amount of first-period and second-period maternal child care the FOCs (10) and (11) would still hold at the same level of child consumption per child as in the case of one child only. However, (8) and (9) no longer hold. It is optimal to increase t_{w2} and t_{w1}. This obviously raises child quality in the first period. It also raises child quality in the second period. First note that in order to produce child quality per twin in the second period at the same level as second-period child quality for the only child, less than double the inputs are needed. Hence producing the same child quality per twin as child quality for the only child when input prices stay the same less than doubles the cost. However the benefit of producing two children of the same child quality as the only child doubles the benefit. Hence child quality in the second period goes up. Moreover due to the decreased opportunity cost of maternal child care due to an increase in first-period maternal child care producing child quality becomes even cheaper. Again this will raise second-period child quality per twin.

Therefore maternal child care in both periods and child quality per child go up compared to the case of an only child.

7.3.2 Proof of Proposition 3: Mother's and nanny's time are rival inputs

The parents' problem – with constraints already incorporated – becomes

$$\max_{t_{w1}^a, t_{w1}^b, t_{w2}^a, t_{w2}^b, x_c^a, x_c^b} \left\{ \begin{array}{c} (w_w - c_n)\left(1 - t_{w1}^a - t_{w1}^b\right) - c_n + w_h + 2\left[q_1^a\left(t_{q1}^a\right) + q_1^b\left(t_{q1}^b\right)\right] \\ +w_w\left(1 + \pi\left(1 - t_{w1}^a - t_{w1}^b\right)\right)\left(1 - t_{w2}^a - t_{w2}^b\right) \\ +w_h(1+\pi) - x_c^a - x_c^b + 2\left[q_2^a\left(t_{w2}^a, x_c^a\right) + q_2^b\left(t_{w1}^b, x_c^b\right)\right] \end{array} \right\}.$$

The FOCs are:

$$\begin{aligned}
\frac{\partial}{\partial t_{w1}^a} &= -[(w_w - c_n) + w_w\pi\left(1 - t_{w2}^a - t_{w2}^b\right)] + 2(p-1)\partial q_1^a/\partial t_{q1}^a = 0 \\
\frac{\partial}{\partial t_{w1}^b} &= -[(w_w - c_n) + w_w\pi\left(1 - t_{w2}^a - t_{w2}^b\right)] + 2(\partial q_1^b/\partial t_{q1}^b) = 0 \\
\frac{\partial}{\partial t_{w2}^a} &= -w_w\left(1 + \pi\left(1 - t_{w1}^a - t_{w1}^b\right)\right) + 2(\partial q_2^a/\partial t_{w2}^a) = 0 \\
\frac{\partial}{\partial t_{w2}^b} &= -w_w\left(1 + \pi\left(1 - t_{w1}^a - t_{w1}^b\right)\right) + 2(\partial q_2^b/\partial t_{w2}^b) = 0 \\
\frac{\partial}{\partial x_c^a} &= -1 + 2(\partial q_2^a/\partial x_c^a) = 0 \\
\frac{\partial}{\partial x_c^b} &= -1 + 2(\partial q_2^b/\partial x_c^b) = 0
\end{aligned}$$

Clearly, the same amounts of maternal child care and child consumption are devoted to each child, i.e. $t_{wk}^a = t_{wk}^b, k = 1, 2$, and $x_c^a = x_c^b$. In comparison with the only-child case, first assume that maternal child care in both periods stays the same per child, so that each child receives the same amount of maternal child care as the only child. In order for the FOCs for the children's private consumption goods to hold, we must have the same amount of per-capita child consumption as before. However, the FOCs with respect to maternal child care are not satisfied. Total maternal child care must increase in both periods because

$-w_w \pi \left(1 - t_{wk}^a - t_{wk}^b\right)$ becomes less negative as we double the maternal child care in each period. Thus child consumption will also increase. Without economies of scope, parents with twins produce a higher average child quality than parents with an only child unless they are time constrained. That is, if the time spent with the only child is more than 1/2 in both periods, it is not feasible for the mother to double maternal child care. In this case, either both parents are at a corner, or the mother spends all her time in child care and the father divides his time between child care and paid labour. Since the father's wage rate is higher than the mother's (by assumption), this may ultimately lead to a lower average child quality for twins than the child quality of the only child. If both parents are at a corner, i.e. the mother does full-time child care while the father works full-time, child quality per twin is certainly lower than the only child's.

