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EFFICIENCY UNITS OF LABOR: LIFE-CYCLE PROFILES ESTIMATES FROM THE CPS 1987-2017

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April, 2018

Abstract

This note provides a set of estimates of the efficiency units profiles that workers supply over the life-cycle in the U.S. labor market. These are helpful for the calibration of OLG models. I rely on data from the March CPS in the 1987-2017 period, and use both non-parametric and parametric estimation methods. Irrespective of the methodology used to obtain the profiles, I find that they differ in essential features from the ones that are typically used in applied work. In terms of the quantitative answers obtained from a life-cycle OLG model with idiosyncratic income shocks and incomplete markets, the discrepancies in the efficiency units profiles are found to be sizable. In terms of consumption equivalent variation, households' welfare decreases by up to 5 percentage points when moving from the standard profiles to the ones estimated here. Quantitatively, the specific profile used have also a substantial impact on the variables that are typically analyzed with this class of models, such as the saving behavior over the life-cycle and the concentration of wealth.

Keywords: Labor efficiency units, Life-cycle models, Calibration.

JEL Classifications: D15, D52, D58, E21.

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This note provides a set of estimates of the efficiency units profiles that workers supply over the life-cycle in the U.S. labor market. These are helpful for the calibration of OLG models. I rely on data from the March CPS in the 1987-2017 period, and use both non-parametric and parametric estimation methods. Irrespective of the methodology used to obtain the profiles, I find that they differ in essential features from the ones that are typically used in applied work. In terms of the quantitative answers obtained from a life-cycle OLG model with idiosyncratic income shocks and incomplete markets, the discrepancies in the efficiency units profiles are found to be of first order. In terms of consumption equivalent variation, households' welfare decreases by up to 5 percentage points when moving from the standard profiles to the ones estimated here. Quantitatively, the specific profile used have also a sizable impact on the variables that are typically analyzed with this class of models, such as the saving behavior over the life-cycle and the concentration of wealth.

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1 Introduction

Life-cycle models used for quantitative work require a reliable measurement of how the profiles of the labor efficiency units evolve as individuals age. In this paper I provide estimates of these profiles using both non-parametric and flexible polynomial methods implemented on Current Population Survey (CPS) data. I use information on both individual and household-level labor earnings from the March CPS, which refer to the 1986-2016 period.[1]

A common choice made by applied macroeconomics researchers is to rely on the estimates computed by Hansen (1993), whose study relied on CPS data covering the 1955-1988 period. Examples of influential papers using Hansen's estimates are Rios-Rull (1996), Imrohoroglu (1998) and Conesa, Kitao and Krueger (2009). Another popular choice is to consider a cross-section of labor income data, in a single year or averaged across multiple years, compute the average income at every age and build an index on the basis of some normalization (e.g., the index averages to 1). Examples of contributions using this approach are Rios-Rull (1994), Imrohoroglu, Imrohoroglu and Joines (1995) and Imrohoroglu, Imrohoroglu and Joines (1998). Another alternative is to use panel data and estimate an econometric model that includes a low-order polynomial in age (or potential labor market experience), setting the efficiency units equal to the fitted age-earnings profile. Examples of contributions using this approach on Panel Study of Income Dynamics (PSID) data are Storesletten, Telmer and Yaron (2004), Heathcote, Storesletten and Violante (2010) and Kaplan (2012).

Although relying on a commonly used profile can be beneficial, for example because it keeps one element of the analysis fixed across different studies, it can also weaken the soundness of the quantitative results. This would be the case if the estimates obtained from older datasets are no longer representative of the actual shape of the profiles of interest. This is one of the main findings of my analysis: compared to the profiles that are typically used in applied work, my estimates differ in some crucial dimensions. In particular, the slope of the profiles in the early stages of the life cycle is markedly flatter. I also document how the age/efficiency units relationship appears to have been changing over time, possibly questioning the suitability of estimates based on the PSID, whose sample is no longer representative of the U.S. workforce.

Quantitatively, the households' are not indifferent to the profiles. In the workhorse life-cycle OLG model with idiosyncratic income shocks and incomplete markets, the discrepancies in the efficiency units profiles are found to have a first order effect on households' welfare. In terms of consumption equivalent changes, welfare decreases by up to 5 percentage points when moving from the standard profiles used in the literature to the ones estimated here. These effects are an order of magnitude larger compared to the welfare effects arising from a number of public policy reforms that are reported in the literature, such as the elimination of UI benefits or a change in the tax code. Moreover, the choice of the efficiency units profiles has a sizable impact on the variables that are typically analyzed with this class of models, such as the consumption/saving behavior over the life-cycle and the concentration of wealth.

The rest of the paper is organized as follows. Section 2 presents the empirical methodology. Section 3

[1] I treat the efficiency units as being exogenous: this is consistent with the assumption of a deterministic learning-by-doing accumulation of human capital. This is the most popular framework in the economic models the efficiency units profiles are used in, and make reduced-form estimation methods appropriate.

discusses the main results. Section 4 concludes. A set of appendices present an in depth discussion of the theoretical model used for the quantitative analysis.

2 Methodology

I estimate the profiles using different methodologies.[2] I start by considering the case with a rigid labor supply, with annual labor earnings as the dependent variable, but I will also provide estimates for the flexible labor supply case, with hourly wages as the dependent variable.

It is worth stressing that I am using the CPS as a "pseudo panel", as I am considering a surrogate of life-cycle analysis. Starting from the year 1987, I select the age 25 respondents, then in 1988 I select the age 26 respondents, and I continue in a similar fashion until I exhaust all the available waves. As I don't have 41 waves (spanning the age 25-65 window needed in the quantitative model) I have to repeat a similar procedure, mixing the selected sample with several different sequences.

As for the estimation stage, I mainly rely on orthogonal polynomials of order 10 in the age of the respondents. This allows to obtain predicted values that change smoothly over the life-cycle, without restricting the shape of the profiles.[3] I also use a non-parametric estimator, which confirms all the results, and whose estimates are virtually identical to the orthogonal polynomials ones.

Since the interpretation of an agent can be different in different models, I provide two sets of estimates. The first one considers the household as a unit of observation, and the profiles are obtained by adding up the labor earnings of all the household members. This is the most common interpretation of the decision makers in OLG models. However, in some instances the researchers specify models of household formation: in this case the unit of observation is an individual, and the relevant labor earnings are the individual ones.

The comparison between the two definitions reveals a stark difference, namely how pronounced the fall in income is close to the typical retirement age of 65.[4]

As mentioned above, a simpler approach would be to consider the conditional average of labor earnings. A feature of my analysis is that in a first stage regression I control for a number of observables that can have an important life-cycle dimension. In particular, I control for educational attainment, number of household members, number of dependent children, race, gender, occupation, and geographical location. Since these characteristics typically are not present in the theoretical model, there are two possible options, and I will present the related estimates for both. One option is to leave these components in the error term, which is

[2] For more details on the implementation, see Appendix B.

[3] Typically researchers fit low order polynomials (e.g., a quartic) in age. Orthogonal polynomials circumvent some drawbacks of standard polynomials, namely the high correlation between terms, which causes multicollinearity issues already with a handful of terms. Orthogonal polynomials don't suffer from these limitations, allowing to include many terms, obtaining more accurate estimates.

[4] In order to gauge the effect of measurement error, I also computed some alternative estimates where the profiles are obtained after trimming the bottom of the earnings distribution at the 5% and 10% thresholds. The profiles are affected but the changes are not particularly pronounced. Moreover, as it is well known in the literature, Heathcote, Perri and Violante (2010), including year dummies leads to an even more pronounced difference between the profiles.

clearly correlated with age because of an omitted variable bias problem. The other option is to control for all the observables available in the dataset, to reduce the impact of potential biases, working with a profile that is consistent with the theoretical model.

Figure 1 provides a visual representation of the mechanism behind the estimation. In essence, I am approximating the profile with a combination of regressions run on sequences of labor incomes, in order to reflect the actual behavior that an individual might experience during their life-cycle.

[Figure 1 about here]

In order to assess the quantitative consequences of the estimated profiles, I work with an Overlapping Generations (OLG) structure. Agents are ex-ante identical, while they differ ex-post, due to idiosyncratic realizations of a series of shocks. The model is an extension of the Huggett (1996) economy, appropriately modified to allow for several sources of heterogeneity in labor income.[5]

3 Results

Figure 2 plots the fitted efficiency units profiles estimated on the sequence of cross-sectional CPS data.[6] All the regressions specify as covariates orthogonal polynomials in age of order 10. This figure is indicative of two things: 1) the behavior of labor income after age 50 has drastically changed over time, as it no longer shows a steep decline, 2) because of earnings real growth over time and compositional effects, a cross sectional regression is not an accurate approximation of an individual's evolution of labor income. The figure suggests that averaging cross sectional estimates does not represent a suitable strategy. Instead, it appears more appropriate to consider a pseudo panel approach, which is consistent with the perspective of a cohort entering the labor market at a specific point in time, and the subsequent evolution of their labor income, captured by different (yet still representative) older individuals that are included in the cross sectional CPS data of the following years. Approximating the profile with a cross sectional regression delivers profiles that do not reflect the actual behavior of what an individual might experience during their life-cycle. The main reason behind this discrepancy is that labor earnings tend to increase over time, because of increases in labor productivity. Most models are solved assuming the stationarity of the environment. However, this represents a choice made for convenience and it does not seem to be warranted.

[Figure 2 about here]

Figure 3 presents the estimates of the efficiency units profiles over the life cycle. These profiles are computed on individual labor earnings from the CPS 1987-2017 data. As a term of reference, the Hansen (1993) profile

[5]For more details on the model, see Appendix A, Rios-Rull (1994), Imrohoroglu, Imrohoroglu and Joines (1995), Huggett (1996), and Storesletten, Telmer and Yaron (2004), among many others.

[6]For readability, I am reporting the profiles at 5 year intervals.

(grey line) is plotted in the figure together with the non-parametric estimate (black line) and the quartic polynomial estimate (blue line). In order to make the comparison even starker, all profiles are normalized to average to 1.

[Figure 3 about here]

From the plot it is immediate to see that these estimates differ in some crucial aspects compared to the Hansen (1993) ones. First, the profiles are less concave in the early stages of an individual's life, and they grow at an almost linear rate until age 45. This implies that the profiles peak much later in life, around age 50. It follows that the presence of liquidity constraints might be much more costly, as the agents have limited opportunities to borrow against resources obtained in the near future. Labor income grows at a much lower pace when the workers enter the labor market and are asset poor. Notice also how the non-parametric estimates are fairly similar to the quartic ones, but at some ages there are some large discrepancies between the two (especially towards the beginning of the agents' lives).

[Figure 4 about here]

Figures 4 plots the estimates of the efficiency units profiles over the life cycle, are computed on households labor earnings from the CPS 1987-2017 data. It is apparent how the general features of these profiles are similar to their counterparts based on the individual labor earnings.

Figures 5 and 6 plot the equilibrium wealth profiles over the life cycle. These refer to a version of the life-cycle OLG model with idiosyncratic income shocks and incomplete markets, and are computed numerically using standard methods.[7]

[Figures 5 and 6 about here]

Although the qualitative behaviors are similar, it is worth mentioning that quantitatively across the different profiles there are differences in the wealth holdings that are $\pm 10\%$ compared to the benchmark.

Table 1 offers some insights on the implications of working with different efficiency units profiles. The first row shows the ex-ante welfare (i.e., the expected utility in the steady state of the newborns) in the economy with the Hansen (1993) profiles. All other rows report the CEV arising from switching to a different profile. The results are stark: the individuals are unequivocally worse off when facing the newly estimated profiles, and these welfare losses are large, being around 5%. In parenthesis, I also report the welfare effects of using different approximation methods, namely the more accurate non-parametric estimates and the orthogonal polynomials, compared to the simple quartic one. The welfare losses in this case are much smaller, but still non-negligible, as they are similar in size to the welfare losses of a number of policy experiments studied with this class of models.

[7]For more details, see Appendices C and D.

[Table 1 about here]

Table 2 reports the welfare effects after implementing a further adjustment to the Hansen (1993) profile. Table 1 above was based on efficiency profiles that are not forced to have the same average. Differently, Table 2 is computed by rescaling the benchmark Hansen (1993) profile. The welfare effects are now much smaller, but in a number of cases they are still sizable, being in the $0.5\% - 1\%$ range.