7.4 Three Periods

7.4.1 Only One Child

Since the child is self-sufficient in the third period, and parents do not consume leisure, they both work full time. Thus, the choice variables here are those in the two-period model, but the third-period implications of earlier maternal child care are taken into account. The full objective function for parents with constraints already incorporated is

$$\begin{aligned} &\max_{t_{w1}, t_{w2}, x_c} (w_w - c_n)(1 - t_{w1}) + w_h + w_w \left(1 + \pi(1 - t_{w1})\right)(1 - t_{w2}) + w_h (1 + \pi) - x_c \\ &+ 2\left[q_1\left(t_{q1}\right) + q_2\left(t_{w2}, x_c\right)\right] + w_w \left(1 + \pi(1 - t_{w1})\right)\left(1 + \pi\left(1 - t_{w2}\right)\right) + w_h (1 + \pi)^2 \end{aligned}$$

The FOCs are:

$$\begin{aligned}
\frac{\partial}{\partial t_{w1}} &= 2(p-1)(\partial q_1/\partial t_{q1}) - (w_w - c_n) - \pi w_w(1-t_{w2}) - \pi w_w(1+\pi(1-t_{w2})) = 0 \\
\frac{\partial}{\partial t_{w2}} &= 2(\partial q_2/\partial t_{w2}) - w_w(1+\pi(1-t_{w1})) - \pi w_w(1+\pi(1-t_{w1})) = 0 \\
\frac{\partial}{\partial x_c} &= -1 + 2(\partial q_2/\partial x_c) = 0
\end{aligned}$$

Compared to the base case, a lower (x_c, t_{w1}, t_{w2}) will be the solution. To see this, note that the only difference between the FOCs of the base case and the FOCs here are the additional negative terms $-\pi w_w(1+\pi(1-t_{wk}))$ in the FOCs with respect to t_{wk} for $k = 1, 2$. This implies that evaluated at $(x_c^*, t_{w1}^*, t_{w2}^*)$ from the base case, the FOCs with respect to t_{wk} for $k = 1, 2$ will not hold with equality and instead the LHS will be negative. Thus decreasing maternal child care in both periods will be necessary to satisfy these two FOCs. At the same time this increases the cost of maternal child care in each period and the cost function of child quality in each period. The benefit function of child quality in each period remains the same, so child quality in each period decreases.

7.4.2 Proof of Proposition 4: Two children, economies of scope for both mother and nanny

With perfect economies of scope, for both parents and nanny, in the second period the older child has full time supervision, so effectively $t_{w2}^a = 1$; the full objective function for parents is

$$\sum_{k=1}^{3}(u_{wk} + u_{hk}) = (x_{p1} + 2q_1^a(t_{q1}^a)) + (x_{p2} + 2q_1^b(t_{q2}^b) + 2q_2^a(x_c^a, 1)) + (x_{p3} + 2q_2^b(t_{w3}, x_c^b))$$

Parents choose the following variables to maximize this function

1. private goods for the parents in each of periods 1, 2 and 3 ($x_{pk}, k = 1, 2, 3$);

2. private goods for the first child in period 2 (x_c^a);

3. private goods for the second child in period 3 (x_c^b) ;

4. division of required unit of child care between the wife and the nanny in periods 1, 2;

5. division of wife's time between child care and labour market participation in period 3.