[Table 2 about here]

4 Conclusions

In this paper I provided updated estimates of the life-cycle labor efficiency units profiles. The quantitative findings based on a rich OLG model with incomplete markets show that the shape of these profiles matter for a number of outcomes of interest. In particular, I have shown that it is sufficient to use different estimation methods for the households to incur welfare changes that are comparable in size to the welfare effects arising from drastic reforms of (say) the Unemployment Insurance scheme, as I found in Cozzi (2014).

These discrepancies are captured by the welfare implications of using different profiles: in terms of consumption equivalent, households' welfare decreases by up to 5 percentage points when moving from the standard profiles to the ones estimated here. Even when the average of every profile is normalized to 1, the welfare effects can still be large. Quantitatively, the specific profile used have also a sizable impact on the variables that are typically analyzed with this class of models, such as the saving behavior over the life-cycle and the concentration of wealth.

Since there is still considerable uncertainty on whether heterogeneous income profiles are the most accurate representation of income dynamics, Hryshko (2012), the quantitative literature mainly focuses on parsimonious specifications of labor income growth over the life cycle. This note provided evidence that this aspect of the analysis can have a first order importance and care should be taken in the choice of how to obtain the efficiency units profiles over the life cycle.

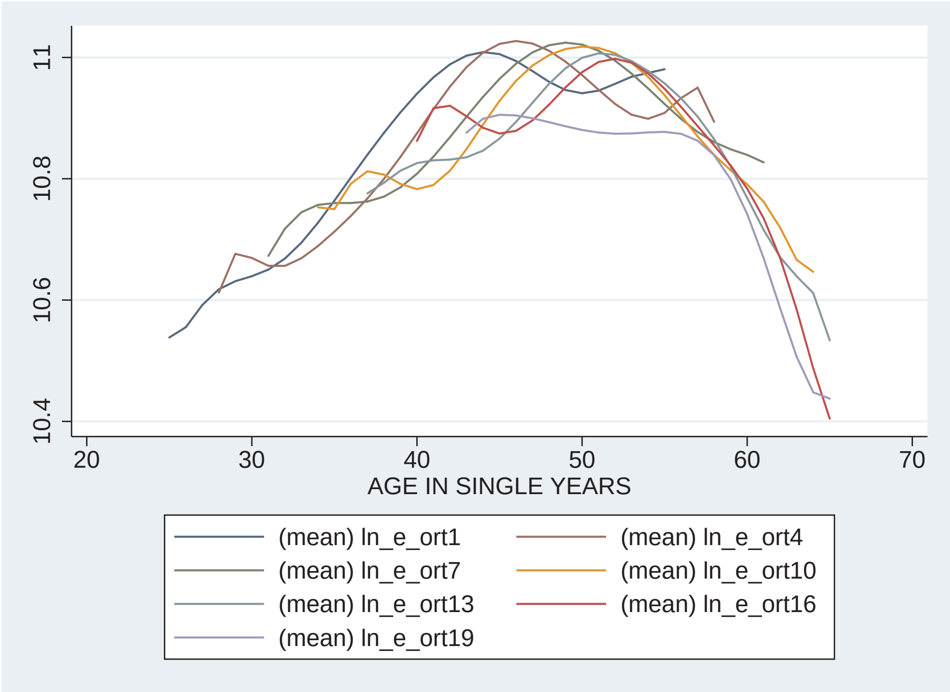

Figure 1: Pseudo Panel Orthogonal Polynomial (order 10) estimates of the Efficiency Units Profiles.

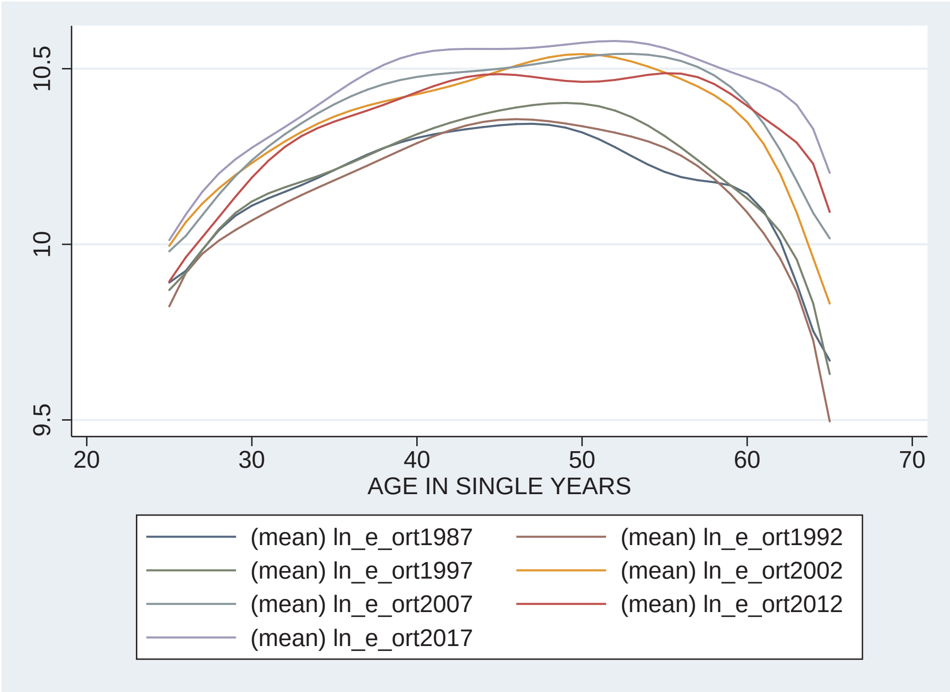

Figure 2: Cross Sectional Orthogonal Polynomial (order 10) estimates of the Efficiency Units Profiles.

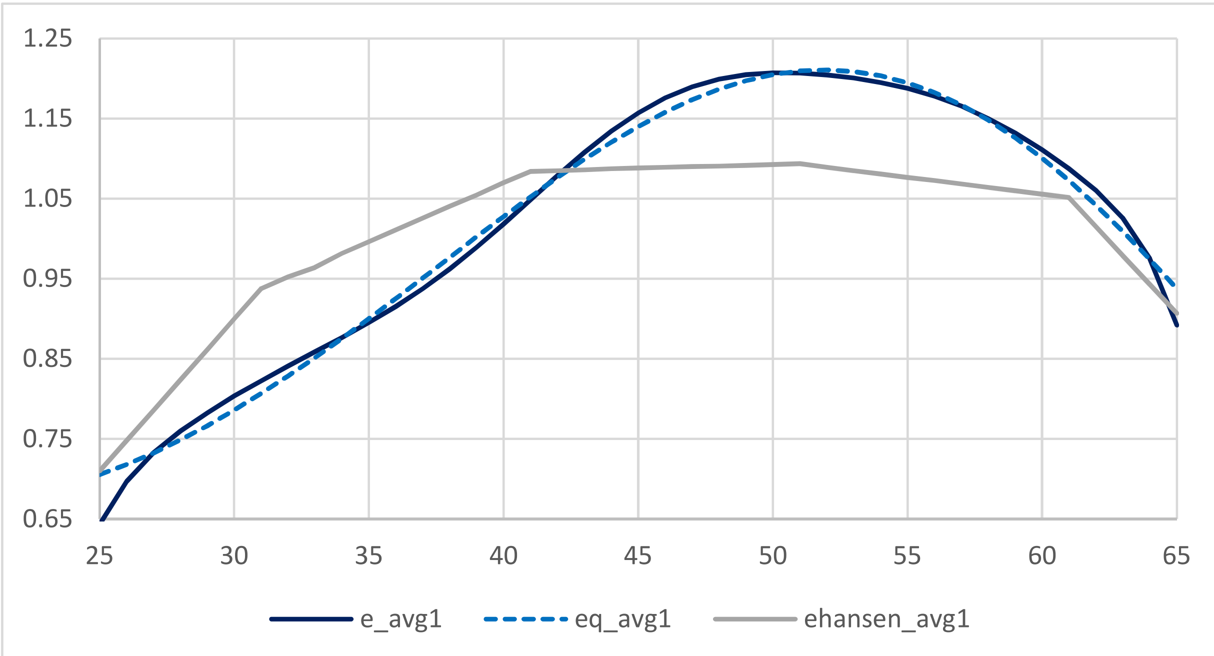

Figure 3: Efficiency Units Profiles over the Life-cycle. Estimates computed on individual labor earnings from the CPS 1987-2017 data. The grey line is the Hansen (1993) profile, the black line is the non-parametric estimate, the blue line is the quartic polynomial estimate. All profiles are normalized to have the same average.

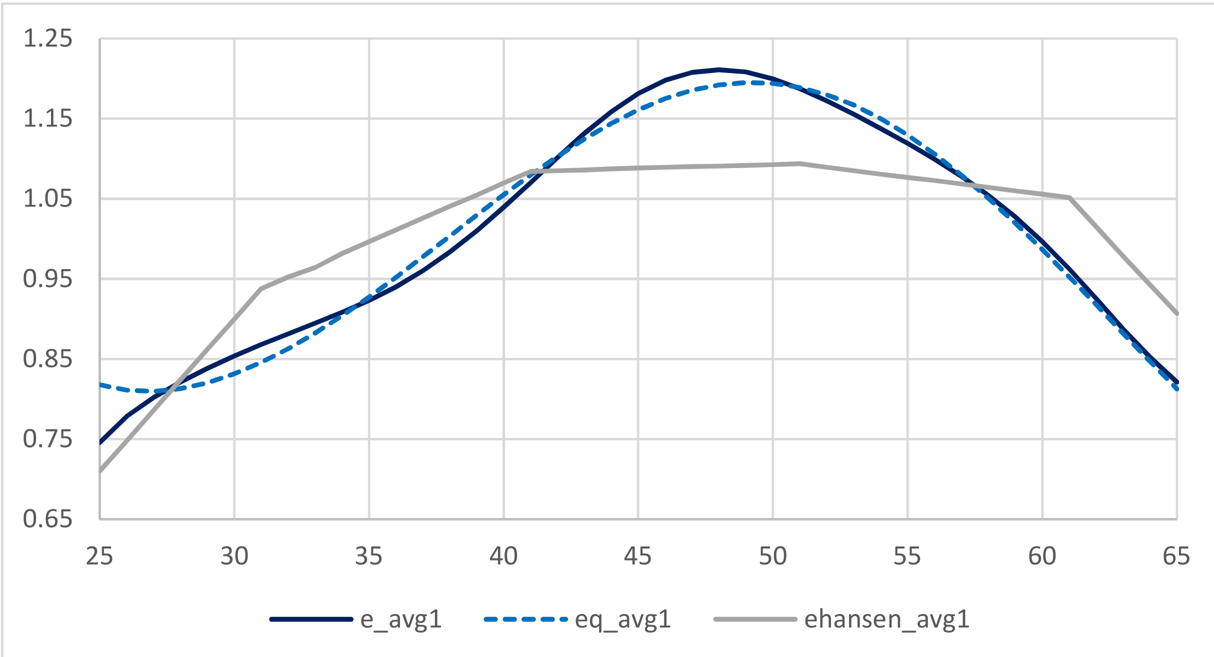

Figure 4: Efficiency Units Profiles over the Life-cycle. Estimates computed on household labor earnings from the CPS 1987-2017 data. The grey line is the Hansen (1993) profile, the black line is the non-parametric estimate, the blue line is the quartic polynomial estimate. All profiles are normalized to have the same average.