Choices are subject to the within-period budget constraints:

$$\begin{array}{ll} \text{period 1:} & x_{p1} \leq w_h + (w_w - c_n)(1 - t_{w1}) \\ \text{period 2:} & x_{p2} + x_c^a \leq w_h(1+\pi) + [w_w(1+\pi(1-t_{w1})) - c_n](1-t_{w2}) \\ \text{period 3:} & x_{p3} + x_c^b \leq w_h(1+\pi)^2 + w_w(1+\pi(1-t_{w1}))(1+\pi(1-t_{w2}))(1-t_{w3}) \end{array}$$

Substituting these budget constraints into the objective function reduces the problem to one of choosing the sequences of maternal child care and private goods for each child: $\{\{t_{wk}\}_{k=1,2,3}, x_c^a, x_c^b\}$ to maximize

$$\sum_{k=1}^{3}(u_{wk} + u_{hk}) = w_h + (w_w - c_n)(1 - t_{w1}) + 2q_1^a\,(t_{q1})$$
$$+w_h(1+\pi) + [w_w(1+\pi(1-t_{w1})) - c_n](1-t_{w2}) - x_c^a + 2q_1^b\,(t_{q2}) + 2q_2^a\,(x_c^a, 1)$$
$$+w_h(1+\pi)^2 + w_w(1+\pi(1-t_{w1}))(1+\pi(1-t_{w2}))(1-t_{w3}) - x_c^b + 2q_2^b(t_{w3}^b, x_c^b)$$

The FOCs are:

$$\begin{aligned}
\frac{\partial}{\partial t_{w1}} &= 2(p-1)(\partial q_1^a/\partial t_{q1}) - (w_w - c_n) - \pi w_w(1-t_{w2}) - \pi w_w(1+\pi(1-t_{w2}))(1-t_{w3}) = 0 \\
\frac{\partial}{\partial t_{w2}} &= 2(p-1)(\partial q_1^b/\partial t_{q2}) - (w_w - c_n) - \pi w_w(1-t_{w1}) - \pi w_w(1+\pi(1-t_{w1}))(1-t_{w3}) = 0 \\
\frac{\partial}{\partial t_{w3}} &= -w_w\left(1+\pi(1-t_{w1})\right)\left(1+\pi(1-t_{w2})\right) + 2(\partial q_2^b/\partial t_{w3}) = 0 \\
\frac{\partial}{\partial x_c^a} &= -1 + 2(\partial q_2^a/\partial x_c^a) = 0 \\
\frac{\partial}{\partial x_c^b} &= -1 + 2(\partial q_2^b/\partial x_c^b) = 0
\end{aligned}$$

Note that the FOC w.r.t. t_{w1} and t_{w2} are completely symmetric and therefore the optimal amount of maternal child care will be the same in both periods. This implies that both children receive the same amount of time when they are young. However, intertemporal child quality differs between siblings: the first-born receives more attention and child quality in her second period of life than does the second-born child. To see this, first recall that $t_{w3}^* < 1$, but the amount of time spent with the first-born in her second period of life is equal to 1. Moreover, child quality of the first-born in her second period of life is cheaper than the same level of child quality for the second-born, because the time input in q_2^a is free up to one unit. Therefore the child born in the first period will have a higher child quality in the second period than will the second-born in the third period.

7.4.3 Proof of Proposition 5: Two children, economies of scope for mother

Here only parental time is investment in child quality in the second period of the child's life. The budget constraints are unchanged. Substituting into the objective function reduces the household problem to one of choosing the sequences of maternal child care and private goods for each child: $\{\{t_{wk}\}_{k=1,2,3}, x_c^a, x_c^b\}$ to maximize

$$\sum_{k=1}^{3}(u_{wk} + u_{hk}) = w_h + (w_w - c_n)(1 - t_{w1}) + 2q_1^a\,(t_{q1})$$
$$+w_h(1+\pi) + [w_w(1+\pi(1-t_{w1})) - c_n](1 - t_{w2}) - x_c^a + 2q_1^b\,(t_{q2}) + 2q_2^a\,(t_{w2}, x_c^a)$$
$$+w_h(1+\pi)^2 + w_w(1+\pi(1-t_{w1}))(1+\pi(1-t_{w2}))(1-t_{w3}) - x_c^b + 2q_2^b\left(t_{w3}, x_c^b\right)$$