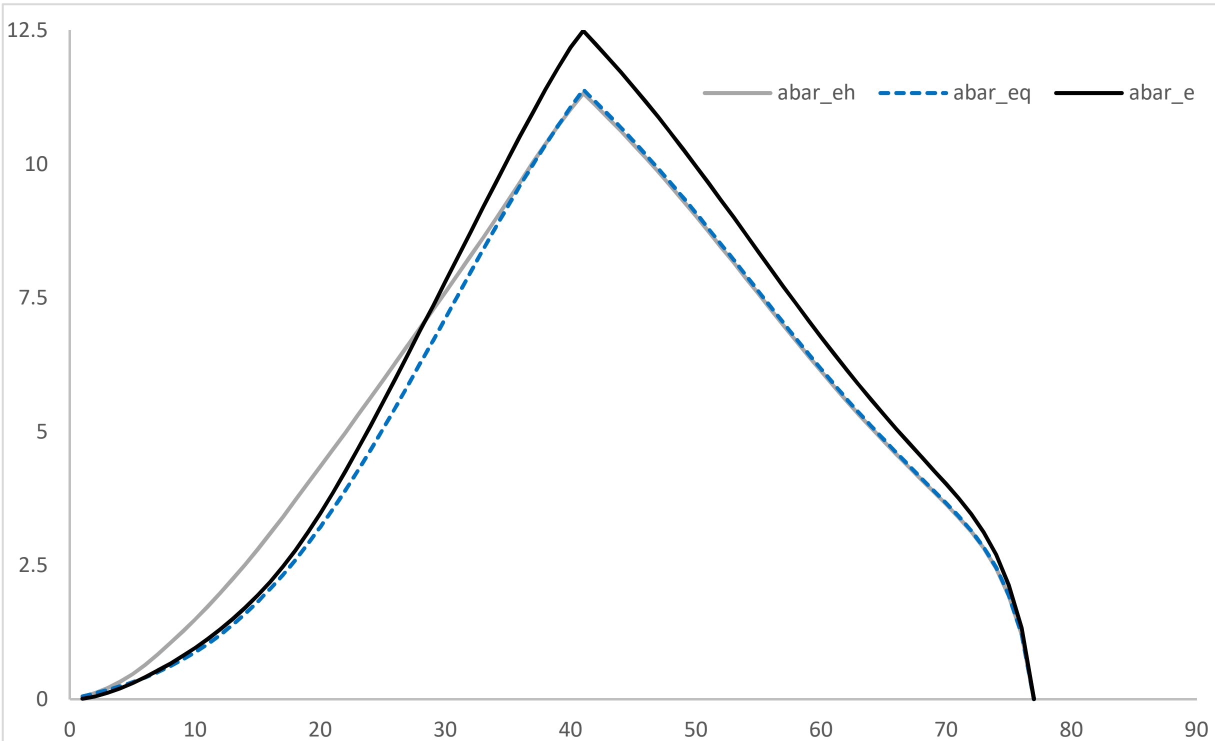

Figure 5: Average Life-cycle profiles of Asset Holdings, with the Efficiency Units Profiles estimated on individual labor earnings. The grey line plots the Asset Holdings with the Hansen (1993) profile, the black line plots the Asset Holdings with the non-parametric estimate, the blue line plots the quartic polynomial estimate.

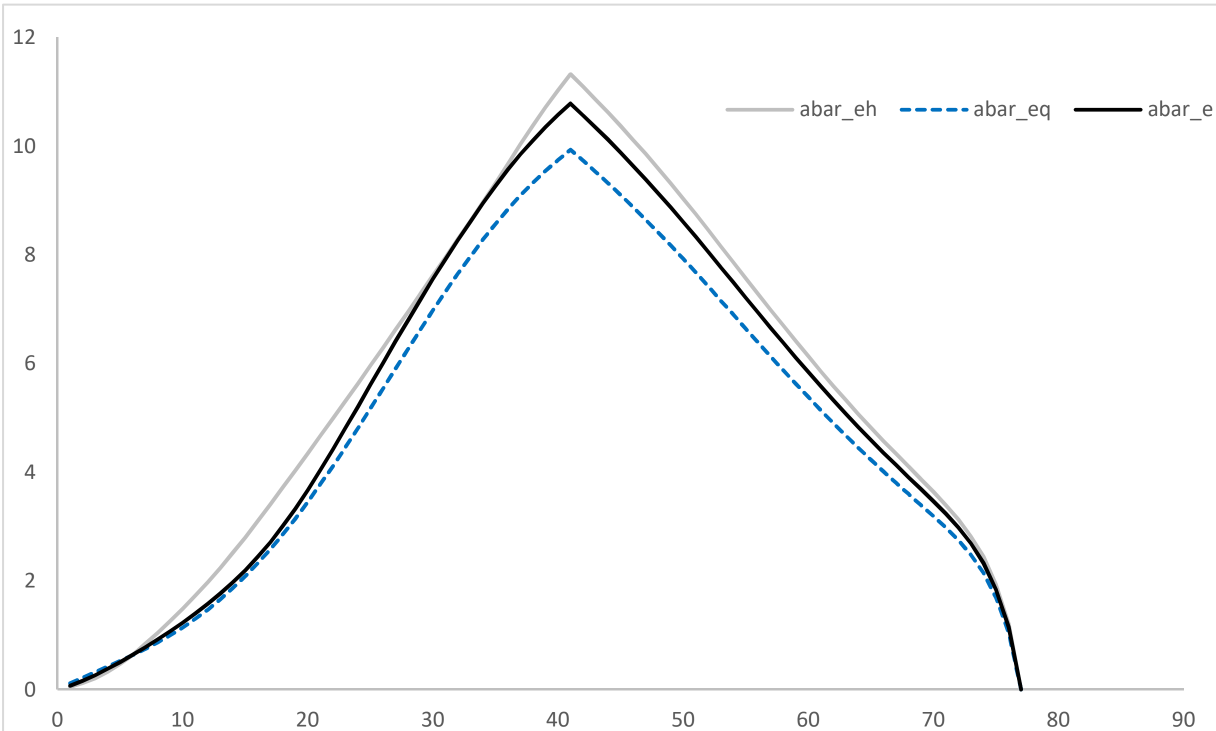

Figure 6: Average Life-cycle profiles of Asset Holdings, with the Efficiency Units Profiles estimated on household labor earnings. The grey line plots the Asset Holdings with the Hansen (1993) profile, the black line plots the Asset Holdings with the non-parametric estimate, the blue line plots the quartic polynomial estimate.

Case	*Household - PE*	*Household - GE*	*Individual - PE*	*Individual - GE*
Hansen (Ex-ante Welfare)	−64.38	−64.38	−64.38	−64.38
Imrohoroglu et al. (1995)	−0.15%	−0.65%	−0.15%	−0.65%
Quartic	−4.74%	−4.81%	−4.33%	−5.66%
Orthogonal	−4.91% (−0.18%)	−5.08% (−0.29%)	−4.51% (−0.19%)	−5.88% (−0.24%)
Non-Parametric	−4.93% (−0.20%)	−5.11% (−0.31%)	−4.59% (−0.27%)	−5.95% (−0.31%)

Table 1: Efficiency Units Profiles and Welfare, CEV with respect to the Benchmark Hansen (1993) Profiles. GE and PE stand for General and Partial Equilibrium. Household (Individual) stands for the estimates obtained from the Household-level (Individual-level) CPS data on labor earnings. The values in parenthesis refer to the value of the CEV with respect to the profiles estimated with a Quartic polynomial in age.

Case	*Household - PE*	*Household - GE*	*Individual - PE*	*Individual - GE*
Hansen (Ex-ante Welfare)	−66.02	−66.02	−66.02	−66.02
Imrohoroglu et al. (1995)	−0.15%	−0.65%	−0.15%	−0.65%
Quartic	0.20%	0.12%	0.62%	−0.77%
Orthogonal	0.02%	−0.17%	0.43%	−1.01%
Non-Parametric	−0.01%	−0.19%	0.35%	−1.08%

Table 2: Efficiency Units Profiles and Welfare, CEV with respect to the Rescaled Hansen (1993) Profiles. GE and PE stand for General and Partial Equilibrium. Household (Individual) stands for the estimates obtained from the Household-level (Individual-level) CPS data on labor earnings.

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Appendix A - The OLG Model and its Recursive Representation

5 Stationary Equilibrium

The economy is a production economy with an endogenous asset distribution, where a government collects taxes to finance both an exogenously given stream of public expenditures and a pension scheme. The model is an extension of the Huggett (1996) economy, appropriately modified to allow for several sources of heterogeneity in labor income. Beside the workers, there is a representative firm that produces the final output relying on a Cobb-Douglas production function on aggregate labor and aggregate capital.

Time is discrete. The economy is populated by finitely lived agents facing an age-dependent death probability π_j^d. Age is denoted with j and there are J overlapping generations, each consisting of a continuum of agents. At age J_R all agents that are still alive become retirees. There is a perfectly competitive annuity market, where the agents share their mortality risk. It follows that there are no accidental bequests. The population grows at rate g_n.

Preferences: Agents' preferences are assumed to be time-separable and represented by the utility function $U(.)$. Agents' utility is defined over stochastic consumption $\{c_j\}_{j=1}^J$ and leisure sequences $\{l_j\}_{j=1}^J$: their aim is to choose how much to consume (c_j), how much to work ($h_j = 1 - l_j$), and how much to save in an interest bearing asset (a_{j+1}) in each period of their lives, in order to maximize their objective function. The agents' problem can be defined as:

$$\max_{\{c_j,l_j,a_{j+1}\}_{j=1}^J} \mathbb{E}_0 U(c_0, c_1, ...; l_0, l_1, ...) = \max_{\{c_j,l_j,a_{j+1}\}_{j=1}^J} \mathbb{E}_0 \sum_{j=1}^{J} \beta^{j-1} \left[\prod_{s=1}^{j} \left(1 - \pi_s^d\right) \right] u(c_j, l_j)$$

where E_0 represents the expectation operator over the idiosyncratic sequences of shocks, and $\beta > 0$ is the subjective discount factor. In the benchmark formulation, I assume that $u(c_j, l_j) = \frac{c_j^{1-\sigma}-1}{1-\sigma}$, that is the per-period utility function is strictly increasing in consumption, strictly concave, satisfies the Inada conditions, and has a constant relative risk aversion. In this case, the labor supply is fixed and equal to the time endowment.

Endowments: Agents differ in their labor endowments $\epsilon_{j,\varepsilon,f}$. There are three channels that contribute to the determination of the total efficiency units that the workers supply in the labor market. First, there is a deterministic age component e_j, which is the same for all agents. Second, there is a stochastic component ε, whose log follows a stationary $AR(1)$ process: $\log \varepsilon_j = \rho_y \log \varepsilon_{j-1} + \xi_j$, with $\xi_j \sim N(0, \sigma_y^2)$. Third, there is a fixed effect component f, with half of the agents being born with the highest realization, and the other half with the lowest.[8] The total efficiency units a worker is endowed with are the product of the three components. It follows that labor earnings are $y_j^w = w\epsilon_{j,\varepsilon,f} = w \times e_j \times \varepsilon \times f$. After the common retirement age J_R, the labor endowment drops to zero, and the agents receive a pension $\overline{y}_R$ paid for with the contributions of the economically active agents. The pension is a fixed replacement rate ϕ_R of the average labor earnings, and agents pay proportional taxes (τ_R) to contribute to the balanced-budget pension scheme. They also finance the

[8] These two values are chosen to match the variance of the fixed effect σ_f^2.

public expenditure G with their income taxes. Agents can insure against their mortality risk. As a consequence, on average agents die with zero wealth. Newborns enter the economy with a zero asset endowment and with the average realization of the stochastic component of labor earnings, which is normalized to 1.

Now the problem of the agents in their recursive representation is defined, then I provide a formal definition of the equilibrium concept used in this model, the recursive competitive equilibrium. The individual state variables are: age $j \in \mathcal{J} = \{1, ..., J\}$, the fixed effect $f \in \mathcal{F} = \{-\sigma_f, +\sigma_f\}$, the persistent shock component of the labor endowment $\varepsilon \in \mathcal{E} = \{\varepsilon_{\min}, ..., \overline{\varepsilon}, ..., \varepsilon_{\max}\}$ and asset holdings $a \in \mathcal{A} = [-b, \overline{a}]$. Notice that ε is discretized with the Rouwenhorst method, using a 7-state Markov chain. The transition function of the labor endowment shocks is represented by the matrix $\Pi(\varepsilon', \varepsilon) = [\pi(v, z)]$, where each element $\pi(v, z)$ is defined as $\pi(v, z) = \Pr\{\varepsilon_{j+1} = z | \varepsilon_j = v\}$, $v, z \in \mathcal{E}$. In every period the exogenous labor endowments are given by $\epsilon_{j,\varepsilon,f} = e_j \varepsilon f$. The stationary distribution of working-age agents is denoted by $\mu_j(a, \varepsilon, f)$ while that of retirees with $\mu_j^R(a)$. Φ_j denotes the share of each cohort j in the total population. These satisfy the recursion $\Phi_{j+1} = \left(\frac{1-\pi_j^d}{1+g_n}\right)\Phi_j$, and are normalized to add up to 1.