The FOCs are:

$$\begin{aligned}
\frac{\partial}{\partial t_{w1}} &= 2(p-1)(\partial q_1^a/\partial t_{q1}) - (w_w - c_n) - \pi w_w(1-t_{w2}) - \pi w_w(1+\pi(1-t_{w2}))(1-t_{w3}) = 0 \\
\frac{\partial}{\partial t_{w2}} &= 2(p-1)(\partial q_1^b/\partial t_{q2}) - (w_w - c_n) - \pi w_w(1-t_{w1}) - \pi w_w(1+\pi(1-t_{w1}))(1-t_{w3}) + 2\frac{\partial q_2^a}{\partial t_{w2}} = 0 \\
\frac{\partial}{\partial t_{w3}} &= -w_w\left(1+\pi(1-t_{w1})\right)\left(1+\pi(1-t_{w2})\right) + 2(\partial q_2^b/\partial t_{w3}) = 0 \\
\frac{\partial}{\partial x_c^a} &= -1 + 2(\partial q_2^a/\partial x_c^a) = 0 \\
\frac{\partial}{\partial x_c^b} &= -1 + 2(\partial q_2^b/\partial x_c^b) = 0
\end{aligned}$$

Evaluated at the optimal values when both mother and nanny were able to achieve perfect economies of scope, the FOC w.r.t. t_{w2} is not satisfied. The LHS becomes positive due to the additional benefit that maternal child care has on the older child and so t_{w2} must increase relative to the previous case. This also implies a higher equilibrium value for t_{w1} as compared to the previous case and a higher value of t_{w3}. Higher values of maternal child care in each period lowers the opportunity cost of maternal child care in each period and hence the second child receives more child quality in both of its periods as a dependant than in the case of economies of scope for both mother and nanny. As before the first-born receives more time devoted to her in the second period of her life than the second-born does in his second period of life but now she receives less than the second-born in his first period. Assuming $\partial q_2^a/(\partial x_c^a \partial t_{w2}) \geq 0$ and hence $\partial q_2^b/(\partial x_c^b \partial t_{w3}) \geq 0$ leads to a higher child quality of second-period childhood for the first-born than for the second-born.

8 Appendix B

Assume the decision process is captured by the symmetric Nash bargaining solution. Then parents choose an intertemporally efficient allocation of time and goods at the beginning of the first period. Denote by U_i^d the intertemporal utility of spouse i in the threat point. The parents as Nash Bargainers choose $\{t_{ik}, x_{ik,} x_c\}, t_{ik}, x_{ik,} x_c$ to maximize $(U_w - U_w^d)(U_h - U_h^d)$ s.t. $(2), (3)$, and (4). Alternatively, the Nash Product can be maximized in two steps. First, solve for the utility possibility frontier. With transferable utility, the intertemporal utility possibility frontier is given by $U_w + U_h = \lambda$,where $\lambda = \max_{t_{ik}, x_{ik,} x_c} \Sigma_{i=w,h} U_i$ s.t. $(2), (3)$, and (4). Second, $\max_{U_w, U_h}(U_w - U_w^d)(U_h - U_h^d)$ s.t. $(U_w, U_h) \in UPF$. The second approach immediately yields $U_i^N = (\lambda - U_w^d - U_h^d)/2 + U_i^d$,where superscript N indicates the Nash Bargaining solution.

In this setting, the distribution of utility shares is independent of decisions and actions during marriage: each person receives a utility, spread over the two periods, equal to their own threatpoint utility plus half of the surplus generated by the marriage.