5.1 Problem of the agents

The model is solved backwards, starting from the terminal age J and with the assumption that the terminal utility value is zero, i.e. $V_{J+1} = 0$.

5.1.1 Problem of the retirees

The value function of an age-j retired agent whose current asset holdings are equal to a is denoted with $V_j^R(a)$. The problem of these agents can be represented as follows:

$$V_j^R(a) = \max_{c,a'} \left\{u(c) + \beta\left(1 - \pi_j^d\right) V_{j+1}^R(a')\right\} \tag{1}$$

$$\begin{aligned} & s.t. \\ & c + a' = (1 + r_j)\, a + \overline{y}_R \\ & c \geq 0, \quad a' > 0 \end{aligned}$$

In the budget constraint notice the presence of the common pension payment $\overline{y}_R$ and the mortality rate adjusted interest rate r_j.

5.1.2 Problem of the workers

The value function of a working-age agent whose current asset holdings are equal to a, whose current efficiency units shock is ε and whose fixed effect is f is denoted with $V_j(a, \varepsilon, f)$. The problem of these agents can be represented as follows:

$$V_j\left(a,\varepsilon,f\right)=\max_{c,a'}\left\{u(c)+\beta\left(1-\pi_j^d\right)\sum_{\varepsilon'}\pi\left(\varepsilon',\varepsilon\right)V_{j+1}\left(a',\varepsilon',f\right)\right\} \tag{2}$$

$$\begin{aligned}
&s.t.\\
&c+a'=\left(1+r_j\right)a+(1-\tau_R-\tau)w\epsilon_{j,\varepsilon,f}\\
&a_0=0,\quad c\geq 0,\quad a'>-b
\end{aligned}$$

Non-retired agents have to set optimally their consumption/savings plans. They enjoy utility from consumption, and face some uncertain events in the future. In the next period they can still be alive, and with probability $\pi\left(\varepsilon',\varepsilon\right)$ they transit from their current efficiency units ε to the value ε'. These agents pay income taxes $\tau w\epsilon_{j,\varepsilon,f}$ to finance public expenditures G. They also pay a proportional tax τ_R on their labor earnings to finance the pension scheme. Finally, they are born with the average shock $\overline{\varepsilon}$, with no wealth and are subject to an exogenous borrowing constraint, $b\geq 0$.

5.2 Recursive Stationary Equilibrium

Definition 1 *For given public policies $\{\tau,\tau_R,G\}$ a recursive stationary equilibrium is a set of decision rules, $\left\{c_j\left(a,\varepsilon,f\right),l_j\left(a,\varepsilon,f\right),a_j'\left(a,\varepsilon,f\right)\right\}_{j=1}^{J_R-1}$ and $\left\{c_j^R\left(a\right),a_j^{R\prime}\left(a\right)\right\}_{j=J_R}^{J}$, value functions, $\left\{V_j\left(a,\varepsilon,f\right)\right\}_{j=1}^{J_R-1}$ and $\left\{V_j^R\left(a\right)\right\}_{j=J_R}^{J}$, prices $\{r,w\}$, and a set of stationary distributions, $\left\{\mu_j\left(a,\varepsilon,f\right)\right\}_{j=1}^{J_R-1}$ and $\left\{\mu_j^R\left(a\right)\right\}_{j=J_R}^{J}$, such that:*

- Given relative prices $\{r,w\}$, taxes and pension benefits $\overline{y}_R$, the individual policy functions $\left\{c_j\left(a,\varepsilon,f\right),h_j\left(a,\varepsilon,f\right),a_j'\left(a,\varepsilon,f\right)\right\}_{j=1}^{J_R-1}$, $\left\{c_j^R\left(a\right),a_t^{R\prime}\left(a\right)\right\}_{j=J_R}^{J}$ solve the household problems (1)-(2), and $\left\{V_j\left(a,\varepsilon,f\right)\right\}_{j=1}^{J_R-1}$, $\left\{V_j^R\left(a\right)\right\}_{j=J_R}^{J}$ are the associated value functions.
- Given relative prices $\{r,w\}$ and public policies, K/L solves the final good sector firm's problem.
- The labor market is in equilibrium, and the labor input L corresponds to the total supply of labor efficiency units

$$L=\sum_{j=1}^{J_R-1}\Phi_j\int\limits_{\mathcal{A}\times\mathcal{E}\times\mathcal{F}}h_j\left(a,\varepsilon,f\right)\epsilon_{j,\varepsilon,f}d\mu_j\left(a,\varepsilon,f\right)$$

- The asset market clears

$$(1+g_n)K=\sum_{j=1}^{J_R-1}\Phi_j\int\limits_{\mathcal{A}\times\mathcal{E}\times\mathcal{F}}a_j'\left(a,\varepsilon,f\right)d\mu_j\left(a,\varepsilon,f\right)+\sum_{j=J_R}^{J}\Phi_j\int\limits_{\mathcal{A}}a_j^{R\prime}\left(a\right)d\mu_j^R\left(a\right)$$

- The goods market clears

$$Y=C+I+G=\sum_{j=1}^{J_R-1}\Phi_j\int\limits_{\mathcal{A}\times\mathcal{E}\times\mathcal{F}}c_j\left(a,\varepsilon,f\right)d\mu_j\left(a,\varepsilon,f\right)+\sum_{j=J_R}^{J}\Phi_j\int\limits_{\mathcal{A}}c_j^R\left(a\right)d\mu_j^R\left(a\right)+\left(\delta+g_n\right)K+G$$

- The government's budget is balanced, that is tax revenues from income taxation are equal to the government purchases G

$$G = \sum_{j=1}^{J_R-1} \Phi_j \int\limits_{\mathcal{A}\times\mathcal{E}\times\mathcal{F}} \tau w \epsilon_{j,\varepsilon,f} d\mu_j (a, \varepsilon, f)$$

- The stationary distributions $\left\{\mu_j (a, \varepsilon, f), \mu_j^R (a)\right\}$ satisfy

$$\mu_{j+1} (a', \varepsilon', f) = \int \nu (a, \varepsilon, f, j, a', \varepsilon') d\mu_j (a, \varepsilon, f) \tag{3}$$

$$\mu_{j+1}^R (a') = \int \nu^R (a, j, a') d\mu_j^R (a) \tag{4}$$

In equilibrium the measure of agents in each state is time invariant and consistent with individual decisions, as given by the above two equations (3)-(4), where $\nu (.)$ and $\nu^R (.)$ are the transition functions.

- The ex-ante welfare measure W is the expected utility of a new-born in the steady-state, namely the value function of the age 1 individuals, evaluated at the initial conditions and integrate with respect to the fixed effect:

$$W = \int\limits_{\mathcal{F}} V_1 (a = 0, \varepsilon = \bar{\varepsilon}, f) d\mu_1 (a = 0, \varepsilon = \bar{\varepsilon}, f) \tag{5}$$

- The social welfare measure W^S is utilitarian, i.e. it weights the agents' lifetime utilities by their mass in the steady-state

$$W^S = \sum_{j=1}^{J_R-1} \Phi_j \int\limits_{\mathcal{A}\times\mathcal{E}\times\mathcal{F}} V_j (a, \varepsilon, f) d\mu_j (a, \varepsilon, f) + \sum_{j=J_R}^{J} \Phi_j \int\limits_{\mathcal{A}} V_j^R (a) d\mu_j^R (a) \tag{6}$$

- The consumption based welfare measure ϖ is the percentage increase in consumption in all states of the world that makes welfare in the counterfactual economy $W^1 (\varpi)$ equal to welfare in the baseline one W^0

$$W^0 = W^1 (\varpi)$$

$$\varpi = \left(\frac{W^1}{W^0}\right)^{\frac{1}{1-\sigma}} - 1 \tag{7}$$

Appendix B - Estimation

- The estimation was performed with STATA SE 15.1. The codes were run on a 64-bit PC platform with Windows 10 Professional and an Intel $i7-6700k$ Quad Core processor clocked at 4.6 Ghz.

- The estimation of the profiles using the non-parametric estimator takes up to 131 hours to complete. This is due to the large number of observations in the dataset ($779,050$ for the family-level analysis, and $667,838$ for the individual-level one, which is lower because the individuals with no income are discarded). The typical STATA command is:

```
npregress kernel income age, kern(gau)
```

  I use a normal kernel rather than the default Epanechnikov option because the estimated profiles are somewhat smoother, which is sometimes desirable for the numerical solution of the calibrated model.

- The regressions whose independent variables are the orthogonal polynomials in age (polage), are typically of order 10. However, in some years the polynomials of order 9 and 10 cannot be included because of the perfect fit of a lower order polynomial. Needless to say, the estimation of the profiles using the orthogonal polynomials takes a fraction of a second to complete. The typical STATA code is:

```
orthpoly age, deg(10) generate(polage*)
regress income polage1-polage10
```

- Notice that, following the discussion in the literature, I compute the life cycle profiles in two different ways. In one case, I run both the non-parametric and polynomial estimations on the residuals of a first-stage regression of income on a whole set of year dummies. In the other case, I set aside the potential role of time effects, which is equivalent to setting the whole set of year dummies to zero.

- The profiles and the codes used to compute them will be available for download at the following website:

  https://sites.google.com/site/marcozzi73/home/research.

Appendix C - Calibration of the OLG Model

Parameter	*Value*	*Target*
Model Period	*Year*	*Frequency of CPS/PSID Data*
J - *Maximum Age*	81	*Certain death at age* 100
J_R - *Maximum Working Age*	46	*Retirement at age* 65
π_j^d - *Death probability*	-	*Bell and Miller* (2002)
g_n - *Population growth*	0.011	*Data*
β - *Rate of time preference*	0.9882	*Interest rate with Hansen (1993) profile* $= 4\%$
σ - *Risk Aversion*	1.5	*Elasticity of Intertemporal Substitution* $= 0.75$
δ - *Capital depreciation rate*	0.0698	*Capital depreciation estimates*
α - *1-Labor share*	0.34	*Labor share of output* $= 66\%$
σ_y^2 - *Var. of the temporary income shocks*	0.015	*Guvenen* (2009)
ρ_y - *Persistence of the temp. income shocks*	0.988	*Guvenen* (2009)
σ_f^2 - *Var. of the fixed effect*	0.058	*Guvenen* (2009)
ξ - *Government Consumption*	0.17	*G/GDP* $= 17\%$
ϕ_R - *Pension Replacement Rate*	0.39	*OECD Data*
b - *Borrowing limit*	0	*No borrowing allowed*

Table 3: Calibration, Benchmark Model

Appendix D - The OLG Model Solution Algorithm

This algorithm represents the computational procedure used to solve the GE OLG model:

1. Generate a discrete grid over the asset space $[-b, ..., a_{\max}]$.
2. Generate a discrete grid over the income shocks with the Rouwenhorst method $[\varepsilon_{\min}, ..., \varepsilon_{\max}]$.
3. Guess the interest rate r_0.
4. Guess the pension benefits $\overline{y}_{R,0}$.
5. Get the capital demand K_0 and wages w_0.
6. Get the saving functions $a'_j(a, \varepsilon, f), a_j^{R\prime}(a)$, the labor supply functions $h_j(a, \varepsilon, f)$ and the value functions $V_j(a, \varepsilon, f), V_j^R(a)$.
7. Get the (transformed) stationary distributions $\mu_j(a, \varepsilon, f), \mu_j^R(a)$.
8. Get the aggregate capital supply and check the asset market clearing; Get r_1.
9. Update r'_0 and $\overline{y}'_{R,0}$ (with a relaxation method).
10. Iterate until asset market clearing and aggregate consistency of the pensions.
11. Get the consumption functions $c_j(a, \varepsilon, f), c_j^R(a)$ and check the final good market clearing.
12. Compute the ex-ante welfare W_{OLG} of a new-born and the social welfare W_{OLG}^S.